ADCHEM 2006

International Symposium on Advanced Control of Chemical Processes
Gramado, Brazil – April 2-5, 2006

EXTENDED ROBUST MODEL PREDICTIVE CONTROL FOR INTEGRATING SYSTEMS

A. H. González*, J. L. Marchetti* and D. Odloak+

* *Institute of Technological Development for the Chemical Industry (INTEC), CONICET - Universidad Nacional del Litoral (U.N.L.). Güemes 3450, (3000) Santa Fe - Argentina.*

\+ *Department of Chemical Engineering, University of São Paulo, Av. Prof. Luciano Gualberto, trv. 3, n. 380, 05508-900 São Paulo, Brazil*

Abstract: This paper proposes a robust model predictive control for systems with stable and integrating poles. The approach combines the methods developed in Odloak (2004) and Carrapiço and Odloak (2005) to obtain a robust controller for integrating systems when multi-plant uncertainty is considered. The key idea in this development is to separate the control problem in two sub-problems, each of which takes into account the required robust constraints. Nominal stability is achieved by using an infinite output horizon, and the whole method is based on a state space model formulation that leads to an offset free MPC. The simulation examples illustrate the performance and robustness of the proposed approach and demonstrate that it can be implemented in real applications.

Keywords: Model Predictive Control, Integrating Systems, Infinite Horizon, Robustness.

1. INTRODUCTION

Robust stability is still one of the main weaknesses of the available MPC technology (Qin and Badgwell, 2003). However, this subject has been extensively treated in the control literature (Kothare et al., 1996, Mayne et al., 2000; Morari and Lee, 1999, Ralhan and Badgwell, 2000, Lee and Yu, 1997). A robust controller is meant to guarantee closed loop stability for different process operating conditions. It is well known that numerous chemical processes are nonlinear but they can be approximated by a set of linear models, where each linear model represents the process locally, around a specific operating condition. If the controller is based on a single linear model, it is desirable to assure that this controller will remain stable for the whole family of models that represent the process.

The standard form to get stability in MPC is the strategy known as infinite horizon. However, this formulation requires additional features in order to achieve offset free tracking. Rodrigues and Odloak (2003) present an incremental state-space model formulation that produces offset free MPC. This formulation adds integrating modes to the system, that must be zeroed at the end of the control horizon in order to keep the infinite cost bounded. If, in addition, the system to be controlled has already integrating modes, a set of constraints must be added to the original problem to compensate both, the original and new unstable modes. As a result, the optimization problem may become infeasible, and the convergence of the cost to zero may be deteriorated.

In this paper, we first present the general model formulation for systems with stable and integrating modes. Then, a single infinite horizon MPC problem is presented as a tutorial to introduce a two-step formulation, which should result more reliable to prevent infeasibilities. This two-step formulation is extended to deal with a multi-model representation of the real plant and the convergence of the method is analyzed. Finally, we provide some simulation results and the conclusions.

2. INFINITE HORIZON MPC FOR INTEGRATING SYSTEMS

2.1 Model Formulation

We assume at first a MIMO system with *nu* inputs and *ny* outputs. For each pair (y_i, u_j), there is a transfer function model

$$G_{i,j}(z)=\frac{b_{i,j,1}z^{-1}+\cdots+b_{i,j,nb}z^{-nb}}{\left(1+a_{i,j,1}z^{-1}+\cdots+a_{i,j,na}z^{-na}\right)\left(1-z^{-1}\right)}, \quad (1)$$

where $\{na, nb \in \mathbb{N}\}$. When the poles of the system are non-repeated, the k^{th} coefficient of the step response can be calculated as follows:

$$S_{i,j}(k) = d_{i,j}^0 + \sum_{l=1}^{na}\left[d_{i,j,l}^d\right] r_l^k + d_{i,j}^i k\Delta t \tag{2}$$

where r_l, $l=1, 2, \ldots, na$ are the non-integrating poles of the system, Δt is the sampling time and the coefficients $d_{i,j}^0, d_{i,j,l}^d, d_{i,j}^i$ are obtained by partial expansion of $G_{i,j}$. A state-space model that produces an offset free MPC can be expressed in the following form:

$$x(k+1) = Ax(k) + B\Delta u(k) \tag{3}$$

$$y(k) = Cx(k) \tag{4}$$

where

$$[x] = \begin{bmatrix} x^s \\ x^d \\ x^i \end{bmatrix} \in \mathbb{C}^{nx}, \quad nx = 2ny + nd, \quad nd = ny.nu.na,$$

$$x^s \in \mathbb{R}^{ny}, \quad x^d \in \mathbb{C}^{ny.nu.na}, \quad x^i \in \mathbb{R}^{ny}$$

$$A = \begin{bmatrix} I_{ny} & 0 & \Delta t\, I_{ny} \\ 0 & F & 0 \\ 0 & 0 & I_{ny} \end{bmatrix} \in \mathbb{C}^{nx\times nx}, B = \begin{bmatrix} D^0 + \Delta t D^i \\ D^d F N \\ D^i \end{bmatrix} \in \mathbb{R}^{nx\times nu},$$

$$[y] = \begin{bmatrix} y_1 & \cdots & y_{ny} \end{bmatrix}^T, \quad C = \begin{bmatrix} I_{ny} & \Psi & 0_{ny} \end{bmatrix},$$

$$\left[x^s\right] = \begin{bmatrix} x_1^s & \cdots & x_{ny}^s \end{bmatrix}^T, \quad \left[x^i\right] = \begin{bmatrix} x_1^i & \cdots & x_{ny}^i \end{bmatrix}^T,$$

$$\left[x^d\right] = \left[x_{1,1,1}^d \; \cdots \; x_{1,1,na}^d \; x_{1,2,1}^d \; \cdots \; x_{1,2,na}^d \right.$$
$$\left. \cdots \; x_{2,1,1}^d \; \cdots \; x_{2,1,na}^d \; \cdots \; x_{ny,1,na}^d \; \cdots \; x_{ny,nu,na}^d \right]^T$$

$$D^0 = \begin{bmatrix} d_{1,1}^0 & \cdots & d_{1,nu}^0 \\ \vdots & \ddots & \vdots \\ d_{ny,1}^0 & \cdots & d_{ny,nu}^0 \end{bmatrix} \in \mathbb{R}^{ny\times nu},$$

$$D^i = \begin{bmatrix} d_{1,1}^i & \cdots & d_{1,nu}^i \\ \vdots & \ddots & \vdots \\ d_{ny,1}^i & \cdots & d_{ny,nu}^i \end{bmatrix} \in \mathbb{R}^{ny\times nu}$$

$$F = \operatorname{diag}\left(r_{1,1,1} \cdots r_{1,1,na} \cdots r_{1,nu,1} \cdots r_{1,nu,na} \cdots \right.$$
$$\left. r_{ny,1,1} \cdots r_{ny,1,na} \cdots r_{ny,nu,1} \cdots r_{ny,nu,na}\right) \in \mathbb{C}^{nd\times nd}$$

$$D^d = \operatorname{diag}\left(d_{1,1,1}^d \cdots d_{1,1,na}^d \cdots d_{1,nu,1}^d \cdots d_{1,nu,na}^d \cdots \right.$$
$$\left. d_{ny,1,1}^d \cdots d_{ny,1,na}^d \cdots d_{ny,nu,1}^d \cdots d_{ny,nu,na}^d\right) \in \mathbb{C}^{nd\times nd}$$

$$N = \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_{ny} \end{bmatrix} \in \mathbb{R}^{nd\times nu}, \; J_i = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \in \mathbb{R}^{nu\,na\times nu},$$

$$i = 1,\ldots,ny, \qquad \Psi = \begin{bmatrix} \Phi & & 0 \\ & \ddots & \\ 0 & & \Phi \end{bmatrix} \in \mathbb{R}^{ny\times nd},$$

$$\Phi = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} \in \mathbb{R}^{nu\,na}.$$

In this model formulation, x^s corresponds to the integrating states introduced by the incremental form of the inputs, x^d corresponds to the stable states and x^i corresponds to the original integrating states of the system.

2.2 *Infinite Prediction Horizon*

The cost function of the infinite horizon MPC can be written as follows

$$\begin{aligned} V_{1,k}(\Delta u_k) = \sum_{j=1}^{\infty} e(k+j)^T Q e(k+j) \\ + \sum_{j=0}^{m-1} \Delta u(k+j)^T R \Delta u(k+j) \end{aligned} \tag{5}$$

where $Q \in \mathbb{R}^{ny\times ny}$ is positive definite and $R \in \mathbb{R}^{nu\times nu}$ is positive semi-definite, $e(k+j) = y(k+j) - y^r$ is the error of the predicted output at sampling time $k+j$ including the effect of future control actions, y^r is the desired output reference and m is the control horizon (as usual in MPC, beyond the control horizon, the input moves are assumed equal to zero). As was described by Carrapiço and Odloak (2005), the cost function (5) would become unbounded since the model formulation described earlier contains integrating modes. To avoid this difficulty, it is necessary to include the following constraints into the optimization problem

$$e^s(k) + \left(D_m^0 - D_{2m}^i\right)\Delta u_k = 0 \tag{6}$$

$$x^i(k) + D_{1m}^i \Delta u_k = 0, \tag{7}$$

where

$$e^s(k) = x^s(k) - y^r$$

$$D_m^0 = \begin{bmatrix} D^0 & \cdots & D^0 \end{bmatrix} \in \mathbb{R}^{ny\times m.nu}$$

$$D_{1m}^i = \begin{bmatrix} D^i & \cdots & D^i \end{bmatrix} \in \mathbb{R}^{ny\times m.nu}$$

$$D_{2m}^i = \begin{bmatrix} 0 & \Delta t D^i & \cdots & (m-1)\Delta t D^i \end{bmatrix} \in \mathbb{R}^{ny\times m.nu}$$

$$\Delta u_k = \begin{bmatrix} \Delta u(k)^T & \cdots & \Delta u(k+m-1)^T \end{bmatrix}^T \in \mathbb{R}^{m.nu}$$

Now, substituting eqs. (6) and (7) into eq. (5) and rearranging the infinite term, the IHMPC can be formulated as follows

Problem P1

$$\begin{aligned} &\min_{\Delta u_k} V_{1,k}(\Delta u_k) = \\ &\sum_{j=0}^{m-1} e(k+j)^T Q e(k+j) + x^d(k+m)^T \bar{Q} x^d(k+m) \\ &+ \sum_{j=0}^{m-1} \Delta u(k+j)^T R \Delta u(k+j) \end{aligned} \tag{8}$$

subject to:
(6), (7), and

$$\Delta u(k+j) \in U, \quad j = 0, 1, \cdots, m-1 \tag{9}$$

$$U = \left\{ \Delta u(k+j) \;\middle|\; \begin{array}{c} -\Delta u^{\max} \le \Delta u(k+j) \le \Delta u^{\max} \\ u^{\min} \le \Delta u(k-1) + \sum_{i=0}^{j} \Delta u(k+i) \le u^{\max} \end{array} \right\}$$

where $\bar{Q}$ is obtained from the solution of the well known Lyapunov equation

$$\bar{Q} - F^T \bar{Q} F = F^T \Psi^T Q \Psi F , \qquad (10)$$

and $\Delta u^{\max}, u^{\min}, u^{\max}$ are: the maximum input increment, and the minimum and maximum input values respectively.

2.3 Preventing Infeasibilities

As long as Problem P1 remains feasible, the convergence of the closed loop system can be guaranteed. However, depending on the size of the disturbance or the set point change, a conflict between constraints (6), (7) and (9) may arise. This is so because in practice the control horizon may be short to reduce the computer effort, and the maximum control move may be small to produce a smooth operation of the system. Carrapiço and Odloak (2005) presented two methods to extend the feasibility range of the infinite horizon controller for integrating systems with incremental state space model. One of these strategies, that includes slack variables to relax the constraints (6), (7) and (9) in Problem P1, is based on an optimization technique developed by Lee and Xiao (2000). The latter propose a two-step approach to solve the problem of including the steady state economic optimization in the conventional MPC of stable and integrating systems represented by step response models. Following this idea, the extended controller is obtained as the solution of the two following problems:

Problem P2a

$$\min_{\delta_k^i, \Delta u_{a,k}} V_{2a,k} = \delta_k^{i\,T} S_2 \delta_k^i + \Delta u_{a,k}^T \bar{R} \Delta u_{a,k}$$

subject to

$$\Delta u_a(k+j) \in U, \; j \geq 0$$

$$x^i(k) + \delta_k^i + D_{1m}^i \Delta u_{a,k} = 0 \qquad (11)$$

where

$\Delta u_{a,k} = \begin{bmatrix} \Delta u_a(k)^T & \cdots & \Delta u_a(k+m-1)^T \end{bmatrix}^T$, $\bar{R}$ and S_2 are positive definite matrices, and $\delta_k^i \in \mathbb{R}^{ny}$ is a vector of slack variables for the integrating states that provides extra degrees of freedom.

Let the optimal solution to Problem P2a be designated $\left(\delta_k^{i*}, \Delta u_{a,k}^*\right)$ and consider the resultant input increment

$$u^*(k+m-1) - u(k-1) = \sum_{j=0}^{m-1} \Delta u_a^*(k+j) . \qquad (12)$$

This optimal input increment is passed to a second problem, which is solved within the same time step:

Problem P2b

$$\min_{\Delta u_{b,k}, \delta_k^s} V_{2b,k} = \sum_{j=0}^{\infty} \left(e\left(k+j\right) + \delta_k^s \right)^T Q \left(e\left(k+j\right) + \delta_k^s \right) + \sum_{j=0}^{m-1} \Delta u_b\left(k+j\right)^T R \Delta u_b\left(k+j\right) + \delta_k^{s\,T} S_1 \delta_k^s$$

subject to:

$$\Delta u_b(k+j) \in \mathrm{U}, \; j \geq 0$$

$$e^s(k) + \delta_k^s + (D_{2,m}^i - D_m^0)\Delta u_{b,k} = 0 \qquad (13)$$

$$u^*(k+m-1) - u(k-1) = \sum_{j=0}^{m-1} \Delta u_b(k+j)$$

where $\Delta u_{b,k} = \begin{bmatrix} \Delta u_b(k)^T & \cdots & \Delta u_b(k+m-1)^T \end{bmatrix}^T$, S_1 is a positive definite matrix and $\delta_k^s \in \mathbb{R}^{ny}$ is a new vector of slack variables.

The control law obtained through the sequential solution of problems P2a and P2b above leads to the convergence of the system output to the reference value.

3. PLANT UNCERTAINTY DESCRIPTION

In order to characterize the model uncertainty, we assume that matrices D^0, D^d and F of the model represented in (3) are not exactly known but they lie within a set Ω. This set is composed by a finite number of integrating models with the same dimensions, that is,

$$\Omega = \left\{ \theta_1 \quad \cdots \quad \theta_L \right\} , \qquad (14)$$

where $\theta \triangleq \left(A, B\right)$ designate each individual plant of the set, and matrices A and B depend on matrices D^0, D^d and F. Note that matrix D^i is assumed to be known, which may be acceptable in many practical applications.

In addition, let us assume that the true plant θ_T lies within the set Ω, and there exist a most likely plant (also laying in Ω), which is named nominal plant (θ_N).

4. COST CONTRACTING MPC FOR INTEGRATING SYSTEMS

Badgwell (1997) developed a robust linear quadratic regulator for the multi-plant uncertainty described in (14). Combining Problems P2a and P2b with Badgwell's results, an extended cost contracting robust MPC for integrating systems can be obtained as the solution of the following sequence of optimization problems:

Problem P3ar

$$\min_{\Delta u_{a,k}, \delta_k^i(\theta_N)} V_{3a,k}\left(\Delta u_{a,k}, \delta_k^i\left(\theta_N\right)\right) = \delta_k^i\left(\theta_N\right)^T S_2 \delta_k^i\left(\theta_N\right) + \Delta u_{a,k}^T \bar{R} \Delta u_{a,k} \qquad (15)$$

subject to:

$$\Delta u_a\left(k+j\right) \in U, \quad j = 0,1,\ldots,m-1 \qquad (16)$$

$$x^i\left(k\right) + \delta_k^i\left(\theta_N\right) + D_{1m}^i\left(\theta_N\right)\Delta u_{a,k} = 0, \qquad (17)$$

where

$$\Delta u_{a,k} = \begin{bmatrix} \Delta u_a(k)^T & \cdots & \Delta u_a(k+m-1)^T \end{bmatrix}^T ,$$

Now, the optimal input increment is passed to the second problem through the constraint

$$\sum_{j=0}^{m-1} \Delta u_b\left(k+j\right) = \sum_{j=0}^{m-1} \Delta u_a^*\left(k+j\right) .$$

Problem P3br

$$\min_{\Delta u_{b,k},\delta_k^s(\theta_N)} V_{3b,k}\left(\Delta u_{b,k},\delta_k^s(\theta_N),\theta_N\right)$$
$$=\sum_{j=0}^{\infty}\left(e(k+j)+\delta_k^s(\theta_N)\right)^T Q\left(e(k+j)+\delta_k^s(\theta_N)\right)$$
$$+\sum_{j=0}^{m-1}\Delta u_b(k+j)^T R\Delta u_b(k+j)+\delta_k^s(\theta_N)^T S_1\delta_k^s(\theta_N) \quad (18)$$

subject to:

$$\Delta u_b(k+j)\in U, \quad j=0,1,...,m-1 \quad (19)$$
$$e^s(k)+\delta_k^s(\theta)+\left(D_m^0(\theta)-D_{2m}^i(\theta_N)\right)\Delta u_{b,k}=0,\ \theta\in\Omega \quad (20)$$
$$\sum_{j=0}^{m-1}\Delta u_b(k+j)=\sum_{j=0}^{m-1}\Delta u_a^*(k+j) \quad (21)$$
$$V_{3b,k}\left(\Delta u_{b,k},\delta_k^s(\theta),\theta\right)\le V_{3b,k}\left(\Delta\tilde{u}_{b,k},\tilde{\delta}_k^s(\theta),\theta\right), \qquad \theta\in\Omega \quad (22)$$

where $\Delta u_a^*(k+i)$ stands for the optimal solution obtained in Problem P3ar,

$\Delta\tilde{u}_{b,k}=\left[\Delta u_b^*(k)^T \quad \cdots \quad \Delta u_b^*(k+m-2)^T \quad 0\right]^T$,

and $\tilde{\delta}_k^s(\theta)$ are such that:

$$e^s(k)+\tilde{\delta}_k^s(\theta)+\left(D_m^0(\theta)-D_{2m}^i(\theta_N)\right)\Delta\tilde{u}_{b,k}=0,\ \theta\in\Omega \quad (23)$$

Remarks

* Note that in Problem P3ar, since we assume that all models have the same matrix D^i, only the nominal one is considered.
* Equations (20), (22) and (23) represent L constraints each (as many as models are considered), that has to be satisfied by the same Δu_k, and by $\delta_k^s(\theta_1)\cdots\delta_k^s(\theta_L)$. Despite both, Δu_k and δ_k^s are optimization variables, only Δu_k is actually independent of the model since a given input sequence generates one output steady state offset per model.
* $x^s(k)$ and $x^i(k)$ are measured states and then correspond to the actual plant θ_T.
* Variable $\Delta\tilde{u}_{b,k}$ is the optimal control sequence obtained at time step *k-1* and translated to time *k*.

4.1 Convergence of the method

The following theorem shows that the control algorithm produced by the solution of Problem P3r provides convergence of the true system output to the reference value.

Theorem 1: Consider an integrating system whose true model is unknown but lies within the set Ω. Assume that in the control objective $V_{3b,k}$, the weight S_1 is large enough to prevent offset in the system output. Assume also that Problem P3r is feasible at time steps *k, k+1, k+2, ...* and the system outputs are not saturated. Then, the control law obtained as the solution of Problem P3r drives the true system to the reference value.

Proof

First stage

Let the optimal solution to Problem P3ar be $\Delta u_{a,k}^*,\delta_k^{i^*}$ [1]; and let the optimal solution to Problem P3br, be $\Delta u_{b,k}^*,\delta_k^{s^*}(\theta),\ \theta\in\Omega$. Note that for every optimal input sequence, there is one slack variable per model. The first control move $\Delta u_b^*(k)$ is injected into the true process and the time is moved to *k+1*. Because of equation (21), it can be shown that $\Delta\tilde{u}_{a,k+1}=\left[\Delta u_b^*(k+1)^T \quad \cdots \quad \Delta u_b^*(k+m-1)^T \quad 0\right]^T$ and $\delta_{k+1}^i=\delta_k^{i*}$ is a feasible solution to Problem P3ar. Now, considering that $\bar{R}$ is negligible in comparison to S_2, the corresponding value of the objective function of Problem P3ar at time k+1 is still $V_{3a,k}^*$. Consequently, the optimal solution of Problem P3ar will be $V_{3a,k+1}^*\le V_{3a,k}^*$. Since we have selected $\bar{R}<<S_2$, the objective function of Problem P3ar will converge to zero, which corresponds to $\delta_k^i=0$.

Second stage

Now, for a large k, $\delta_k^i(\theta_T)=0$; that is[2]:

$$x^i(k)+D_{1m}^i\Delta u_{a,k}^*=0. \quad (24)$$

Take again the solution $\Delta u_{b,k}^*,\delta_k^{s^*}(\theta),\ \theta\in\Omega$. For the true model the corresponding cost is:

$$V_{3b,k}\left(\Delta u_{b,k}^*,\delta_k^{s^*}(\theta_T),\theta_T\right)$$
$$=\sum_{j=1}^{\infty}\left(e(k+j)+\delta_k^s(\theta_T)\right)^T Q\left(e(k+j)+\delta_k^s(\theta_T)\right)$$
$$+\sum_{j=0}^{m-1}\Delta u_b(k+j)^T R\Delta u_b(k+j)+\delta_k^s(\theta_T)^T S_1\delta_k^s(\theta_T)$$

Assume that we inject the first control action $\Delta u_b^*(k)$ into the true plant and move time to *k+1*. At this time, the objective for $\left(\Delta\tilde{u}_{b,k+1},\tilde{\delta}_{k+1}^s(\theta_T),\theta_T\right)$, is:

$$V_{3b,k+1}\left(\Delta\tilde{u}_{b,k+1},\tilde{\delta}_{k+1}^s(\theta_T),\theta_T\right)=$$
$$V_{3b,k}\left(\Delta u_{b,k}^*,\delta_k^{s*}(\theta_T),\theta_T\right)-\left(e(k+1)+\delta_k^{s*}(\theta_T)\right)^T$$
$$Q\left(e(k+1)+\delta_k^{s*}(\theta_T)\right)-\Delta u_b^*(k)^T R\Delta u_b^*(k),$$

---

[1] There is only one slack δ_k^i for all the models because D^i is known.

[2] Note that who decides the value of the slack $\delta_k^i(\theta_T)$ (which is zero in this case) is the complete increment $\sum_{j=0}^{m-1}\Delta u_a(k+j)$, and not the individual increments $\Delta u_a(k+j)$.

(25)

where

$$\Delta\tilde{u}_{b,k+1} = \left[\Delta u_b^*(k+1)^T \quad \cdots \quad \Delta u_b^*(k+m-1)^T \quad 0\right]^T$$

and $\tilde{\delta}_{k+1}^s(\theta_T)$ is such that

$$e^i(k+1) + \tilde{\delta}_{k+1}^s(\theta_T) + \left(D_m^0(\theta_T) - D_{2m}^i\right)\Delta\tilde{u}_{b,k+1} = 0.$$

Note that, since the state at time *k+1* corresponds to the true plant, and $\delta_k^i = 0$; then $\tilde{\delta}_{k+1}^s(\theta_T) = \delta_k^{s*}(\theta_T)$. Since no new input increments are added in $\Delta\tilde{u}_{b,k+1}$, the predicted output (in the absence of integrating modes) will be the same as in the case of using $\Delta u_{b,k}^*$, for the true models.

Now assume that the optimal solution to problem P3br is found at time *k+1*. We know that the plant lies in the family Ω, so the robustness constraint (22) must be satisfied for the true plant at time *k+1*. That is:

$$V_{3b,k+1}\left(\Delta u_{b,k+1}^*, \delta_{k+1}^{s*}(\theta_T), \theta_T\right) \le V_{3b,k+1}\left(\Delta\tilde{u}_{b,k+1}, \tilde{\delta}_{k+1}^s(\theta_T), \theta_T\right) \tag{26}$$

Combining (25) with (26) we obtain:

$$V_{3b,k+1}\left(\Delta u_{b,k+1}^*, \delta_{k+1}^{s*}(\theta_T), \theta_T\right) - V_{3b,k}\left(\Delta u_{b,k}^*, \delta_k^{s*}(\theta_T), \theta_T\right) \le -\left(e(k+1) + \delta_k^{s*}(\theta_T)\right)^T Q\left(e(k+1) + \delta_k^{s*}(\theta_T)\right) - \Delta u_b^*(k)^T R \Delta u_b^*(k)$$

This shows that the sequence of optimal cost is non-increasing. Finally, since we assume that S_1 is large enough to prevent output offset, the error converges to zero for the true plant.

5. SIMULATION RESULTS

The system adopted to test the robust controller is based on the ethylene oxide reactor system presented by Rodrigues and Odloak (2003). This is a typical example of the chemical process industry that exhibits stable and integrating poles. The following transfer matrix represents the system

$$G(s) = \begin{bmatrix} \dfrac{-0.19}{s} & \dfrac{-1.7}{19.5s+1} \\ \dfrac{-0.763}{31.8s+1} & \dfrac{0.235}{s} \end{bmatrix},$$

and the translation into the model formulation presented in section 2, can be seen in Carrapiço and Odloak (2005).

We focus on a case in which both, the stable gains and the time constants are uncertain parameters. The true plant has a larger gain and a smaller time constant than the nominal model used in the controller, which is a quite critical situation. The set Ω contains the five models indicated in Table 1.

The differences between the nominal model and the true plant were selected so that the nominal controller becomes unstable.

Table 1: Parameters of the Models used in the Test.

	Mo 1	Mo 2	Mo 3	Mo 4	Nom	True
d^0_{12}	-4.68	-4.68	-0.17	-0.17	-1.7	-4.68
d^0_{21}	-2.10	-0.08	-2.10	-0.08	-0.76	-2.10
r_{12}	.8787	.9738	.9262	.9738	.9500	.8787
r_{21}	.8963	.9448	.9932	.9932	.9690	.8963

Figures I, II and III show the simulation responses when a set point change of 2 and 3 units is made on output 1 and 2 respectively. In Figure I, it can be seen that the input constraint $\Delta u_{\max}$, u_{min} and u_{max} become active during the transient states. This shows the capability of the controller to handle input constraints, together with the additional robust requirements. Another property to remark is the flexibility of the controller to tolerate a short control horizon. This is a critical parameter to reduce the computational cost.

Figures IV and V, show the responses for the same system when the robust constraints are not added to the MPC formulation (nominal controller). In this case the manipulated variables saturates giving oscillatory system behaviour. The tuning parameters of the robust controller are shown in Table 2.

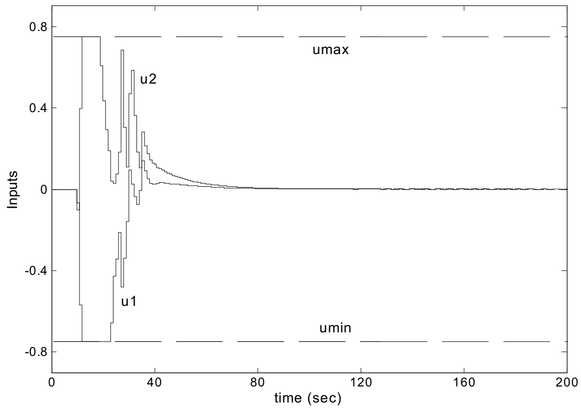

Figure I. Input responses of the robust controller.

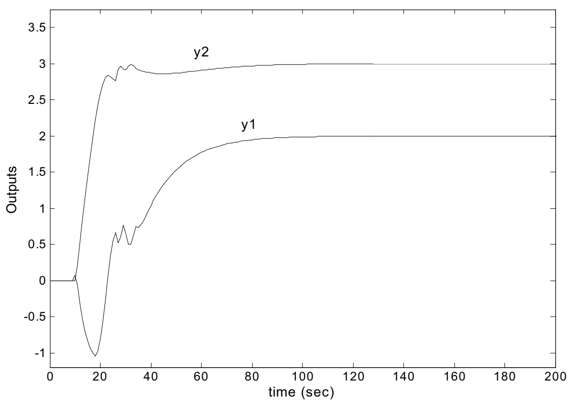

Figure II. Output responses of the robust controller.

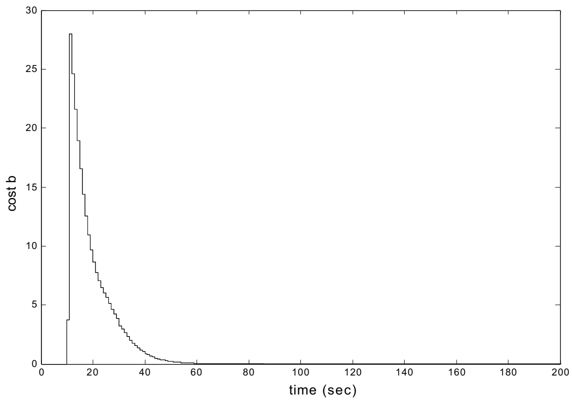

Figure III. Secondary cost of the robust controller.

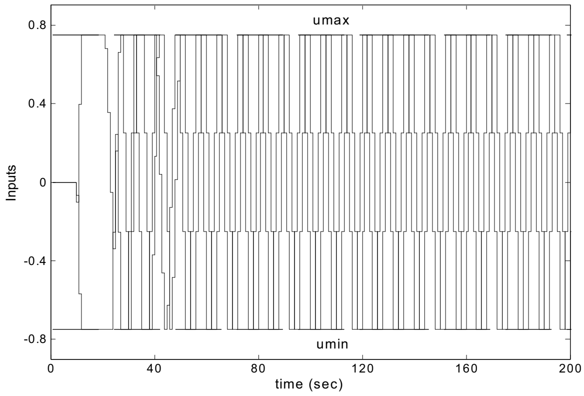

Figure IV. Input responses of the nominal controller, maintaining the same tuning parameters.

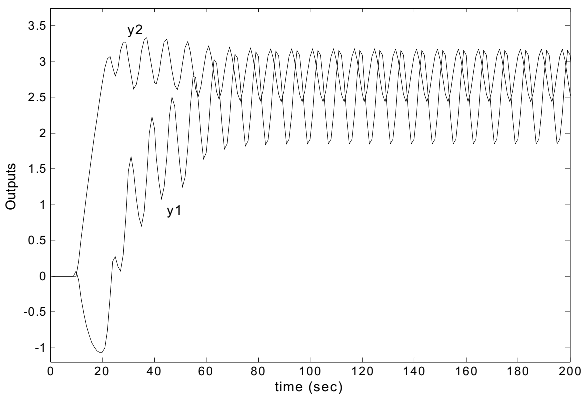

Figure V. Output responses of the nominal controller, maintaining the same tuning parameters.

Table 2: Controller parameters.

T	m	Δu_{max}	u_{max}	u_{min}
1	3	0.5	0.75	-0.75

Q	R	$\bar{R}$	S_1	S_2
0.1	0.1	0.1	2.5	1000

6. CONCLUSION

In this paper we have presented a method to extend a particular robust MPC controller to the case of systems containing stable and integrated modes. Robust stability is achieved by assembling cost contracting constraints with the constraints necessary to compensate the unstable modes of the system. On the other hand, the control formulation allows dealing with problems that cannot be reduced to the regulator problem due to unknown disturbances or model non-linearities, and can be directly implemented in real applications. A representative example shows the capability of the controller to handle significant uncertainty in both, the stable gains and the time constants of the system, in the case that inputs constraints become active during the transient states.

REFERENCES

Badgwell T.A. (1997). Robust model predictive control of stable linear systems. *International Journal of Control*, 68, 797-818.

Carrapiço O.L. and D. Odloak. (2005). A stable model predictive control for integrating process. *Computers and Chemical Engineering,* 29, 1089-1099.

Kothare M.V., V. Barakrishnan and M. Morari. (1996). Robust constrained model predictive control using linear matrix inequalities. *Automática*, 32, 1361-1379.

Lee J.H. and J. Xiao. (2000). Use of two-stage optimization in model predictive control of stable and integrating systems. *Computer and Chemical Engineering*, 23, 1591-1596.

Lee J.H. and Z. Yu. (1997). Worst case formulation of model predictive control for systems with bounded parameters. *Automatica*, 33(5), 763-781.

Mayne, D.Q., J. B. Rawlings, C.V. Rao and P. O. Scokaert. (2000). Constrained model predictive control: stability and optimality. *Automatica*, 36, 789–814.

Morari M. and J.H. Lee. (1999). Model predictive control: past, present and future. *Computer and Chemical Engineering,* 23, 667-682.

Odloak D. (2004). Extended Robust Model Predictive Control. *AIChE Journal*., 50 (8) 1824-1836.

Qin S.J. and T.A. Badgwell. (2003). A Survey of Industrial Model Predictive Control Technology. *Control Engineering Practice*, 11 (7) 733-764.

Ralhan S. and T.A. Badgwell. (2000). Robust control of stable linear systems with continuous uncertainty. *Computer and Chemical Engineering*, 24, 2533-2544.

Rodrigues, M.A. and D. Odloak. (2003). An infinite horizon model predictive control for stable and integrating processes. *Computer and Chemical Engineering,* 27, 1113-1128.