Adaptive Backstepping Nonlinear Control of Bioprocesses

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Abstract

This paper deals with the design of adaptive and non adaptive nonlinear controllers based on backstepping techniques applied to bioprocess models. Backstepping techniques are known to be appropriate in particular to handle the control of systems with relative degree higher than one. In the present paper, we shall concentrate on one illustrative case study : the control of the biomass concentration by acting on the influent substrate concentration. We propose the following sequence of controllers : we start with a non adaptive backstepping controller, then redesign the controller to include parameter estimation of uncertain model parameters. The design includes by construction the stability and convergence analysis of both controllers, whose performance are further illustrated in numerical simulations.

Key words: Bioprocesses, nonlinear systems, backstepping, adaptive control.

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1 Introduction

The control of bioprocesses is typically facing the following questions (see e.g. [2]). The models that describe the dynamics of bioprocesses (based on mass balance considerations) are basically nonlinear. Because of the time-varying nature of the micro-organisms but also the high complexity of the underlying biochemical reactions, the dynamical models used in the design of control algorithms can be perceived as being non stationary. Indeed, even when performed carefully, the identification of the parameters of the bioprocess dynamical models often result in parameter values that may exhibit large confidence intervals. Besides, the selection of appropriate kinetic models is always a difficult and largely heuristic task. Finally it is worth reminding that industrial applications of bioprocesses is, generally speaking, characterized by a lack of reliable sensors to measure on-line all the key process variables. For all these reasons, it is obvious that an appropriate choice for a control structure can be one that account for the well-known process nonlinearities while handling the model dynamics uncertainty and the lack of on-line measurement via on-line estimation of the poorly known parameters and of the unmeasured variables. Such an approach has been largely developed in [2] and subsequent works (e.g. [7] [6]) and followed by several applications (e.g. [11] [4] [5] [3]) in a large spectrum of bioprocess applications. This approach is based in particular on input-output linearization of nonlinear systems, a technique largely popularized by Isidori [8]. If the approach remains rather simple for systems with relative degree one, it may become easily very complex once the relative degree is larger than one. This can be a major limitation of this approach when applied to bioprocesses due to the large uncertainty of their dynamical models.

Constructive nonlinear control schemes that considers a step-by-step design of the controller based on successive Lyapunov functions [10] [12] have been introduced and developed as a potential alternative to the state feedback and input-output linearization approaches. Among these, backstepping techniques can be used to handle systems with relative degree larger than one. So far the application of backstepping techniques to the control of (bio)chemical processes has not been widely used (e.g. [9], [1]). In this paper, we propose to design a backstepping scheme to control a bioprocess with relative degree two. We have selected as a case study the control of the biomass concentration by acting on the influent substrate concentration. Two controllers are proposed. The first one is a non-adaptive version. The second one is an adaptive version in which the maximum specific growth rate is estimated on-line. The design includes by construction the stability and convergence analysis of both controllers. Their performance is illustrated via numerical simulations.

2 Case study : CSTR with simple microbial growth

In order to limit the complexity of the calculations, we shall concentrate on one case study for both controllers : the control of the biomass concentration X by acting on the influent substrate concentration in a bioprocess characterized by a single microbial growth reaction. The model dynamics in a CSTR (continuous stirred tank reactor) are given by the following equations :

$$\dot{X} = \mu X - DX \quad (1)$$
$$\dot{S} = -k_1 \mu X + DS_{in} - DS \quad (2)$$

where k_1, μ, D, S, S_{in} represent the yield coefficient, the specific growth rate (h^{-1}), the dilution rate (h^{-1}), and the substrate concentration (g/l) in the reactor and in the influent, respectively. Many different models of specific growth rates are available in the literature (e.g. [2]). In this paper, we shall consider the rather general situation when the specific growth is a function of the limiting substrate only. We also rewrite the specific growth rate as the product of a kinetic constant k_0 and a specific reaction rate $r(S)$: $\mu = k_0 r(S)$. Then the above equations (1)(2) can be rewritten as follows :

$$\dot{X} = k_0 r(S) X - DX \quad (3)$$
$$\dot{S} = -k_1 k_0 r(S) X + DS_{in} - DS \quad (4)$$

With respect to the control problem defined here above, the output y and input u variables are defined as follows :

$$y = X \ \text{ et } \ u = S_{in} \quad (5)$$

For the ease of the backstepping design, let us consider the following change of coordinates and variables :

$$t \rightarrow \tau = tD \Rightarrow X \rightarrow x_1 \ et \ S \rightarrow x_2 \quad (6)$$

This implies that the dynamical equations (3)(4) of the CSTR become :

$$\dot{x}_1 = \frac{k_0}{D} r(x_2) x_1 - x_1 \quad (7)$$

$$\dot{x}_2 = -k_1 \frac{k_0}{D} r(x_2) x_1 - x_2 + u \quad (8)$$

Let us now define the parameter θ_1 :

$$\theta_1 = \frac{D}{k_0} \quad (9)$$

Then the dynamical equations are equal to :

$$\dot{x}_1 = \theta_1^{-1} r(x_2) x_1 - x_1 \quad (10)$$

$$\dot{x}_2 = -k_1 \theta_1^{-1} r(x_2) x_1 - x_2 + u \quad (11)$$

3 Non-adaptive backstepping controller

We shall concentrate on the design of the non-adaptive backstepping regulator. First we define the control error z_1 and the auxiliary variable z_2:

$$z_1 = x_1 - x_1^* \quad (12)$$

$$z_2 = r(x_2) x_1 - \alpha_1 \quad (13)$$

(with x_1^* the desired set point) and we select the following Lyapunov candidate function :

$$V_1 = \theta_1 \frac{z_1^2}{2} \quad (14)$$

Its time derivative is equal to :

$$\dot{V}_1 = z_1[r(x_2)x_1 - \theta_1 x_1] \quad (15)$$

$$= z_1[z_2 + \alpha_1 - \theta_1 x_1] \quad (16)$$

Let us select α_1 as follows, i.e. with a term proportional to the control error plus a term that cancels the term $\theta_1 x_1$:

$$\alpha_1 = -C_1 z_1 + \theta_1 x_1, \quad C_1 > 0 \quad (17)$$

Then the time derivative of V_1 is equal to :

$$\dot{V}_1 = z_1 z_2 - C_1 z_1^2 \quad (18)$$

Let us now look at the dynamics of the variable z_2 :

$$\dot{z}_2 = r(x_2)\dot{x}_1 + \frac{\partial r}{\partial x_2}\dot{x}_2 + C_1 \dot{z}_1 - \theta_1 \dot{x}_1 \quad (19)$$

$$= [r(x_2) - \theta_1 + C_1][\theta_1^{-1} r(x_2) x_1 - x_1] + \frac{\partial r}{\partial x_2} x_1 [-k_1 \theta_1^{-1} r(x_2) x_1 - x_2] + \frac{\partial r}{\partial x_2} x_1 u \quad (20)$$

$$= f(x_1, x_2) + \frac{\partial r}{\partial x_2} x_1 u \quad (21)$$

The choice of the following control law will allow to have arbitrary stable first order dynamics for z_2 :

$$u = \frac{1}{\frac{\partial r}{\partial x_2} x_1}[-C_2 z_2 - f(x_1, x_2)], \quad C_2 > 0 \quad (22)$$

Remark : the above control law must handled with care for specific growth rate models that exhibit extrema with respect to S (like the Haldane model) to avoid division by zero.

Let us now consider the second step of the design, i.e. the selection of the following Lyapunov candidate function

$$V_2 = V_1 + \frac{z_2^2}{2} \quad (23)$$

whose time derivative is equal to :

$$\dot{V}_2 = z_1 z_2 - C_1 z_1^2 + z_2 \dot{z}_2 \quad (24)$$

$$= z_1 z_2 - C_1 z_1^2 - C_2 z_2^2 \quad (25)$$

If we select C_1 as follows : $C_1 = C_{11} + C_{12}$ ($> \frac{1}{2}$) et $C_{22} > \frac{1}{2}$, then :

$$\dot{V}_2 = -C_{11} z_1^2 - C_{12} z_1^2 + z_1 z_2 - C_2 z_2^2 \quad (26)$$

$$\leq -C_{11} z_1^2 - \frac{1}{2}(z_1 - z_2)^2 \quad (27)$$

This implies that asymptotically : $z_1 \rightarrow 0$ and $z_2 \rightarrow 0$.

The performance of the non-adaptive backstepping controller are illustrated on Figure 1 under a square wave variation of the desired set point for the biomass (every 20 hours between 3 g/l and 4 g/l). The figure at the top gives the control input, the inlet substrate concentration S_{in}, the figure in the middle shows the controlled output, the biomass concentration X, while the substrate concentration in the tank S is shown on the figure at the bottom. A Monod model has been considered in the numerical simulations :

$$\mu = \frac{\mu_{max} S}{K_S + S} \tag{28}$$

The initial conditions, model parameters and the design parameters have been set to the following values :

$$\begin{aligned} X(0) &= 2\,g/l,\ S(0) = 0.9,\ D = 0.05\,h^{-1}, \\ K_S &= 5\,g/l,\ \mu_{max} = 0.33\,h^{-1},\ k_1 = 2 \\ X^* &= 3\,g/l,\ C_1 = 4,\ C_2 = 4 \end{aligned}$$

4 Adaptive backstepping controller

In the above non-adaptive controller, we have assumed a perfect knowledge of the process model. In practice this can be a constraint that might lead to bad performance in presence of model uncertainty (e.g. with respect to the process kinetics). We shall see how to modify the design to include the on-line adaptation of the kinetic parameter k_0.

First of all, we modify the definition of α_1 as follows :

$$\alpha_1 = -C_1 z_1 + \hat{\theta}_1 x_1 \tag{29}$$

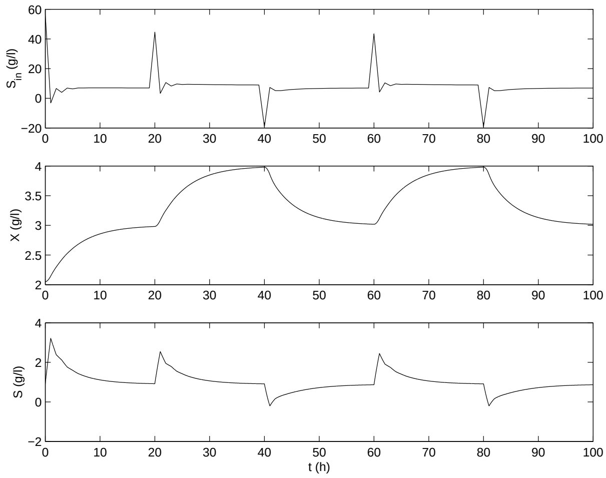

Figure 1: *Backstepping nonlinear control of a bioreactor*

and consider the parameter estimation error $\tilde{\theta}_1$:

$$\tilde{\theta}_1 = \hat{\theta}_1 - \theta_1 \tag{30}$$

This leads to the selection of the following Lyapunov candidate function V_{s1} :

$$V_{s1} = V_1 + \frac{\tilde{\theta}_1^2}{2\gamma_1},\ \ \gamma_1 > 0 \tag{31}$$

whose time derivative is equal to :

$$\dot{V}_{s1} = z_1 z_2 - C_1 z_1^2 + \tilde{\theta}_1 [z_1 x_1 + \frac{1}{\gamma_1}\dot{\hat{\theta}}_1] \tag{32}$$

Now the dynamics of z_2 ($= C_1 z_1 + r(x_2)x_1 - \hat{\theta}_1 x_1$) are equal to :

$$\begin{aligned} \dot{z}_2 = & [r(x_2) - \hat{\theta}_1 + C_1][\theta_1^{-1} r(x_2) x_1 - x_1] \\ & + \frac{\partial r}{\partial x_2} x_1 [-k_1 \theta_1^{-1} r(x_2) x_1 - x_2] \\ & + \frac{\partial r}{\partial x_2} x_1 u - x_1 \dot{\hat{\theta}}_1 \end{aligned} \tag{33}$$

$$
\begin{aligned}
&= \theta_1^{-1} r(x_2) x_1 [r(x_2) - \hat{\theta}_1 + C_1 - k_1 \frac{\partial r}{\partial x_2}] \\
&\quad - x_1 [r(x_2) - \hat{\theta}_1 + C_1 + x_2 \frac{\partial r}{\partial x_2}] \\
&\quad + \frac{\partial r}{\partial x_2} x_1 u - x_1 \dot{\hat{\theta}}_1 \qquad (34) \\
&= \theta_1^{-1} h(x_1, x_2, \hat{\theta}_1) - f(x_1, x_2, \hat{\theta}_1) \\
&\quad + \frac{\partial r}{\partial x_2} x_1 u - x_1 \dot{\hat{\theta}}_1 \qquad (35)
\end{aligned}
$$

If we select the following control law :

$$
\begin{aligned}
u &= \frac{1}{\frac{\partial r}{\partial x_2} x_1} [-C_2 z_2 + x_1 \dot{\hat{\theta}}_1 - \hat{\theta}_2 h(x_1, x_2, \hat{\theta}_1) \\
&\quad + f(x_1, x_2, \hat{\theta}_1)] \qquad (36)
\end{aligned}
$$

with $\theta_2 = \theta_1^{-1}$, then the dynamics of z_2 becomes:

$$
\dot{z}_2 = -C_2 z_2 + h(x_1, x_2, \hat{\theta}_1) \tilde{\theta}_2 \qquad (37)
$$

($\tilde{\theta}_2 = \hat{\theta}_2$ - θ_2). Let us now proceed to the second step of the backstepping design and select the following Lyapunov candidate function V_2 :

$$
V_2 = V_{s1} + \frac{1}{2} z_2^2 + \frac{\tilde{\theta}_2^2}{2\gamma_2}, \quad \gamma_2 > 0 \qquad (38)
$$

whose time derivative is equal to :

$$
\begin{aligned}
\dot{V}_2 &= z_1 z_2 - C_1 z_1^2 + \tilde{\theta}_1 [z_1 x_1 + \frac{1}{\gamma_1} \dot{\hat{\theta}}_1] \\
&\quad + z_2 [-C_2 z_2 - h \tilde{\theta}_2] + \frac{\tilde{\theta}_2}{\gamma_2} [\dot{\hat{\theta}}_2] \qquad (39) \\
&= z_1 z_2 - C_1 z_1^2 + \tilde{\theta}_1 [z_1 x_1 + \frac{1}{\gamma_1} \dot{\hat{\theta}}_1] \\
&\quad - C_2 z_2^2 + \tilde{\theta}_2 [\frac{\dot{\hat{\theta}}_2}{\gamma_2} - h(x_1, x_2, \hat{\theta}_1)] \qquad (40)
\end{aligned}
$$

If we choose the following adaptation laws for $\hat{\theta}_1$ and $\hat{\theta}_2$:

$$
\begin{aligned}
\dot{\hat{\theta}}_1 &= -\gamma_1 z_1 x_1 \qquad (41) \\
\dot{\hat{\theta}}_2 &= \gamma_2 h z_2 \qquad (42)
\end{aligned}
$$

the time derivative of V_2 becomes :

$$
\dot{V}_2 = -C_1 z_1^2 + z_1 z_2 - C_2 z_2^2 \qquad (43)
$$

which is negative definitive via an appropriate choice of the design parameters C_1 and C_2.

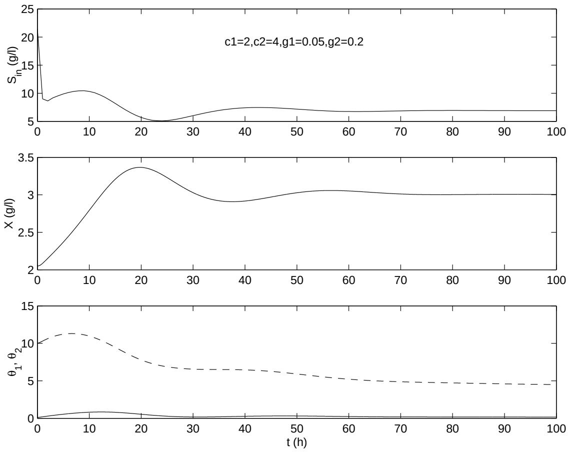

Figure 2: *Adaptive backstepping control of a bioreactor*

The performance of the adaptive backstepping controller are illustrated on Figure 2 for a desired set point for the biomass equal to 3 g/l with a Monod model. The figure at the top gives the control input S_{in}, the figure in the middle shows the controlled output X while the estimates of both parameters θ_1 (straight line) and θ_2 (dotted line) are shown on the figure at the bottom. The value of θ_1 corresponds to the ratio of the dilution rate D over the maximum specific growth μ_{max}. The initial conditions and model parameters are te same as those in Figure 1 (non-adaptive control), and the design parameters and initial estimates have been set to the following values :

$$
C_1 = 2, \; C_2 = 4, \; \gamma_1 = 0.05, \; \gamma_2 = 0.2
$$

$$\hat{\theta}_1(0) \quad = \quad 0.1, \hat{\theta}_2(0) = 10 \tag{44}$$

5 Conclusions

If this paper can be viewed as preliminary results, the following primary conclusions can be drawn. One of the main attractive feature of the backstepping design is its constructive nature since it is based on a step-by-step design based on successive Lyapunov functions. Yet if it appears that here the backstepping design is rather straightforward, it may become rapidly very complex once the system has high relative degree. And its potential application would require a more exhaustive performance analysis, in particular in presence of several disturbances and uncertainties.

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