ROBUST STABLE ADAPTIVE CONTROL OF UNCERTAIN BILINEAR PLANTS AND IT'S APPLICATION FOR DISTILLATION COLUMN

Cheng Shao†, Guangjun Luo†, and Furong Gao‡

†Research Center of Information and Control, Dalian University of Technology, Dalian, Liaoning, P.R. China

Email:cshao@dlut.edu.cn

‡ Department of Chemical Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong

Abstract: A robust adaptive control design is considered for a class of bilinear plants with unmodeled high-order dynamics and bounded disturbances. A basic optimal control law is first introduced by the generalized minimum variance control strategy, followed by a modification of introducing the modeling error estimate to the control law. Modified least-squares scheme with a relative dead zone is developed to form a novel robust adaptive controller. The resulting closed loop system is proven theoretically to have a zero average tracking error and robust stability. Furthermore, the simulation and an application in controlling of sensitive plate temperature of distillation column demonstrate the effectiveness of the algorithm. *Copyright © 2003 IFAC*

Keywords: robust stability, adaptive control, bilinear system, unmodeled dynamics, distillation column.

1. INTRODUCTION

Attempts to invent, design and build systems capable of controlling unknown plants or adapting to unpredictable changes in the environment resulted in the emergence of adaptive control in the 1950s, and since then, adaptive control has been in the mainstream of control research and development with numerous papers and books published and successful applications every year (Åström 1983, Ljung and Söderstrom 1983, Goodwin and Sin 1984, Åström and Wittenmark 1995, Kanellakopoulos and Koktovic 1995, and the References therein). Significant contributions have been made with the stability establishment of adaptive control theory (Egardt 1979, Gawthrop 1980, Goodwin and Sin 1984, Middleton *et al.* 1988, 1989). They, however, have been limited to the ideal cases, such as linear plants of disturbance-free, random noises, etc. A stable adaptive control algorithm may not be necessarily robust stable (Egardt 1979, Rohrs *et al.* 1982), and the disturbances in an adaptive control system may be inherently related to the plant inputs and outputs (Krisselmeier and Anderson 1986). This led to the recent interest of the robust stability research for adaptive control systems. In the presence of nonlinear uncertainties, unmodeled dynamics and/or bounded disturbances, it has been shown that for the linear plants, robust stability can be ensured by combining the normalization, σ-modification and relative dead zone parameter estimation algorithms with the control strategies of minimum variance, generalized minimum variance or pole placement (Clarke and Gawthrop 1975, 1979, Gawthrop and Lim 1982, Middleton *et al.* 1988, 1989, Shao 1991, 1996). For the bilinear systems, adaptive control stability has been studied recently with bounded external disturbances (Sun and Rao 1999), but the robust stability has not been considered for the systems with the presence of unmodeled dynamics.

This paper examines the robust stable adaptive control problem of the bilinear system in the presence of unmodeled plant uncertainties and bounded disturbances. The considered plant is a class of high-order bilinear systems with unknown and perturbed parameters. By means of minimizing a generalized variance function, a basic control law is first derived and then modified by introducing model error feedback. A self-tuning controller is proposed by combining the control law with a modified least squares parameter estimator with relative bilinear dead zone. With the proposed self-tuning controller, it is proven that robust stability for the bilinear system can be ensured with respect to the unmodeled high-order nonlinear dynamics and bounded disturbances, without any static state tracking error in an average sense.

Distillation column is a kind of important fraction equipment in chemistry industry. Due to the complex structure of the equipment and the different effects caused by the fluids, the whole system is of essential nonlinearity and of large delay. The control of distillation process is quite concerned with the products quality. In the control system designing for it, in order to satisfy the high quality demand, the sensitive plate temperature is usually selected as the controlled variable. Bilinear system can be used to model many industrial process. In this paper ,a plant of sensitive plate temperature modeled as bilinear system is presented and used as application model. Because of nonlinear characteristics of distillation column, it is naturally to develop robust nonlinear adaptive control algorisms to solve this problem. The experimental results suggested in the paper demonstrated the effectiveness of the proposed bilinear adaptive algorithm for control of distillation column also.

2. THE PLANT DESCRIPTION

Consider a class of bilinear plants with uncertain perturbation and bounded disturbances

$$y(t) = q^{-d} G_1 u(t) + q^{-d} G_2 y(t) u(t) + v(t) \qquad (2.1a)$$

$$G_1 = \frac{B(1+\mu B')}{A(1+\mu A')} \qquad (2.1b)$$

$$G_2 = \frac{C(1+\mu C')}{A(1+\mu A')} \qquad (2.1c)$$

where y and u are the scalar output and input, respectively, v is a bounded output disturbance, $d \geq 1$ is the plant delay, A, A', B, B', C and C' are polynomials of delay operator q^{-1} of orders n_A, $n_{A'}$, n_B, $n_{B'}$, n_C and $n_{C'}$, respectively, and $\mu \geq 0$ is a singular uncertain perturbation scalar, by which the unmodeled high-order dynamics will be brought. In fact from (2.1):

$$y(t) = \frac{B}{A} u(t-d) + \frac{C}{A} u(t-d) y(t-d) + \eta_P(t) \qquad (2.2a)$$

$$\eta_P(t) = \mu \frac{B(B'-A')}{A(1+\mu A')} u(t-d) + \mu \frac{C(C'-A')}{A(1+\mu A')} u(t-d) y(t-d) + v(t) \qquad (2.2b)$$

Consequently, a singular perturbation from $\mu > 0$ to $\mu = 0$ results in the reduced-order model

$$y(t) = \frac{B}{A} u(t-d) + \frac{C}{A} u(t-d) y(t-d) + v(t) \qquad (2.3)$$

The designer is assumed to be given only the reduced-order model (2.3), without the knowledge of the coefficients of A, B and C. Therefore, the modeling errors $\eta_P(t)$ which includes the unmodeled high-order dynamics related to $u(t)$, $y(t)$ and their products, has to be considered in designing of adaptive controller and the robust stability of the resulting closed loop system must be ensured. Model representation of (2.3) has been effectively employed for modeling a combustion process with one input (flow of air) and one output (Oxygen content) in discrete time and many other industrial processes, such as nuclear fission, convective heat-transfer, and turbo-pump dynamics may be also effectively modeled by a bilinear system (Mohler 1991, Aganović and Gajić 1995). The analysis of the model is made with the following assumptions for the model polynomials A, B and C, in this paper.

Assumption 1: *A is monic and coprime with B.*

Assumption 2: *n_A, n_B, n_C and delay d are known.*

Remark 1: Assumption 1 implies that the reduced-model is controllable, the pole placement control design can, therefore, be applied to the processes that are unstable and/or non-minimum phase. Assumption 2 provides a necessary structure parameter frame for constructing a self-tuning controller.

3. A NOVEL SELF-TUNING CONTROLLER

Our objective is to design a self-tuning controller based on the reduced-order model, or the structural knowledge of A, B and C, so that the application of such a controller to the plant (2.1) results in a robust stable closed loop system tracking the desired output in the presence of the unmodeled dynamics and bounded disturbances.

Let P be an arbitrary monic polynomial in q^{-1} of order n_P. Introduce the polynomial identity

$$P = AF + q^{-d} G \qquad (3.1)$$

where F and G are polynomials in q^{-1} of orders $n_F = d-1$ and $n_G = \max\{n_A - 1, n_P\}$, respectively, and F is monic also. Multiplying (2.2a) by AF gives

$$Py(t+d) = Gy(t) + FBu(t) + FCu(t)y(t) + AF\eta_P(t+d) \qquad (3.2)$$

Define

$$\phi(t) = Py(t)$$

$$X^T(t) = (y(t), \ldots, y(t-n_G), u(t), \ldots, u(t-n_B-d+1), u(t)y(t), \ldots, u(t-n_C-d+1)y(t-n_C-d+1)) \quad (3.3)$$

$$\eta(t) = AF\eta_P(t) \quad (3.4)$$

Then a regression form of (3.2) can be given as follows

$$\phi(t+d) = \theta^T X(t) + \eta(t+d) \quad (3.5)$$

where θ is the parameter vector composed of the coefficients of *G*, *FB* and *FC*. It should be noted that though the plant is modeled linearly in θ, the nonlinearity exists in the multicity of measured inputs and outputs, and the high-order unmodeled dynamics $\eta(t)$. The following lemmas are given to establish a relative upper bound of $\eta(t)$.

Lemma 1: Let $D(q^{-1})$ be a polynomial in q^{-1} with finite order n_D. For arbitrary $\sigma \in (0,1)$ there exists $\mu_0 > 0$ such that $D_\mu(z^{-1}) = 1 + \mu D(z^{-1}) \neq 0$ for all $|z| > \sigma$ and $\mu \in [0, \mu_0]$, that is $D_\mu(q^{-1})$ is strictly *Hurwitz* uniformly in $\mu \in [0, \mu_0]$.

The proof is given in Shao (1996).

Lemma 2: There exist non-negative constants K_1 and K_2 independent of μ and μ_1 such that for all $\mu \in [0, \mu_1]$

$$|\eta(t)| \le \mu K_1 \{\max_{0\le\tau\le t} |y(\tau)| + \max_{0\le\tau\le t-d} |u(\tau)y(\tau)|\} + K_2 \quad (3.6)$$

Proof: Substituting (2.2*a*) into (2.2*b*) results in:

$$\eta_P(t) = \mu\frac{B'-A'}{1+\mu B'}y(t) + \mu\frac{C(C'-B')}{A(1+\mu B')}u(t-d)y(t-d) + \frac{1+\mu A'}{1+\mu B'}v(t) \quad (3.7)$$

The result follows from (3.7) and (3.4) by applying Lemma 1 to $B'_\mu = 1 + \mu B'$ and referring to the proof of Lemma 2 of Shao (1996).

To achieve a basic optimal control law, suppose that $\{\eta(t)\}$ is a white noise sequence, the generalized minimum variance control strategy of Clarke-Gawthrop type (Clarke and Gawthrop 1975, 1979) then may be employed by minimizing the following quadratic cost function with respect to $u(t)$:

$$J = E\{P(y(t+d) - y^*(t+d)) + \tilde{Q}u(t)\}^2 \quad (3.8)$$

where P and $\tilde{Q}$ are constant weighting polynomials in q^{-1} with $\tilde{Q} = (1-q^{-1})Q$, and $y^*(t)$ is bounded desired output. It follows from (3.5) that

$$J = E\{(\theta^T X(t) - Py^*(t+d)) + \tilde{Q}u(t)\}^2 + D\eta \quad (3.9)$$

It is obvious that an optimal control law can be given by

$$\theta^T X(t) + \tilde{Q}u(t) = Py^*(t+d) \quad (3.10)$$

The preceding control law (3.10) is not suitable for our purpose, as $\eta(t)$ includes unmodeled dynamics. In fact for the self-tuning case, replacing θ in (3.10) with its estimate $\hat{\theta}(t)$ and then applying (3.10) to (3.5) one obtains

$$P(y(t+d) - y^*(t+d)) = (\theta - \hat{\theta}(t))^T X(t) - \tilde{Q}u(t) + \eta(t+d) \quad (3.11)$$

which means that due to the existence of $\eta(t)$ the tracking error $e(t) = y(t) - y^*(t)$ will not converge to zero even when the parameter estimates $\hat{\theta}(t)$ approach to their true values. To remove the effect of unmodeled dynamics, the control law (3.10) is modified by introducing an estimate of $\eta(t)$:

$$\hat{\eta}(t) = \phi(t) - \hat{\theta}(t)^T X(t-d) \quad (3.12)$$

which results in a novel self-tuning control law

$$\hat{\theta}(t)^T X(t) + \tilde{Q}u(t) = Py^*(t+d) - \hat{\eta}(t) \quad (3.13)$$

The parameter estimates are given by the modified least squares scheme with relative dead zone (Shao, 1996) by changing the parameter $\lambda(t)$ as follows:

$$\lambda(t) = \begin{cases} 0 & if |\varepsilon(t)| \le 2\beta[\mu^*(\max_{0\le\tau\le t}|y(\tau)| + \max_{0\le\tau\le t-d}|u(\tau)y(\tau)|) + 1] \\ \gamma & otherwise, \gamma \in [\sigma_0, 3(1-\sigma_0)/4] \\ & 0 < \sigma_0 < 3/7 \end{cases} \quad (3.14)$$

where β is positive user adjustable parameter with $\beta \ge \max\{K_1, K_2\}$ (see (3.6)), and $\{W(t)\}$ is a matrix sequence with arbitrary initial $W(-1) > 0$.

Remark 2: It can be shown that for a linear plant (*i.e.* *C* equals to zero in (2.1)) the quadratic cost function (3.8) is equivalent to the generalized minimum variance function of the Clarke-Gawthrop type (Clarke and Gawthrop 1975, 1979, Gawthrop 1980, Gathrop and Lim 1982, Shao 1996)

$$J = E\{[P(y(t+d) - y^*(t+d)]^2 + [\tilde{Q}u(t)]^2\} \quad (3.15)$$

And here choice of weighting polynomial $\tilde{Q}$ in the form of $\tilde{Q} = (1-q^{-1})Q$, to be seen in the sequel, will remove static state tracking error in an average sense.

Remark 3: It can be observed that a relative bilinear dead zone method is employed in this paper. Despite the appearance of the bilinear term in the control law, $u(t)$ is always solvable from (3.13) by choosing proper $\lambda(t)$ and/or α. The singularity problem in solving $u(t)$ from (3.13) can hence be avoided.

Remark 4: The condition $\beta \geq \max\{K_1, K_2\}$ is not crucial. In practice, one can start with a large initial value, and then reduce β when the closed-loop system approaches the steady state, to improve control accuracy.

The following assumption is made on P and $\widetilde{Q}$.

Assumption 3: The off-line choices of P and $\widetilde{Q}$ are such that

$$f(q^{-1}) = P(q^{-1})B(q^{-1}) + \widetilde{Q}(q^{-1})A(q^{-1}) \tag{3.16}$$

is stable, that is $f(z) \neq 0$, $|z| \leq 1$.

Remark 5: Assumption 3 is often made for linear control systems with the pole placement design. Here it is made, however, for the reduced-order model with the design consideration of the robust stable adaptive control of a bilinear system with unmodeld high-order dynamics. The linear part of the plant (2.2) may be unstable and/or non-minimum phase, without any further constraints on *A* and *B*.

The only assumption on the unmodeled dynamics is made as the following:

Assumption 4: A sufficiently small upper bound μ^* of μ is available. (*The meaning of 'sufficiently small' will be elaborated later.*)

Remark 6: This is a condition often used to construct relative dead zone adaptation algorithms for solving the linear robust adaptive control problems (Kreisslmeier and Andson 1986, Shao 1996). Extension has been made here to bilinear nonlinear systems with unmodeled high-order dynamics.

4. ROBUST STABILITY ANALYSIS

The following lemmas are given for the robust stability of the resulting closed loop system.

Lemma 3: If μ^* is sufficiently small such that $\mu^* \leq \mu_1$, the application of parameter estimation scheme (3.14) to (3.5) for all $\mu \in [0, \mu^*]$ has the following properties.

(1) $$\lim_{t\to\infty} \frac{\lambda(t)^{1/2}\varepsilon(t)}{[\alpha + X(t-d)^T W(t-2) X(t-d)]^{1/2}} = 0 \tag{4.1}$$

(2) $\left|[\hat{\theta}(t-1) - \hat{\theta}(t-d)]^T X(t-d)\right| \leq h(t)\|X(t-d)\|$, $h(t) \to 0$ as $t \to \infty$ (4.2)

where $\|\cdot\|$ denotes the vector-Euclidean norm.

(3) $\hat{\theta}(t)$ is bounded.

The proof may be referred to that of Lemma 3 in Shao (1996), and is thus omitted here.

Lemma 4: The tracking error and the input dynamics satisfy

$$\begin{aligned}(PB + \widetilde{Q}A)e(t) = {} & \widetilde{Q}Cy(t-d)u(t-d) + A\widetilde{Q}\eta_P(t) \\ & + B\Delta_d\varepsilon(t) + \delta_1(t)\end{aligned} \tag{4.3}$$

$$\begin{aligned}(PB + \widetilde{Q}A)u(t-d) = {} & -PCy(t-d)u(t-d) - AP\eta_P(t) \\ & + A\Delta_d\varepsilon(t) + \delta_2(t)\end{aligned} \tag{4.4}$$

where $\Delta_d = 1 - q^{-d}$ and

$$\begin{aligned}\delta_1(t) = {} & B[\hat{\theta}(t-1) - \hat{\theta}(t-d)]^T X(t-d) \\ & + B[\hat{\theta}(t-d) - \hat{\theta}(t-d-1)]^T X(t-2d) \\ & - A\widetilde{Q}y^*(t)\end{aligned} \tag{4.5}$$

$$\begin{aligned}\delta_2(t) = {} & A[\hat{\theta}(t-1) - \hat{\theta}(t-d)]^T X(t-d) \\ & + A[\hat{\theta}(t-d) - \hat{\theta}(t-d-1)]^T X(t-2d) \\ & + APy^*(t)\end{aligned} \tag{4.6}$$

Proof: Using (3.13) and (3.14*d*) gives

$$\begin{aligned}Pe(t) &= \phi(t) - \hat{\eta}(t-d) - \hat{\theta}(t-d)^T X(t-d) - \widetilde{Q}u(t-d) \\ &= \Delta_d\varepsilon(t) + [\hat{\theta}(t-1) - \hat{\theta}(t-d)]^T X(t-d) \\ &\quad + [\hat{\theta}(t-d) - \hat{\theta}(t-d-1)]^T X(t-2d) - \widetilde{Q}u(t-d)\end{aligned} \tag{4.7}$$

From (2.2*a*) one obtains

$$\begin{aligned}Ae(t) = {} & Bu(t-d) + Cu(t-d)y(t-d) \\ & + A\eta_P(t) - Ay^*(t)\end{aligned} \tag{4.8}$$

A summation of (4.8) multiplied by $\widetilde{Q}$ and (4.7) multiplied by *B* results to (4.3) with $\delta_1(t)$ of (4.5). In the same fashion, a summation of (4.8) multiplied by *P* and (4.7) multiplied by -*A* leads to (4.4) with $\delta_2(t)$ of (4.6).

Lemma 5: Subject to Assumptions 1-4, there exist sufficiently small $\mu^* > 0$ and non-negative constants L_1', L_2' and L_3' that are independent of μ and polynomial *C* such that for all $\mu \in [0, \mu^*]$

$$\max_{0\le\tau\le t-d} |u(\tau)| \le L_1'(\mu_{\tilde{Q}}+\mu_P) \max_{0\le\tau\le t-d} |Cu(\tau)y(\tau)| + L_2' \max_{0\le\tau\le t} |\varepsilon(\tau)| + L_3' \quad (4.9)$$

$$\max_{0\le\tau\le t} |y(\tau)| \le L_1'(\mu_{\tilde{Q}}+\mu_P) \max_{0\le\tau\le t-d} |Cu(\tau)y(\tau)| + L_2' \max_{0\le\tau\le t} |\varepsilon(\tau)| + L_3' \quad (4.10)$$

where $\mu_{\tilde{Q}} = \sum_{i=0}^{n_{\tilde{Q}}} |\tilde{q}_i|$, $\mu_P = \sum_{i=0}^{n_P} |p_i|$, $\tilde{q}_i$ and p_i are coefficients of polynomials $\tilde{Q}$ and P, respectively.

The following assumption is further made on weighting polynomials P and $\tilde{Q}$ to ensure the resulting closed loop system with robust stability.

Assumption 5: The norms μ_P, $\mu_{\tilde{Q}}$ of P and $\tilde{Q}$, respectively, are relatively small.

Remark 7: Assumption 5 gives the specific requirement on the weighting polynomials P and $\tilde{Q}$ for the robust stabilization of bilinear systems, in addition to the general pole placement method.

Theorem 1: Subject to Assumptions 1-5, there exists sufficiently small $\mu^* > 0$ such that the application of self-tuning control algorithm (3.14) to plant (2.1) ensures that

(1) The resulting closed loop system is globally robust stable in the sense that u and y are bounded for arbitrary bounded initial conditions and all $\mu \in [0, \mu^*]$

(2) The tracking error satisfies

$$\lim_{N\to\infty} \frac{1}{N} \sum_{t=0}^{N} e(t) = 0 \quad (4.11)$$

(3) In particular, if the disturbance v is constant (not necessarily equals to zero) and the reference signal y^* is fixed, then

$$\lim_{t\to\infty} [y(t) - y^*] = 0 \quad (4.12)$$

The proof is omitted here.

5. NUMERICAL SIMULATION

To demonstrate the effectiveness of the proposed adaptive control algorithm some numerical simulation results are given below.

EXAMPLE

Select
$A = 1 + q^{-1} + q^{-2}$, $B = 1$, $C = 1$; $A' = 1 + q^{-1}$, $B' = 1$, $C' = 1$. Then a bilinear system plant is given as below:

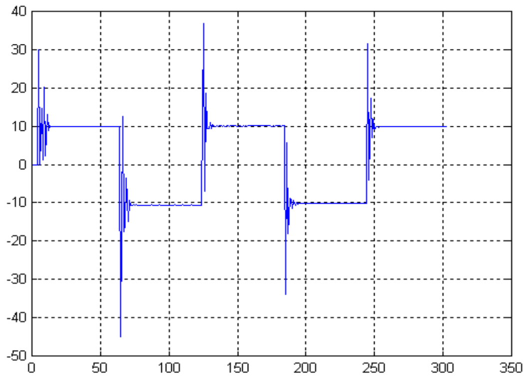

Fig.1 The plant output

$$y(k) = -y(k-1) - y(k-2) + u(k-2) * y(k-2) + u(k-2) - \mu * y(k-1)/(1+\mu) + (1+2\mu)/(1+\mu) * e(k)$$

$e(k)$ is a random gaussian distribution sequence with mean zero and variance 0.01; For designing controller:
$P = 6 + 5q^{-1} + q^{-2}$, Q=1, $\alpha = 1$, $\beta = 4$, $\gamma = 0.5$, $\mu = 0.01$, and the initial conditions:
$\hat{\theta}(0) = [-2, 0.4, 3, -0.01, 4, 0.3]$, $W(1) = aI > 0$, $(a = 0.8)$; As shown in figure 1, the controller worked well with the presence of unmodeled dynamics.

6. APPLICATION OF THE ALGORITHM

A certain loop in a industrial distillation column can be modeled as follows:

$$T(k+1) = 0.3848T(k) + 0.0767T(k)u(k) + 0.5 * \sin(u(k-1) * y(k-1)) - 1.2663u(k) + e(k)$$

$T(k)$ $({}^oC)$ is the temperature of the sensitive plate, $u(k)$ $(kmol/h)$ is the charge in flow.

The sampling cycle T_s =1s. $e(k)$ is a random gaussian distribution sequence with mean zero and variance 0.01.The simulation result is shown in figure 2. From the results, it can be seen that the tracking error is zero and the tracking velocity is satisfactory. The robust adaptive control algorithm worked very well and improved the quality of the controlled system.

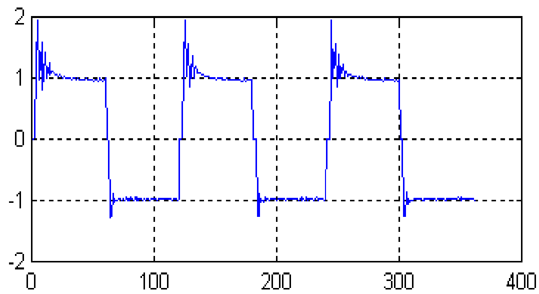

Fig.2 The plant output

7. CONCLUSIONS

A new self-tuning control algorithm has been developed in this paper for a class of bilinear systems with uncertain perturbation and bounded disturbances. The generalized minimum variance control strategy has been extended to suit our purpose together with a modified least squares estimation with relative nonlinear dead zone. The robust stability of the resulting closed loop system has been established with respect to unmodeled high-order dynamics related to the plant input and output, and to bounded disturbances. Simulation example and application for control of sensitive plate temperature of distillation column showed that the proposed algorithm is available for robust control of a class of nonlinear systems with uncertain disturbances and high degree unmodeled dynamics.

ACKNOWLEDGEMENT

The authors would like to acknowledge the supports of Foundation for University Key Teacher by the Ministry of Education of China, the State Science and Technology Pursuit Plan Project of China (2001BA204B01), and the Hong Kong Research Grant Council (HK UST RIG95/96. EG03).

REFERENCES

Åström, K. J., and Wittenmark, B. (1995). *Adaptive Control*. Addison-Wesley Publishing Company, Inc. Boston, MA, U.S.A.

Åström, K. J. (1983). Theory and applications of adaptive control-a survey. *Automatica*, **19**, 471.

Aganović, Z., and Gajić, Z. (1995). *Linear Optimal Control of Bilinear Systems*, Lecture Notes in Control and Information Sciences . Springer-Verlag, Berlin.

Clarke, D. W., and Gawthrop, P.J. (1975), Self-tuning controller. *Proceedings of the Institution of Electrical Engineers*, **122**, 929; (1979). Self-tuning control. *Ibid*., **126**, 633.

Egardt, B. (1979). *Stability of Adaptive Controllers*, Lecture Notes in Control and Information Sciences. Springer-Verlag, Berlin.

Gawthrop, P. J. (1980). On the stability and convergence of a self-tuning controller. *International Journal of Control*, **31**, 973.

Gawthrop, P. J., and Lim, K. W. (1982). Robustness of self-tuning controllers. *Proceedings of the Institution of Electrical Engineers*, Pt D, **129**, 21.

Goodwin, G. C., and Sin, K. S. (1984). *Adaptive Filtering Prediction and Control* . Prentice-Hall ,Englewood Cliffs, New Jersey, U.S.A..

Kreisselmeier, G. and Anderson, B. D. O. (1986). Robust model reference adaptive control. *IEEE Transactions on Automatic Control*, **31**, 127.Krstic, M.,

Kanellakopoulos, I., and Koktovic P. (1995). *Nonlinear and Adaptive Control Design.* John Wiley & Sons, Inc New York, U.S.A..

Ljung, L., and Söderstrom, T. (1983). *Theory and Practice of Recursive Identification.* MIT Press ,London, U.K..

Middleton, R. H., Goodwin, G. C., Hill, D. J. and Mayne, D. Q. (1988). Design issues in adaptive control. *IEEE Transactions on Automatic Control*, **33**, 50.

Middleton, R. H., Goodwin, G. C. and Wang, Y. (1989). On the robustness of adaptive controllers using relative deadzones. *Automatica*, **25**, 889.

Mohler, R.R. (1991). *Nonlinear Systems: Volume II, Applications to Bilinear Control* . Prentice-Hall, Englewood Cliffs, New Jersey, U.S.A..

Rohrs, C. E., Valavani, L., Athans, M., and Stein, G.(1982). Robustness of adaptive control algorithms in the presence of unmodeled dynamics,. *Proceedings of. 21st IEEE Conference on Decision and Control*, Orlando, FL. p.3.

Shao, C. (1991). Stable adaptive control system subject to bounded disturbances. *International Journal of Adaptive Control and Signal Processing*, **5**, 121.

Shao, C.(1996). On the robust stability of a Clarke-Gawthrop type of self-tuning controller. *International Journal of Control.* **64**, 721.

Sun, X. and Rao, M.(1999). Multivariable adaptive control of bilinear systems in the presence of bounded disturbances. *Preprints of 14th Triennial World Congress*, Beijing, P.R. China, **I**, 301.