AN INTERNAL MODEL CONTROL FOR MAX-PLUS LINEAR SYSTEMS WITH LINEAR PARAMETER VARYING STRUCTURE

Shiro Masuda * **Hiroyuki Goto** ** **Takashi Amemiya** *
Kazuhiro Takeyasu **

* *Dep. of Production,Information and Systems Engineering, Tokyo Metropolitan Institute of Technology,*
6-6, Asahigaoka, Hino, Tokyo, Japan, 191-0065, smasuda@cc.tmit.ac.jp
** *The Japan Research Institute, Limited; 16 Ichiban-cho, Chiyoda-ku, Tokyo 102-0082, JAPAN*

Abstract. The max-plus-linear (MPL) system is a state-space description for a certain class of discrete-event-systems, and it has remarkable analogous features to the conventional linear state-space description in the modern control theory. Hence, several control techniques in the modern control theory have been extended so that they could be applied to MPL systems. In the research context, the internal model control (IMC) for MPL systems has been proposed by Boimond et al. and it succeeds to realize feedback control techniques for discrete-event-systems described in MPL systems. In this paper, the IMC control for MPL systems is extended to the case where the controlled systems are given as MPL systems with linear parameter varying structure, which is called LPV-MPL systems. In the LPV-MPL systems, the systems parameters are explicitly represented in the systems description. Hence, the obtained IMC control law can utilize the additive information on the parameters variations effectively when the parameters are measured on-line, or the variation of the parameters are scheduled beforehand. The effectiveness of the proposed IMC is shown through a numerical example where it is applied to a two-inputs, two-outpus production system with four machines.

Keywords. max-plus-linear systems, linear parameter varying, internal model control, discrete-event-systems

1. INTRODUCTION

The researches on modeling and control of discrete-event-systems using max-plus algebra have been reported recently(Cohen *et al.*, 1989; Baccelli *et al.*, 1992). The basic operations of max-plus algebra are maximization and addition, which have a remarkable analogy with ones of conventional algebra. Especially, state-space descriptions in the max-plus algebra for a certain class of discrete-event-systems become linear representations which are similar to state-space equations in the traditional modern control theory (van den Boom and Schutter, 2001*a*). Hence, the several researches on control design for the max-plus-linear (MPL) systems have been reported from the viewpoint of the analogy (Boimond and Ferrier, 1996; van den Boom and Schutter, 2001*a*; van den Boom and Schutter, 2001*b*).

The internal model control (IMC) for MPL systems has been proposed by (Boimond and Ferrier, 1996) in the research context. It succeeds to realize feedback control techniques for discrete-event-systems described in MPL systems. In the IMC control, however, it takes much time to recover from the output delays because the input signals are modified just after the output errors are observed. Hence, it would be desirable that the information on the parameters variation would be collected beforehand, and it could be utilized effectively.

On the other hand, the MPL systems with linear parameter varying structure, which is called LPV-MPL systems was proposed, and the design method for inverse systems of LPV-MPL systems was developed (Masuda *et al.*, 2002). In the LPV-MPL systems, the systems parameters are explicitly represented. Hence, the obtained control law can utilize the additive information on the parameters variations effectively when the parameters are measured on-line, or the variation of the parameters are scheduled beforehand.

Therefore, in this paper, the IMC control is extended so that it can be applied to the LPV-MPL systems. In the proposed control law, the information on the parameters variations in addition to the feedback signals are effectively utilized for recovery from the output delays due to large parameters variation. Furthermore, owing to the IMC control law, the proposed method has robust property even when the the information on the parameters variations has some errors.

The effectiveness of the proposed IMC is shown through a numerical example where it is applied to a two-inputs, two-outpus production system with four machines.

2. MATHEMATICAL PRELIMINARIES

The basic operations of max-plus algebra are addition denoted by $\oplus$ and mulitiplication denoted by $\otimes$, which are defined as follows.

$$x \oplus y = \max(x, y), \quad x \otimes y = x + y, \quad x, y \in \boldsymbol{R}_\varepsilon$$

where $\boldsymbol{R}_\varepsilon = \boldsymbol{R} \cup \{-\infty\}$, and $\boldsymbol{R}$ stands for the real field. Let ε be defined as $-\infty$, which is the unit element of the addition $\oplus$, and let e be defined as 0, which is the unit element of the multiplication $\otimes$. We also define the following operations.

$$x \wedge y = \min(x, y), \quad x \backslash y = -x + y \tag{1}$$

The above operations are extended to the matrices calculation whose elements belong to $\boldsymbol{R}_\varepsilon$. So, if $\boldsymbol{A}, \boldsymbol{B} \in \boldsymbol{R}_\varepsilon^{m\times n}$, $\boldsymbol{C} \in \boldsymbol{R}_\varepsilon^{n\times p}$, then

$$[\boldsymbol{A} \oplus \boldsymbol{B}]_{ij} = [\boldsymbol{A}]_{ij} \oplus [\boldsymbol{B}]_{ij} = \max\left([\boldsymbol{A}]_{ij},\ [\boldsymbol{B}]_{ij}\right) \tag{2}$$

$$[\boldsymbol{A} \wedge \boldsymbol{B}]_{ij} = [\boldsymbol{A}]_{ij} \wedge [\boldsymbol{B}]_{ij} = \min\left([\boldsymbol{A}]_{ij},\ [\boldsymbol{B}]_{ij}\right) \tag{3}$$

$$1 \le i \le n, \quad 1 \le j \le m$$

$$\begin{aligned}[\boldsymbol{A} \otimes \boldsymbol{C}]_{ij} &= \bigoplus_{k=1}^{n} \left([\boldsymbol{A}]_{ik} \otimes [\boldsymbol{C}]_{kj}\right) \\ &= \max_{k=1,\cdots,n} \left([\boldsymbol{A}]_{ik} + [\boldsymbol{C}]_{kj}\right) \end{aligned} \tag{4}$$

where $[\,\cdot\,]_{ij}$ stands for the element in the i-th row, j-th column of the matrix, and

$$\bigoplus_{k=1}^{n} a_k = \max(a_1, a_2, \cdots, a_n)$$

. If $d \in \boldsymbol{R}_\varepsilon$, $\boldsymbol{A} \in \boldsymbol{R}_\varepsilon^{m\times n}$, then

$$[d \otimes \boldsymbol{A}]_{ij} = d \otimes [\boldsymbol{A}]_{ij} \tag{5}$$

Furthermore, we define the operator $\odot$ in the following way.

$$\begin{aligned}[\boldsymbol{A} \odot \boldsymbol{C}]_{ij} &= \bigwedge_{k=1}^{n} \left([\boldsymbol{A}]_{ik} \backslash [\boldsymbol{C}]_{kj}\right) \\ &= \min_{k=1,\cdots,n} \left(-[\boldsymbol{A}]_{ik} + [\boldsymbol{C}]_{kj}\right) \end{aligned} \tag{6}$$

where

$$\bigwedge_{k=1}^{n} a_k = \min(a_1, a_2, \cdots, a_n)$$

. In the subsequent discussions, $\boldsymbol{a} \le \boldsymbol{b}$ implies $[\boldsymbol{a}]_i \le [\boldsymbol{b}]_i$ $1 \le i \le n$ for $\boldsymbol{a},\ \boldsymbol{b} \in \boldsymbol{R}_\varepsilon^n$.

3. THE LPV-MPL SYSTEM

Consider the following MPL systems.

$$\boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) \oplus \boldsymbol{B}\boldsymbol{u}(k+1) \tag{7}$$

$$\boldsymbol{y}(k) = \boldsymbol{C}\boldsymbol{x}(k) \tag{8}$$

where $\boldsymbol{A} \in \boldsymbol{R}_\varepsilon^{n\times n}$, $\boldsymbol{B} \in \boldsymbol{R}_\varepsilon^{n\times p}$, $\boldsymbol{C} \in \boldsymbol{R}_\varepsilon^{q\times n}$. And $\boldsymbol{x}(k) \in \boldsymbol{R}_\varepsilon^n$, $\boldsymbol{u}(k+1) \in \boldsymbol{R}_\varepsilon{}^p$, $\boldsymbol{y}(k) \in \boldsymbol{R}_\varepsilon^q$ are state variables, control inputs and controlled outputs respectively. These variables represent time instants at which the representing events occur at k-times. According to the custom, the operation of multiplication denoted by $\otimes$ is omitted.

In LPV-MPL systems(Masuda *et al.*, 2002), the system matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ in (7) and (8) are replaced by the parameter affine form $\boldsymbol{A}(\boldsymbol{d})$, $\boldsymbol{B}(\boldsymbol{d})$ and $\boldsymbol{C}(\boldsymbol{d})$, which are defined as

$$\boldsymbol{A}(\boldsymbol{d}) = d_0\boldsymbol{A}_0 \oplus d_1\boldsymbol{A}_1 \oplus \cdots \oplus d_l\boldsymbol{A}_l = \bigoplus_{i=0}^{l} d_i\boldsymbol{A}_i$$

$$\boldsymbol{B}(\boldsymbol{d}) = d_0\boldsymbol{B}_0 \oplus d_1\boldsymbol{B}_1 \oplus \cdots \oplus d_l\boldsymbol{B}_l = \bigoplus_{i=0}^{l} d_i\boldsymbol{B}_i$$

$$\boldsymbol{C}(\boldsymbol{d}) = d_0\boldsymbol{C}_0 \oplus d_1\boldsymbol{C}_1 \oplus \cdots \oplus d_l\boldsymbol{C}_l = \bigoplus_{i=0}^{l} d_i\boldsymbol{C}_i$$

where $\boldsymbol{A}_i$, $\boldsymbol{B}_i$ and $\boldsymbol{C}_i$,$i = 1, \cdots l$ are matrices whose elements are either ε or e and the size are the same as $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$, respectively, and $\boldsymbol{d}$ is the parameter vector whose elements are $d_0, d_1, \cdots, d_l$ as is defined in the next.

$$\begin{aligned}\boldsymbol{d} &= [d_0,\ d_1,\ d_2,\ \cdots,\ d_l], \\ & d_0 = e,\ \ d_i > 0,\ i = 1, \cdots l\end{aligned}$$

Hence, the LPV-MPL system can be described as

$$\boldsymbol{x}(k+1) = \boldsymbol{A}(\boldsymbol{d})\boldsymbol{x}(k) \oplus \boldsymbol{B}(\boldsymbol{d})\boldsymbol{u}(k+1) \quad (9)$$

$$\boldsymbol{y}(k) = \boldsymbol{C}(\boldsymbol{d})\boldsymbol{x}(k) \quad (10)$$

In general, the elements of the matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ in the system representation consists of e and ε and real numbers. The elements of e and ε depend on the system structure such as the connection among the machines in the case where the production systems are modelled based on the MPL system. While parameters e and ε are expected to be unchanged even as time goes by, it should be considered that the real parameters might be the varying ones.

4. THE INTERNAL MODEL CONTROL (IMC)

The internal model control (IMC), which is a popular control technique in the field of chemical industries. The block diagram is given in Figure 1..

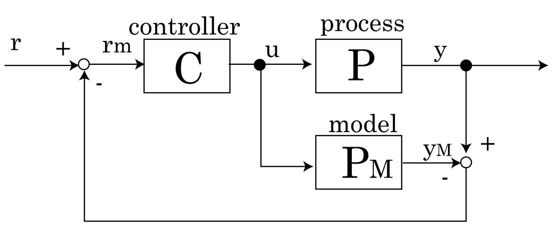

Fig. 1. The Block Diagram of IMC

In Figure 1., P stands for the real process, and P_M stands for the process model. y and y_M are the controlled process outputs and the model outputs, respectively. u and r are control input and reference signals, respectively. r_M is modified reference signals, which satisfy the following equation

$$r_M = r - (y - y_M) \quad (11)$$

Hence, if the control input is designed so that the model outputs y_M should be equal to the modified reference signals r_M, the controlled process outputs follow the given reference signals. Therefore, by using the inverse systems of the model P_M for the controller C in the IMC, we can get robust tracking of the process outputs to the reference signals even in the presence of model-plant mismatch.

Addition to the IMC control, this paper considers utilizing additive information on the parameter variation of the controlled process, depected in Figure 2.

In Figure 1., θ stands for the parameters of process model. In the conventional IMC control, it takes much time to recover from the output delays because the input signals are modified just after the output errors are observed. On the other hand, in the proposed IMC, it can be expected that we can get better performance because the information on the parameters variation would be utilized effectively.

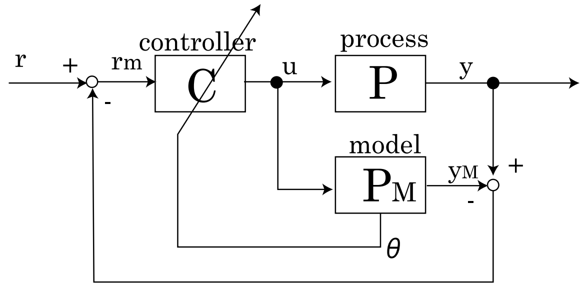

Fig. 2. The Block Diagram of the Proposed IMC

However, the conventional controller requires recalculation of the inverse system of the MPL system according as the parameters changes because the relation between controller's parameters and the MPL system's parameters is not represented explicitly. Therefore, this paper utilizes the inverse system of LPV-MPL systems (Masuda *et al.*, 2002). Since the system's parameters are explicitly represented in the LPV-MPL systems, the additive information on the parameters variations can be utilized effectively when the parameters are measured on-line, or the variation of the parameters are scheduled beforehand.

5. THE INVERSE SYSTEM FOR LPV-MPL

As is shown in **4.**, the controller of IMC systems is designed for the model, so we will give the model equation of LPV-MPL system besides the real process model (9) and (10).

$$\boldsymbol{x}_M(k+1) = \boldsymbol{A}_M(\boldsymbol{d}_M)\boldsymbol{x}_M(k) \oplus \boldsymbol{B}_M(\boldsymbol{d}_M)\boldsymbol{u}(k+1) \quad (12)$$

$$\boldsymbol{y}_M(k) = \boldsymbol{C}_M(\boldsymbol{d}_M)\boldsymbol{x}_M(k) \quad (13)$$

This section gives the inverse system for the model equation of LPV-MPL system (Masuda *et al.*, 2002) in (12) and (13).

The first, let the predition equation be derived for the preparation of the inverse system. By using (9) and (10), we can get

$$\begin{bmatrix} y_{M1}(k+\delta_1+1) \\ \vdots \\ y_{Mq}(k+\delta_q+1) \end{bmatrix} = \boldsymbol{\Gamma}_M(\boldsymbol{d}_M)\boldsymbol{x}_M(k) \oplus \boldsymbol{\Delta}_M(\boldsymbol{d}_M)\boldsymbol{u}(k+1) \quad (14)$$

where

$$\boldsymbol{\Gamma}_M(\boldsymbol{d}_M) = \begin{bmatrix} \boldsymbol{c}_M^1(\boldsymbol{d}_M)\boldsymbol{A}_M(\boldsymbol{d}_M)^{\delta_1+1} \\ \vdots \\ \boldsymbol{c}_M^q(\boldsymbol{d}_M)\boldsymbol{A}_M(\boldsymbol{d}_M)^{\delta_q+1} \end{bmatrix}, \qquad (15)$$

$$\boldsymbol{\Delta}_M(\boldsymbol{d}_M) = \begin{bmatrix} \boldsymbol{c}_M^1(\boldsymbol{d})\boldsymbol{A}_M(\boldsymbol{d}_M)^{\delta_1}\boldsymbol{B}_M(\boldsymbol{d}_M) \\ \vdots \\ \boldsymbol{c}_M^q(\boldsymbol{d}_M)\boldsymbol{A}_M(\boldsymbol{d}_M)^{\delta_q}\boldsymbol{B}_M(\boldsymbol{d}_M) \end{bmatrix} \qquad (16)$$

$\boldsymbol{c}_M^h(\boldsymbol{d}_M)$, $h = 1 \cdots q$ is the h-th row vector of $\boldsymbol{C}_M(\boldsymbol{d}_M)$. δ_h are called the characteristic numbers (Boimond and Ferrier, 1996), which imply that δ_h-th outputs are firstly influenced after the k-th input, and they are defined as:

$$\begin{aligned} \boldsymbol{\varepsilon} &= \boldsymbol{c}_M^h(\boldsymbol{d}_M)\boldsymbol{B}_M(\boldsymbol{d}_M) = \boldsymbol{c}_M^h(\boldsymbol{d}_M)\boldsymbol{A}_M(\boldsymbol{d}_M)\boldsymbol{B}_M(\boldsymbol{d}_M) \\ &= \cdots = \boldsymbol{c}_M^h(\boldsymbol{d}_M)\boldsymbol{A}_M(\boldsymbol{d}_M)^{\delta_h-1}\boldsymbol{B}_M(\boldsymbol{d}_M) \qquad (17) \\ \boldsymbol{\varepsilon} &\neq \boldsymbol{c}_M^h(\boldsymbol{d}_M)\boldsymbol{A}_M(\boldsymbol{d}_M)^{\delta_h}\boldsymbol{B}_M(\boldsymbol{d}_M), \quad h = 1, \cdots q \qquad (18) \end{aligned}$$

$\boldsymbol{\varepsilon}$ is the vector whose elements are ε. When the desired reference signals are defined as

$$\boldsymbol{r}(k) = [r_1(k+\delta_1+1), \cdots, r_q(k+\delta_q+1)]^{\mathrm{T}},$$

it is considered that the control law for the inverse system should be satisfied with the following equation replaced the predicted output vector in (14) with the desired reference signals.

$$\boldsymbol{r}(k) = \boldsymbol{\Gamma}_M(\boldsymbol{d}_M)\boldsymbol{x}_M(k) \oplus \boldsymbol{\Delta}_M(\boldsymbol{d}_M)\boldsymbol{u}(k+1) \quad (19)$$

(19) is considered to be a linear matrix equation in max-plus algebra. Hence, let the equation be solved based on the linear equation theory in dioid(Cohen *et al.*, 1989). According to the theory, after (19) is transformed into

$$\boldsymbol{\Delta}_M(\boldsymbol{d}_M)\boldsymbol{u}(k+1) = \boldsymbol{r}(k) \oplus \boldsymbol{\Gamma}_M(\boldsymbol{d}_M)\boldsymbol{x}_M(k) \quad (20)$$

the greatest subsolution of (20) is calculated. In (Masuda *et al.*, 2002), the following control law is introuduced for the inverse systems of LPV-MPL systems.

$$\boldsymbol{u}(k+1) = \bigwedge_{i=1}^{I} \Big\{ \boldsymbol{\Delta}_i^{\mathrm{T}} \boldsymbol{N}_i(\boldsymbol{d}_M) \odot \Big(\boldsymbol{r}(k) \oplus \bigoplus_{i=1}^{I} (\boldsymbol{M}_i(\boldsymbol{d}_M)\boldsymbol{\Gamma}_i)\boldsymbol{x}_M(k) \Big) \Big\} \quad (21)$$

Here,

$$\boldsymbol{\Gamma}_M(\boldsymbol{d}_M) = \bigoplus_{i=1}^{I} (\boldsymbol{M}_i(\boldsymbol{d}_M)\boldsymbol{\Gamma}_i) \qquad (22)$$

$$\boldsymbol{\Delta}_M(\boldsymbol{d}_M) = \bigoplus_{i=1}^{I} (\boldsymbol{N}_i(\boldsymbol{d}_M)\boldsymbol{\Delta}_i) \qquad (23)$$

where $I = (l+1)^{\bar{\delta}+2}$, $\bar{\delta} = \max_h \delta_h$ $\boldsymbol{\Gamma}_i$ and $\boldsymbol{\Delta}_i$,$i = 1, \cdots I$ are matrices whose elements are all ε and e. The size of $\boldsymbol{\Gamma}_i$ and $\boldsymbol{\Delta}_i$,$i = 1, \cdots I$ are the same as $\boldsymbol{\Gamma}$ and $\boldsymbol{\Delta}$, respectively. $\boldsymbol{M}_i(\boldsymbol{d})$ and $\boldsymbol{N}_i(\boldsymbol{d})$ are diagonal matrices.

Therefore, the control law for the proposed IMC can be obtained as

$$\boldsymbol{u}(k+1) = \bigwedge_{i=1}^{I} \Big\{ \boldsymbol{\Delta}_i^{\mathrm{T}} \boldsymbol{N}_i(\boldsymbol{d}_M) \odot \Big(\boldsymbol{r}_M(k) \oplus \bigoplus_{i=1}^{I} (\boldsymbol{M}_i(\boldsymbol{d}_M)\boldsymbol{\Gamma}_i)\boldsymbol{x}_M(k) \Big) \Big\} \quad (24)$$

$$\boldsymbol{r}_M(k) = \boldsymbol{r}(k) - (\boldsymbol{y}(k) - \boldsymbol{y}_M(k)) \qquad (25)$$

where $\boldsymbol{r}_M(k)$ is the modified reference signals.

The main feature of the control law (24) is that it explicitly includes the parameters of controlled MPL systems as free parameters. Hence, when the parameters are measured on-line, or the variation of the parameters are scheduled beforehand, the proposed control law can utilize the additive information on the parameters variations effectively.

6. A SIMULATION EXAMPLE

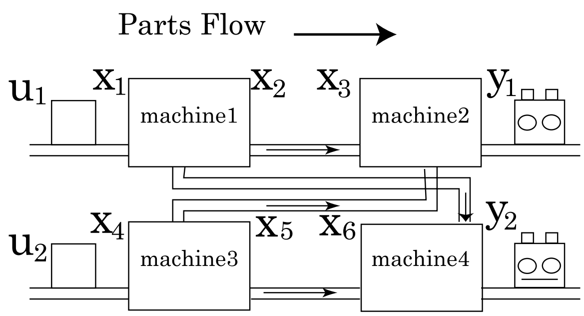

Fig. 3. Two-Inputs and Two-Outputs Production System

Consider a two-inputs, two-outpus production system with four machines depicted in Figure 3.. The inputs $u_i(k+1), i = 1, 2$ are defined as time instants at which the $k+1$-th manufactured parts are fed into the input stock in the line i. The outputs $y_i(k), i = 1, 2$ are time instants at which k-th finished products leaves the output stock in the line i. The state variables $x_1(k), x_3(k), x_4(k), x_6(k)$ are time instants at which k-th processing unit starts working in the machine 1, 2, 3 and 4, respectively. The state variables $x_2(k)$ and $x_5(k)$ are time instants at which k-th processing unit finished working in the machine 1 and 3, respectively. $d_i, i = 1, \cdots, 4$ are the working time in the machine 1, 2, 3 and 4, respectively.

The working times for each machine are $d_1 = 0.7$, $d_2 = 0.4$, $d_3 = 0.3$, $d_4 = 0.6$ for the first 15 parts, but the working times are changed into $d_1 = 1.0$,

$d_2 = 0.9$, $d_3 = 1.2$, $d_4 = 1.2$ after 16-th parts. It is assumed that the information on the parameter variations are given beforehand. Namely, the model parameter $d_{Mi} = d_i, \quad i = 1, \cdots, 4$. However, after 16-th parts, the information on d_3 has error, so the model parameter d_{M3} is assumed to be set to $d_{M3} = 0.8$.

The reference signal is given as follows

$$\begin{aligned} r_1(i+1) &= r_1(i) + 1.6, \quad r_2(i+1) = r_2(i) + 1.4 \\ & \qquad 0 \le i \le 9 \\ r_1(i+1) &= r_1(i) + 1.5, \quad r_2(i+1) = r_2(i) + 1.5 \\ & \qquad 10 \le i \le 30 \end{aligned}$$

Then, the proposed control law is applied to the production system. The following control law can be derived as is designed in section 4.

$$\boldsymbol{u}(k+1) = \bigwedge_{i=1}^{2} \Big\{ \boldsymbol{\Delta}_i^{\mathrm{T}} \boldsymbol{N}_i(\boldsymbol{d}_M) \odot \Big(\boldsymbol{r}_M(k) \oplus \bigoplus_{i=1}^{4} (\boldsymbol{M}_i(\boldsymbol{d}_M)\boldsymbol{\Gamma}_i)\boldsymbol{x}_M(k) \Big) \Big\} \tag{26}$$

$$\boldsymbol{r}_M(k) = \boldsymbol{r}(k) - (\boldsymbol{y}(k) - \boldsymbol{y}_M(k)) \tag{27}$$

where

$$\boldsymbol{N}_1(\boldsymbol{d}_M) = \begin{bmatrix} d_{M1} + d_{M2} & \varepsilon \\ \varepsilon & d_{M3} + d_{M4} \end{bmatrix},$$

$$\boldsymbol{N}_2(\boldsymbol{d}_M) = \begin{bmatrix} d_{M2} + d_{M3} & \varepsilon \\ \varepsilon & d_{M1} + d_{M4} \end{bmatrix},$$

$$\boldsymbol{M}_1(\boldsymbol{d}_M) = \begin{bmatrix} d_{M1} + d_{M2} & \varepsilon \\ \varepsilon & d_{M1} + d_{M4} \end{bmatrix}$$

$$\boldsymbol{M}_2(\boldsymbol{d}_M) = \begin{bmatrix} 2d_{M2} & \varepsilon \\ \varepsilon & \varepsilon \end{bmatrix}$$

$$\boldsymbol{M}_3(\boldsymbol{d}_M) = \begin{bmatrix} d_{M2} + d_{M3} & \varepsilon \\ \varepsilon & d_{M3} + d_{M4} \end{bmatrix}$$

$$\boldsymbol{M}_4(\boldsymbol{d}_M) = \begin{bmatrix} \varepsilon & \varepsilon \\ \varepsilon & 2d_{M4} \end{bmatrix}$$

$$\boldsymbol{\Delta}_1 = \begin{bmatrix} e & \varepsilon \\ \varepsilon & e \end{bmatrix}, \quad \boldsymbol{\Delta}_2 = \begin{bmatrix} \varepsilon & e \\ e & \varepsilon \end{bmatrix}$$

$$\boldsymbol{\Gamma}_1 = \begin{bmatrix} \varepsilon & e & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & e & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{bmatrix}, \quad \boldsymbol{\Gamma}_2 = \begin{bmatrix} \varepsilon & \varepsilon & e & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{bmatrix}$$

$$\boldsymbol{\Gamma}_3 = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & e & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & e & \varepsilon \end{bmatrix}, \quad \boldsymbol{\Gamma}_4 = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & e \end{bmatrix}$$

The simulation results are shown in Figure 4..

In Figure 4., the output errors, which imply $\boldsymbol{e}(k) = \boldsymbol{y}(k) - \boldsymbol{r}(k)$ with using both IMC control law and the information on the parameter variation are shown.

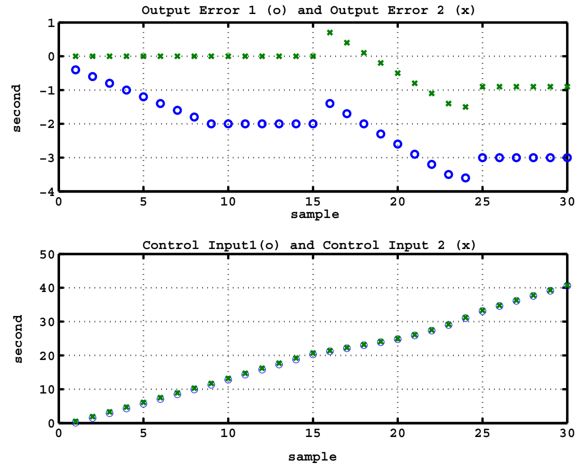

Fig. 4. Plots of the 1st output error (o) and the 2nd output error (x) with using both IMC control law and the information on the parameter variations (above) and the plots of the control input (below)

For the comparison, the output errors with only using the information on the parameter variation are shown in .

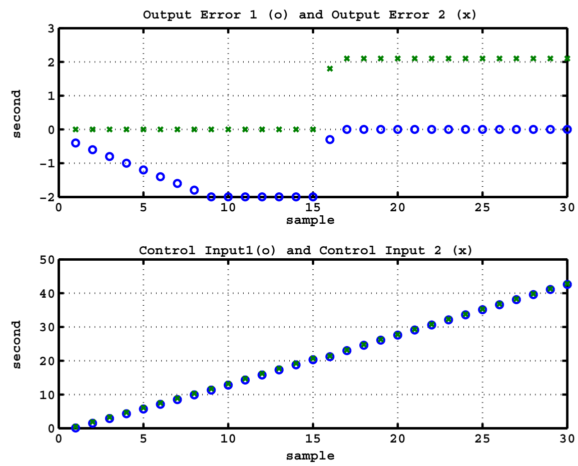

Fig. 5. Plots of the 1st output error (o) and the 2nd output error (x) with only using IMC control law (without using the information on the parameter variations) (above) and the plots of the control input (below)

From Figure 4., in the case with using both IMC control law and the information on the parameter variation, the output delays, which mean the output errors are positive value, does not occur except during 3 samples after 16-th sample at which the working time is changed. From Figure 5. and Figure 6., however, the output delays occur after the 16-th sample due to the change of the working time when either IMC control law or the information on the parameter variation is not utilized.

Therefore, it follows from the simulation result that the utilization of both IMC control law and the information on the parameter variations improves the performance of the control system.

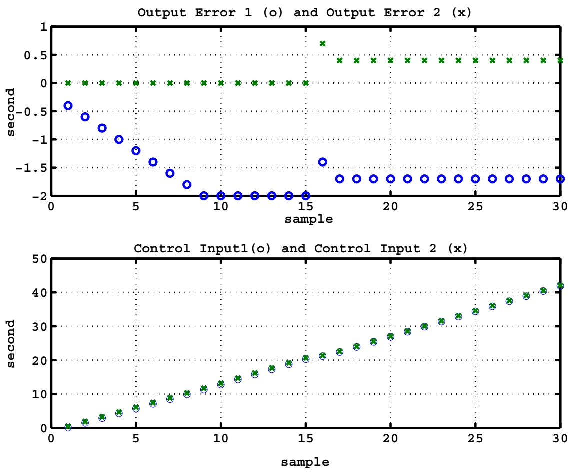

Fig. 6. Plots of the 1st output error (o) and the 2nd output error (x) with only using the information on the parameter variations (without using both IMC control law) (above) and the plots of the control input (below)

Therefore, we can see that the proposed control law shows better performance than the conventional IMC control law and the inverse systems for LPV-MPL systems with using the information on the parameter variations.

7. CONCLUDING REMARKS

This paper proposed the IMC control for MPL systems in the case where the controlled systems are given as MPL systems with linear parameter varying structure, which is called LPV-MPL systems. In the LPV-MPL systems, the systems parameters are explicitly represented in the systems description. Hence, the obtained IMC control law can utilize the additive information on the parameters variations effectively when the parameters are measured on-line, or the variation of the parameters are scheduled beforehand.

Furthermore, owing to the IMC control law, the proposed method has robust property even when the the information on the parameters variations has some errors.

The effectiveness of the proposed IMC is shown through a numerical example where it is applied to a two-inputs, two-outpus production system with four machines.

8. REFERENCES

Baccelli, F., G. Cohen, G. J. Olsder and J. P. Quadrat (1992). *Synchronization and Linearity*. New York: Jhon Wiley & Son.

Boimond, J. L. and J. L. Ferrier (1996). Internal model control and max-algebra: controller design. *IEEE Trans. on Automatic Control* **AC-41**(3), 457–461.

Cohen, G., P. Moller, J. P. Quadrat and M. Viot (1989). Algebraic tools for the performance evaluation of discrete event systems. *Proceedings of the IEEE* **77**(1), 39–58.

Masuda, S., H. Goto, T. Amemiya and K. Takeyasu (2002). An inverse system for linear parameter-varying max-plus-linear systems. In: *Proceedings of IEEE 2002 Conference on Decision and Control*.

van den Boom, T. and B. De Schutter (2001*a*). Model predictive control for max-plus-linear systems. *Automatica* **37**(7), 1049–1056.

van den Boom, T. and B. De Schutter (2001*b*). Mpc for perturbed max-plus-linear systems. In: *Proceedings of the European Control Conference 2001*. pp. 3783–3788.