HYBRID CONTROL: IMPLEMENTING OUTPUT FEEDBACK MPC WITH GUARANTEED STABILITY REGION [1]

Prashant Mhaskar, Nael H. El-Farra and Panagiotis D. Christofides

Department of Chemical Engineering
University of California, Los Angeles, CA 90095-1592

Abstract: In this work, a hybrid control scheme that employs switching between bounded control and model predictive control (MPC) is proposed for the output feedback stabilization of linear time-invariant systems with input constraints. Initially, we design a bounded output feedback controller for which the region of constrained closed-loop stability is explicitly characterized and an MPC controller that minimizes a given performance objective subject to constraints. Switching laws are derived to orchestrate the transition between the two controllers in a way that reconciles their respective stability and optimality properties, and guarantees asymptotic closed-loop stability for all initial conditions within the stability region of the bounded controller. The hybrid scheme is shown to provide a safety net for the practical implementation of output feedback MPC by providing *a priori* knowledge, through off-line computations, of a large set of initial conditions for which closed-loop stability is guaranteed. The proposed hybrid control approach is illustrated through a simulation example.

Keywords: Hybrid control, Bounded control, MPC, State observers, Input constraints.

1. INTRODUCTION

1.1 *Classical vs. hybrid control*

The conventional, or “classical", approach to control has been that of modelling the process (up–to the requisite level of detail) and then designing an appropriate controller to achieve the desired control objective. The salient feature of the classical approach is the absence of a discrete component in the control structure.

In contrast, a hybrid control structure (Figure 1) involves, by design, a blend of continuous (i.e., the classical controllers) and discrete components (i.e., the logic–based supervisor that switches between the controllers). The controllers within the control block could be of similar structure (but with different gains or parameters), or could be structurally different (for example, an analytic and a model predictive controller). The switching between multiple classical controllers is orchestrated by the supervisor for the purpose of either meeting an objective that cannot be achieved by the individual controllers or to reconcile the different (complementary) strengths and capabilities of different control approaches.

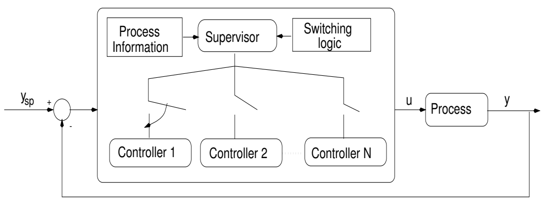

Fig. 1. A schematic representation of a hybrid control structure.

The general idea of hybrid control, manifested through switching between different controllers, has been used in the literature in a variety of contexts. Examples include gain–scheduling (e.g., see (Rugh and Shamma, 2000)) as a tool for control of nonlinear systems, multiple linear models for transition control (e.g., see

[1] Financial support from NSF, CTS-0129571, is gratefully acknowledged

(Banerjee and Arkun, 1998; Sun and Hoo, 1999)) and scheduled predictive control (e.g., see (Aufderheide *et al.*, 2001)) of nonlinear processes. A recurrent theme in most of the work on hybrid control has been that of switching between different models (which results in an implicit switching between different controllers), or that of using multiple *structurally similar* controllers.

In (El-Farra *et al.*, 2002), we proposed a hybrid control structure, that employs switching between two *structurally different* controllers – an MPC controller and a bounded analytical controller – for the state feedback stabilization of linear systems with input constraints. The bounded controller was used to provide a safety net for the implementation of MPC within a well–defined region of guaranteed stability. The proposed hybrid control structure was extended to address the problems of robust control of linear systems with uncertainties (El-Farra *et al.*, 2003*b*) and constrained stabilization of nonlinear systems (El-Farra *et al.*, 2003*a*). In this work, a hybrid control scheme, uniting bounded control with MPC, is proposed for the output feedback stabilization of linear time-invariant systems with input constraints.

1.2 *Background*

Input constraints arise as a manifestation of the physical limitations inherent in the capacity of control actuators (e.g., bounds on the magnitude of valve opening). Input constraints automatically impose limitations on our ability to steer the dynamics of the closed-loop system at will, and can cause severe deterioration in the nominal closed-loop performance and may even lead to closed-loop instability if not explicitly taken into account at the stage of controller design.

One of the key limitations imposed by input constraints is the restriction on the set of initial states of the closed-loop system that can be steered to the origin with the available control action. The absence of an *a priori* explicit characterization of this set (or an appropriate estimate thereof) can have an impact on the practical implementation of the given control policy by requiring extensive closed-loop simulations over the whole set of possible initial conditions, to check for closed-loop stability, or by limiting operation within an unnecessarily small and conservative neighborhood of the desired equilibrium point. These considerations have motivated significant work on the design of stabilizing bounded control laws that provide explicitly-defined, large regions of attraction for the closed-loop system (e.g., see (Lin and Sontag, 1991; Teel, 1992; El-Farra and Christofides, 2001; El-Farra and Christofides, 2003)).

Currently, MPC, also known as receding horizon control, is a widely used control method for handling constraints within an optimal control setting. Within MPC, the control action is obtained by solving repeatedly, on-line, a finite-horizon constrained open-loop optimal control problem. The industrial success of MPC has spurred numerous research investigations into the stability properties of MPC controllers and led to a plethora of MPC formulations that focus on closed-loop stability (e.g., see (Rawlings and Muske, 1993; Allgower and Chen, 1998; Mayne *et al.*, 2000) for extensive surveys of these developments). The significant progress in characterizing the stability properties of MPC notwithstanding, the issue of obtaining, *a priori* (i.e. before controller implementation), an analytic characterization of the region of constrained closed-loop stability for MPC controllers remains to be adequately addressed. This difficulty can have an impact on the practical implementation of MPC by requiring extensive closed-loop simulations over the whole set of possible initial conditions to check for closed-loop stability, or by potentially limiting operation within an unnecessarily small neighborhood of the nominal equilibrium point.

In addition to the problem of input constraints, the problem of output feedback stabilization of constrained systems has been the subject of numerous research studies. Examples include scalar output feedback control of linear systems (Shamma and Tu, 1998), stability analysis of a composite system comprising of a moving horizon regulator and a moving horizon observer for control of nonlinear systems (Michalska and Mayne, 1995) and moving horizon estimation as an extension of Kalman filtering, for constrained and nonlinear processes (Rao and Rawlings, 2002). In these works, however, the stability region of the constrained closed–loop system is not explicitly characterized.

Motivated by the above considerations, we propose in this paper a controller switching strategy that extends the hybrid control structure in (El-Farra *et al.*, 2002) to the case of output feedback. The guiding principle in realizing this strategy is that of using a suitable state observer design which, in conjunction with the bounded controller, yields an explicitly characterized stability region within which the operation of the MPC controller can be embedded by devising suitable switching rules (see (Mhaskar *et al.*, 2003) for a detailed analysis of the theoretical issues involved and the mathematical proofs of the results). The rest of the paper is organized as follows: in section 2, we present some preliminaries that describe the class of systems considered and review briefly the methodology of designing the state observer, and how the constrained control problem is addressed in both bounded control and model predictive control. In section 3, we formulate the controller switching problem under output feedback and propose a switching scheme that addresses the problem. Finally, in section 4, numerical simulations are presented to demonstrate the implementation of the switching scheme and test the robustness of the proposed approach with respect to measurement noise.

2. PRELIMINARIES

In this work, we consider the problem of output feedback stabilization of continuous-time linear time-invariant (LTI) systems with input constraints, with the following state-space description:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \\ u(t) &\in \mathcal{U} \subset \mathbb{R}^m \end{aligned} \quad (1)$$

where $x = [x_1, \cdots, x_n]' \in \mathbb{R}^n$ denotes the vector of state variables, $y = [y_1, \cdots, y_k]' \in \mathbb{R}^k$ denotes the vector of output variables, $u = [u_1, \cdots, u_m]'$ is the vector of manipulated inputs, taking values in a compact and convex subset, $\mathcal{U}$, of $\mathbb{R}^m$ that contains the origin in its interior. The matrices A, B and C are constant $n \times n$, $n \times m$ and $k \times n$ matrices, respectively. The pairs (A, B) and (C, A) are assumed to be controllable and observable, respectively. Throughout the paper, the notation x' denotes the transpose of x.

2.1 *State observer design*

For the system of Eq.1, we use a standard Luenberger observer described by

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}) \quad (2)$$

where $\hat{x} = [\hat{x}_1, \cdots, \hat{x}_n]' \in \mathbb{R}^n$ denotes the vector of estimates of the state variables, L is a constant $n \times k$ matrix chosen such that the eigenvalues of $A - LC$ are placed at $-\beta a_1$, $-\beta a_2 ... -\beta a_n$ with $\beta \geq 1$ and $a_i \neq a_j \geq 1$. In the closed–loop system, the estimation error, defined as $e = x - \hat{x}$, evolves independently of the controller according to $\dot{e} = (A - LC)e$

Note that the dynamics of the error equation can be manipulated at will by appropriate choice of the design parameters a_i and β. This state estimator design guarantees convergence of the error in a way that for larger values of the parameter β, the error decreases faster (i.e., given any $e_m > 0$, one can find a T_d such that $\|e(t)\| \leq e_m \ \forall \ t \geq T_d$). However, for larger values of β, the error could possibly increase to large values before eventually decaying. The design, therefore, includes the possibility of "peaking" of state estimates, where the observer generates incorrect estimates for short times. This, however, does not pose a problem in our design because the physical constraints on the manipulated input prevent transmission of the incorrect estimates to the plant.

2.2 *Model predictive control*

For the sake of illustration, we consider here the following nominally-stabilizing finite-horizon MPC formulation with terminal equality constraints:

$$\begin{aligned} J_s(x, t, u(\cdot)) &= \int_t^{t+T} (x'(s)Qx(s) + u'(s)Ru(s))ds \\ u(\cdot) &= argmin\{J_s(x, t, u(\cdot)) | u(\cdot) \in S\} \\ s.t. \ \ \dot{x}(t) &= Ax(t) + Bu(t), \ \ x(0) = x_0 \\ u(\cdot) &\in S, \ \ x(t+T) = 0 \end{aligned} \quad (3)$$

where $S = S(t, T)$ is the family of piecewise continuous functions, with period Δ, mapping $[t, t+T]$ into the set of admissible controls, where T is the horizon length. A control $u(\cdot)$ in S is characterized by the sequence $\{u[k]\}$ where $u[k] := u(k\Delta)$. A control $u(\cdot)$ in S satisfies $u(t) = u[k]$ for all $t \in [k\Delta, (k+1)\Delta)$. J_s is the performance index and R and Q are strictly positive definite, symmetric matrices. Feasibility of the formulation in Eq.3 can be ensured by relaxing the terminal equality constraint; however, closed loop stability then cannot be guaranteed.

One of the issues that arise in the implementation of MPC formulations of the form of Eq.3 is the difficulty in obtaining an explicit characterization of the stability region, which depends on a complex interplay between several factors, including the constraints, the initial condition, and the horizon length. Faced with these difficulties, the current industrial implementation of MPC relies heavily on extensive simulations to test the stability of MPC controllers.

2.3 *Bounded Lyapunov-based control*

Consider the Lyapunov function candidate $V = x'Px$, where P is a positive-definite symmetric matrix that satisfies the Riccati equation

$$A'P + PA - PBB'P = -\bar{Q} \quad (4)$$

for some positive-definite matrix $\bar{Q}$. Using this Lyapunov function, we can construct, using a modification of Sontag's formula for bounded controls proposed in (Lin and Sontag, 1991) (see also (El-Farra and Christofides, 2003)), the following bounded nonlinear controller

$$u(x) = -2k(x)B'Px := b(x) \quad (5)$$

where $k(x) =$

$$\left(\frac{L_f^*V + \sqrt{\left(L_f^*V\right)^2 + (u_{max}\|(L_gV)'\|)^4}}{\|(L_gV)'\|^2 \left[1 + \sqrt{1 + (u_{max}\|(L_gV)'\|)^2}\right]} \right)$$

with $L_f^*V = x'(A'P + PA)x + \rho x'Px$, $(L_gV)' = 2B'Px$, $\rho > 0$. One can show that whenever the closed-loop state trajectory evolves within the state–space region described by the set:

$$\Phi(u_{max}) = \{x \in \mathbb{R}^n : L_f^*V \leq u_{max}\|(L_gV)'\|\} \quad (6)$$

the resulting control action respects the constraints (i.e., $\|u\| \leq u_{max}$) and asymptotically stabilizes the origin of the closed-loop system. Note that the size of the set Φ depends on the magnitude of the constraints in a way such that the tighter the constraints, the smaller the region described by this set. Using this set, an estimate of the stability region of the controller of Eqs.5–6 can be obtained by considering an invariant subset of Φ, preferably the largest, which we denote by $\Omega(u_{max})$. A common way of doing this is using the level sets of V.

Using Lyapunov arguments, one can derive bounds on the estimation errors (e_m), with respect to which the state feedback bounded controller ensures that the closed–loop state trajectory does not escape the state feedback stability region (see Figure 3). By initializing the states and the state estimates sufficiently inside $\Omega(u_{max})$ and choosing a consistent observer gain matrix L, one can ensure that the norm of the estimation error decays to a value less than the tolerable measurement error before the states have a chance to reach the boundary of the state feedback stability region. For a given choice of $\Omega_b \subset \Omega(u_{max})$, therefore, one can choose a value β for the observer gain parameter that guarantees stability for all initial conditions within Ω_b under output feedback control.

3. IMPLEMENTING OUTPUT FEEDBACK MPC WITH GUARANTEED STABILITY REGION

While the bounded controller possesses a well-defined region of initial conditions that guarantee closed-loop stability in the presence of constraints, the performance of this controller is not guaranteed to be optimal with respect to an arbitrary performance criterion. On the other hand, the MPC controller is well-suited for handling constraints within an optimal control setting; however, the analytical characterization of its set of initial conditions, for which closed–loop stability is guaranteed, is a more difficult task than it is through bounded control. The lack of state measurements introduces another level of complexity in implementing the controllers designed with the assumption of state feedback. In this section, we show how to reconcile the two control approaches by means of a switching scheme that combines the desirable properties of both approaches.

3.1 *Problem formulation and overview of solution*

Consider the linear time-invariant system of Eq.1, subject to input constraints $\|u\| \leq u_{max}$, and for which the observer of Eq.2, the bounded controller of Eqs.5-6 and the MPC controller of Eq.3 have been designed. We formulate the control problem as that of designing a set of switching laws that orchestrate the transition between the MPC controller and the bounded controller under output feedback in a way that guarantees asymptotic stability of the origin of the closed-loop system starting from any initial condition in an explicitly characterized set $\Omega_b \subset \Omega(u_{max})$, respects input constraints, and accommodates the optimality requirements whenever possible. In the remainder of this section, we present a switching scheme that addresses the problem.

3.2 *Controller switching scheme*

The four main components of the hybrid control structure include the observer, the bounded controller, the MPC controller, and a higher-level supervisor that orchestrates the switching between the two controllers. A schematic representation of this structure is shown in Figure 2. The design procedure for the hybrid control structure and the implementation of the controller switching scheme is as follows:

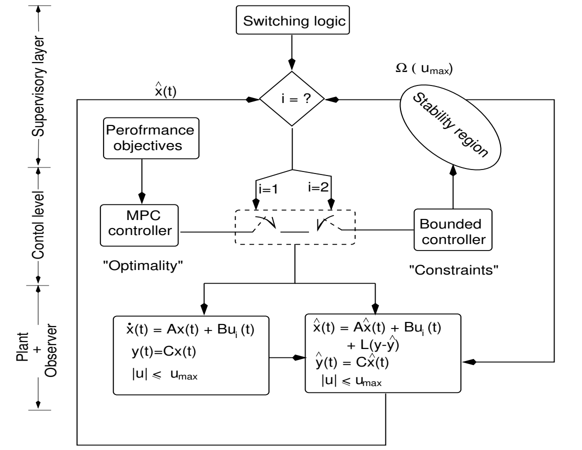

Fig. 2. A schematic representation of the hierarchical hybrid control structure merging the bounded and MPC controllers under output feedback

(1) Given the system of Eq.1 and the performance objective, design the bounded controller and the MPC controller.
(2) Compute the stability region estimate for the bounded controller under state feedback, $\Omega(u_{max})$, using Eq.6 and, for the state observer design, choose a β consistent with the choice of the output feedback stability region Ω_b.
(3) Compute the region $\Omega_s \subset \Omega_b$ and T_d such that if the norm of the error is less than a given tolerance, then $\hat{x} \in \Omega_s \Rightarrow x \in \Omega_b$ for all times greater than T_d.
(4) Initialize the closed–loop system at an initial condition, $x(0)$ within Ω_b, under the bounded controller using an initial guess for the state $\hat{x}(0)$ within Ω_b.
(5) After a time T_d, once $\hat{x} \in \Omega_s$, test the feasibility of the MPC controller using values of the estimates generated by the state observer.
(6) If the MPC controller is feasible, implement it for as long as $\hat{x} \in \Omega_s$ and $V(\hat{x})$ keeps decaying, else switch to the bounded controller.

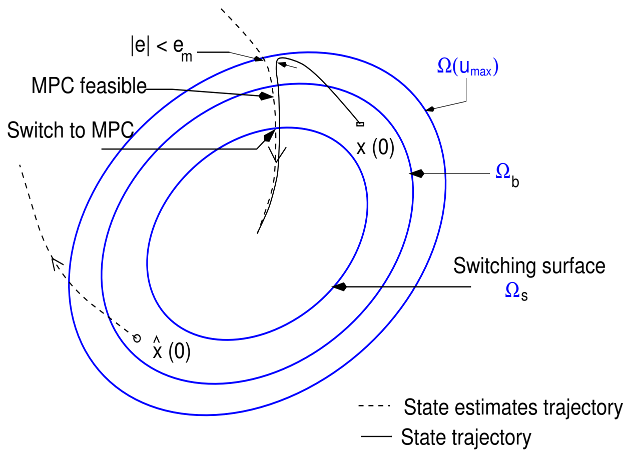

Fig. 3. A schematic representation of the implementation of the proposed controller switching scheme.

Remark 1: Figure 3 shows a representative sketch of the closed–loop system under the controller switching scheme. The states are initialized at x_0 while the state estimates are initialized at $\hat{x}_0$. The state estimator design ensures that the norm of the error is under the allowable error before (and if) the state trajectory reaches the boundary of the state feedback stability region, $\Omega(u_{max})$. After a time T_d (by which time, the state estimator design ensures that the estimation error has gone down to a small value), the supervisor implements MPC in closed–loop only if it is feasible *and* the state estimates are in Ω_s, while monitoring the evolution of the Lyapunov function value. If the switching rules are satisfied, MPC is implemented in closed–loop for the remaining time, else the supervisor switches back to the bounded controller to stabilize the closed–loop system.

Remark 2: For a given choice of the output feedback stability region, an estimate of the necessary observer gain can be obtained; however, this estimate is typically conservative. In practice, having chosen Ω_s, one can choose a 'sufficiently' large gain based on simulations or experience. The stability region under output feedback Ω_b can be made as close to the one under state feedback, Ω as desired by increasing the gain parameter β.

4. A NUMERICAL EXAMPLE

In this section, we demonstrate an application of the proposed hybrid control structure to a three dimensional linear system where only two of the states are measured. Specifically, we consider an exponentially unstable linear system of the form of Eq.1 with $A = \begin{bmatrix} 0.55 & 0.15 & 0.05 \\ 0.15 & 0.40 & 0.20 \\ 0.10 & 0.15 & 0.45 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where both inputs u_1, u_2 are constrained in the interval $[-1, 1]$. We initially used Eqs.5 to design a bounded controller and construct its stability region via Eq.6. The matrix P was chosen as:

$$P = \begin{bmatrix} 6.5843 & 4.2398 & -3.830 \\ 4.2398 & 3.6091 & -2.667 \\ -3.830 & -2.667 & 2.8033 \end{bmatrix} \quad (7)$$

and the observer gain parameter was chosen to be $\beta = 500$ to ensure closed–loop stability for all initial conditions within $\Omega_b \subset \Omega$. For the MPC controller, the parameters in the objective function of Eq.3 were chosen as $Q = qI$, with $q = 1$ and $R = rI$, with $r = 0.1$. We also chose a horizon length of $T = 1.5$ in implementing the MPC controller of Eq.3. The resulting quadratic program was solved using the MATLAB subroutine QuadProg, and the set of ODEs integrated using the MATLAB solver ODE45.

In the first simulation run (solid lines in Figs.4–5), the states were initialized at $x_0 = [0.75 - 0.5\ 1.0]'$ while the observer states were initialized at $\hat{x}_0 = [0\ \ 0\ \ 0]'$ (which belong to the stability region of the bounded controller, Ω_b). The supervisor employs the bounded controller, while continuously checking MPC feasibility. At $t = 5.45$, the MPC becomes feasible and is implemented in the closed-loop to stabilize the system. Note that feasibility of MPC can be achieved by increasing the horizon length to $T = 3.5$ (dashed lines in Figs.4–5). However, this conclusion could not be reached *a priori*, i.e. before running the closed-loop simulation in its entirety to check whether the choice $T = 3.5$ is appropriate. In contrast, closed–loop stability starting from the given initial condition under the proposed hybrid control structure is guaranteed.

In the second set of simulation results, we demonstrate the need for a choice of observer gain consistent with the choice of Ω_b. To this end, we consider now an observer design with a low gain ($\beta = 0.5$). With the low observer gain, the estimates take a long time to converge to the true state values, resulting in the implementation of 'incorrect' control action, by which time, even though the states and state estimates are initiated within Ω_b, the states have escaped $\Omega(u_{max})$, thereby resulting in closed–loop instability (dotted lines in Figs.4–5).

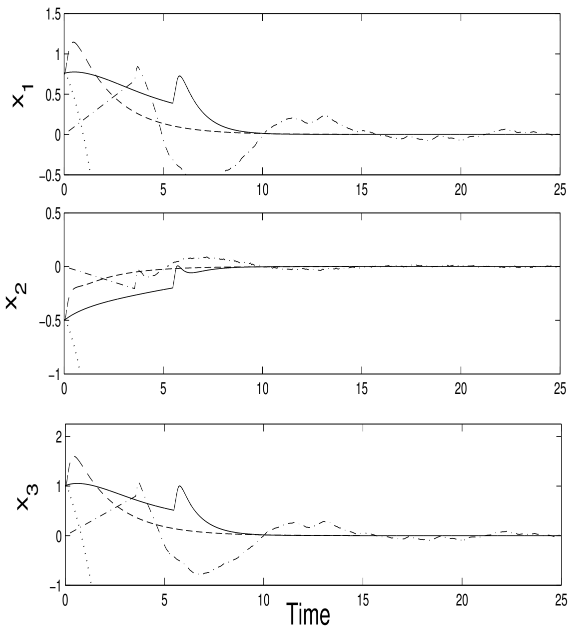

Fig. 4. Closed-loop state trajectory with $T = 1.5$ (solid line), $T = 3.5$ (dashed line), with the low observer gain (dotted line) and using observer switching (dash-dotted lines).

To recover, as closely as possible, the state feedback stability region, large values of the observer gain are needed. However, it is well known that high observer gains can amplify measurement noise and induce poor performance. This points to a fundamental tradeoff that cannot be resolved by simply changing the estimation scheme. For example, if the observer gain consistent with the choice of the output feedback stability region is abandoned, the noise problem may disappear, but then stability cannot be guaranteed. One approach to avoid this scenario in practice is to initially use an observer gain to ensure quick decay of the

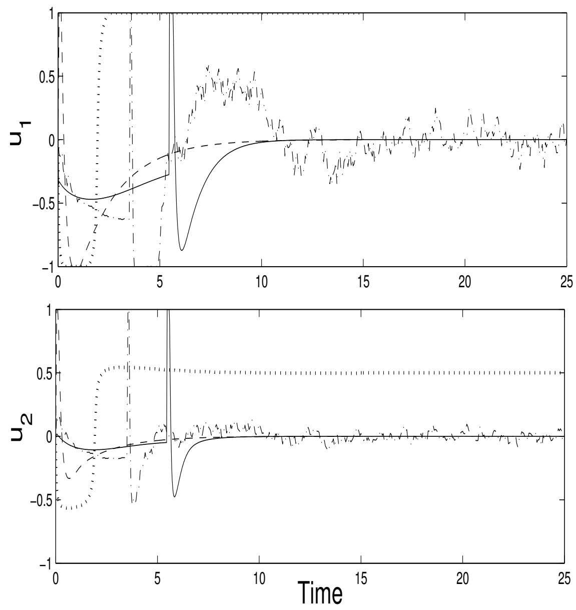

Fig. 5. Manipulated input trajectory with $T = 1.5$ (solid line), $T = 3.5$ (dashed line), with the low observer gain (dotted line) and using observer switching (dash-dotted lines).

initial estimation error, and then to switch to a “low" observer gain. In the following simulation, we show how switching between an observer with a high gain and a low gain in conjunction with switching between controllers can be used to mitigate the undesirable effects of measurement noise. To illustrate the point, we use switching between the high and low observer gains used in the first two simulation runs and demonstrate the attenuation of noise.

Specifically, we consider the nominal system described by Eq.1, together with model uncertainty and measurement noise. The model matrix A_m is assumed to be within five percent error of the process matrix A and the sensors are assumed to introduce noise in the measured outputs as $y(t) = Cx(t) + \delta(t)$ where $\delta(t)$ is a random gaussian noise with zero mean and a variance of 0.01. As seen by the dash-dotted lines in Fig.4, starting from the initial condition, $x_0 = [0.75 - 0.5\ 1.0]'$, using a high observer gain followed by a switch to the low gain observer at $t = 1.0$, and a switch from bounded control to MPC at $t = 3.5$, the supervisor is still able to preserve closed–loop stability, while at the same time resulting in a smooth enough control action (see Fig.5).

5. REFERENCES

Allgower, F. and H. Chen (1998). Nonlinear model predictive control schemes with guaranteed stability. In: R. Berber and C. Kravaris (Eds.)*, NATO ASI on Nonlinear Model Based Process Control*. Dordrecht: Kluwer. pp. 465–494.

Aufderheide, B., V. Prasad and B. W. Bequette (2001). A comparison of fundamental model-based and multiple model predictive control. In: *Proceedings of 40th IEEE Conference on Decision and Control*. Orlando, FL. pp. 4863–4868.

Banerjee, A. and Y. Arkun (1998). Model predictive control of plant transitions using a new identification technique for interpolating nonlinear models. *J. of Process Contr.* **8**, 441–457.

El-Farra, N. H. and P. D. Christofides (2001). Integrating robustness, optimality and constraints in control of nonlinear processes. *Chem. Eng. Sci.* **56**, 1841–1868.

El-Farra, N. H. and P. D. Christofides (2003). Bounded robust control of constrained multivariabe nonlinear processes. *Chem. Eng. Sci.,* to appear.

El-Farra, N. H., P. Mhaskar and P. D. Christofides (2002). Uniting bounded control and MPC for stabilization of constrained linear systems, accepted. *Automatica*.

El-Farra, N. H., P. Mhaskar and P. D. Christofides (2003*a*). Hybrid control: A paradigm for implementing predictive control to nonlinear systems with guaranteed stability regions. In: *Proceedings of American Control Conference*. Denver, CO.

El-Farra, N. H., P. Mhaskar and P. D. Christofides (2003*b*). Hybrid control of uncertain process systems. In: *8th International Symposium on Process Systems and Engineering*. Kumming, P.R. China.

Lin, Y. and E. D. Sontag (1991). A universal formula for stabilization with bounded controls. *Syst. & Contr. Lett.* **16**, 393–397.

Mayne, D. Q., J. B. Rawlings, C. V. Rao and P. O. M. Scokaert (2000). Constrained model predictive control: Stability and optimality. *Automatica* **36**, 789–814.

Mhaskar, P., N. H. El-Farra and P. D. Christofides (2003). Hybrid predictive control of process systems, in preperation.

Michalska, H. and D. Q. Mayne (1995). Moving horizon observers and observer-based control. *IEEE Trans. Automat. Contr.* **40**, 995–1006.

Rao, C. V. and J. B. Rawlings (2002). Constrained process monitoring: Moving-horizon approach. *AIChE Journal* **48**, 97–109.

Rawlings, J. B. and K. R. Muske (1993). The stability of constrained receding horizon control. *IEEE Trans. Automat. Contr.* **38**, 1512–1516.

Rugh, W. J. and J. S. Shamma (2000). Research on gain scheduling. *Automatica* **36**, 1401–1425.

Shamma, J. S. and K. Tu (1998). Output feedback control for systems with constraints and saturations: Scalar control case. *Syst. & Contr. Lett.* **25**, 1–11.

Sun, D. and K. A. Hoo (1999). Dynamic transition control structure for a class of SISO nonlinear systems. *IEEE Trans. Contr. Syst. Tech.* **7**, 622–629.

Teel, A. (1992). Global stabilization and restricted tracking for multiple integrators with bounded controls. *Syst. & Contr. Lett.* **18**, 165–171.