A BMI-BASED DESIGN OF SWITCHED PID CONTROLLERS

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Abstract: This paper provides a design method for two-degrees-of-freedom PID controllers including switched PD compensator based on bilinear matrix inequalities (BMIs). Two design specifications based on $\mathcal{H}_2$ norm are formulated in BMIs, and PID parameters can be exactly obtained by solving the BMI problems via branch and bound algorithms. A set of PD compensators can be obtained simultaneously using proposing design method. The most effective parameter is selected out of the set of PD compensator based on the switching criterion which obtained from estimated system conditions using recursive least square algorithms. Numerical example is also shown.

Keywords: PID controllers

1. INTRODUCTION

PID controllers play a critical role in 80-90 percent of chemical process systems [1]. They are widely used because of their simple structures which consist of only three parameters, that is, proportional parameter, integral parameter, and derivative parameter. It is, however, difficult to tune those parameters practically since the process dynamics often change due to changes in operating conditions or various disturbances. We have to design controllers such that they have both robustness for changes in conditions of the systems and good tracking properties. PID controllers with one-degree-of-freedom can not have robustness and good tracking properties since they are contrary properties. In order to design the controller with robustness and good tracking properties, this paper deals with two-degrees-of-freedom PID control systems, which have a PID control system and a PD compensator.

The design of many conventional control systems has resulted in an optimization problem, which can be solved by numerical computation based on powerful computer support. One of the most useful tools is bilinear matrix inequality (BMI), which is a flexible framework for analysis and synthesis of control systems. Although checking the solvability of BMI problems is NP hard [2], it is not hard to obtain an exact solution of a BMI problem via branch and bound algorithms if it has a few parameters. Fortunately, a design problem of PID controller has only three parameters, so that we can design PID controller based on BMI.

This paper formulates the design problem of PID controllers with two-degrees-of-freedom as a BMI problem. The aim of the control design is to make the control system has both robustness and good tracking properties. In order to reduce the conservativeness of the control system, this paper deal with PD compensator which has switching structure. This switching structure is constructed from a system estimator using recursive least squares algorithms, the switching criterion based on stationary gain of the estimated system and a set of pre-specified PD parameters corresponding to the switching criterion.

This paper is organized as follows. The system description, problem formulations and the design method of PID controller with two-degrees-of-freedom based on BMI are given in Section 2. In Section 3, for more effective PD compensator, a switching structure based on adaptive control method is constructed. Section 4 provides branch and bound algorithms in order to obtain an exact solution of BMI problems. Finally, numerical simulation examples are presented in Section 5.

2. CONTROLLER DESIGN BESED ON BMI

2.1 System description

Consider a system described by the following continuous-time model:

$$G(s) = \frac{K_0}{1+Ts} e^{-Ls} \tag{1}$$

where K_0 expresses the system gain, T is the time-constant and L refers to the delay. By using the first order Padé approximation of the delay, the system is approximated as

$$G(s) \cong \frac{K_0}{1+Ts} \cdot \frac{1-\frac{Ls}{2}}{1+\frac{Ls}{2}} \tag{2}$$

Here, utilizing multiples of the sampling time period T_s in the equation (2), the continuous-time model is transformed to the following discrete-time model:

$$A(z^{-1})y(t) = z^{-1}B(z^{-1})u(t) + \frac{1}{\Delta}\xi(t) \tag{3}$$

where

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + a_2 z^{-2} \\ B(z^{-1}) &= b_0 + b_1 z^{-1} \end{aligned} \tag{4}$$

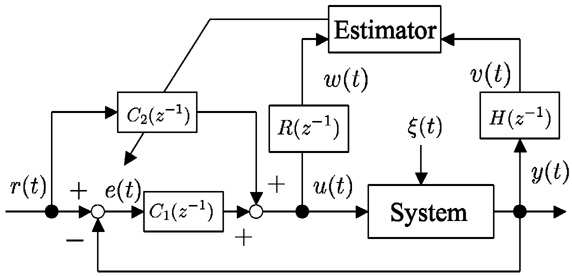

Fig. 1. Closed-loop system with two-degrees-of-freedom.

and $u(t)$, $y(t)$ and $\xi(t)$ denote the control input signal, the corresponding output signal and the stochastic noise, respectively. The operator z^{-1} denotes a backward shift, that is, $z^{-1}y(t) = y(t-1)$, and Δ denotes the differencing operator defined as $1 - z^{-1}$. This paper deals with the descrete-time model (3) as the control object instead of the continuous-model (1).

Next, consider the control system represented by the PID controller with two-degrees-of-freedom in Fig.1, where $r(t)$ and $e(t)$ refer to the reference signal and the control error, respectively. $H(z^{-1})$ and $R(z^{-1})$ denote a low pass filter, and where $C_1(z^{-1})$ and $C_2(z^{-1})$ denote the PID controller and the PD compensator, respectively. And they are given by

$$C_1(z^{-1}) = k_c + \frac{k_i}{\Delta} + \Delta k_d \tag{5}$$

$$C_2(z^{-1}) = -k_\alpha - \Delta k_\beta \tag{6}$$

The two-degrees-of-freedom PID controller in (5) and (6) includes five parameters: propotional gains k_c and k_α, integral gain k_i and derivative gains k_d and k_β. The one-degree-of-freedom PID controller $C_1(z^{-1})$ is required to satisfy the design specification for the system perturbation and the stochastic noise by using fixed PID parameters which are obtained from the BMI solution discussed in Section 4. And the PD compensator $C_2(z^{-1})$ which has a set of pre-specified PD parameters corresponding to the divided small perturbations, is required to satisfy the good tracking property by using switching structure based on the estimator discussed in Section 3.

2.2 Problem fomulation

This paper deals with the $\mathcal{H}_2$ norms which represent the integral squared errors (ISE) of the control system. They can evaluate the two design specifications which require the robustness for the control system and the tracking property for the reference signal. Moreover these evaluation measures result in the optimization problem which is represented by matrix inequalities.

First, we consider the error transfer function of the control system in Fig.1. In order to evaluate the tracking property for the step reference signal, $E_r(z^{-1})$ is defined as the transfer function from $r(t)$ to $e(t)$. Since a step input is given by $r(z^{-1}) = 1/(1-z^{-1})$ and $\xi(z^{-1}) = 0$, $E_r(z^{-1})$ can be expressed as

$$E_r(z^{-1}) = \frac{A(z^{-1}) - z^{-1}B(z^{-1})C_2(z^{-1})}{\Delta A(z^{-1}) + z^{-1}B(z^{-1})\Delta C_1(z^{-1})} \quad (7)$$

Similarly, in order to evaluate the influence of the stochastic noise $\xi(t)$, $E_d(z^{-1})$ is defined as the transfer function from $\xi(t)$ to $e(t)$. We assume that $\xi(z^{-1})$ is a white noise which is represented by $\xi(z^{-1}) = 1$ and $r(z^{-1}) = 0$, then $E_d(z^{-1})$ can be expressed as follows.

$$E_d(z^{-1}) = \frac{-1}{\Delta A(z^{-1}) + z^{-1}B(z^{-1})\Delta C_1(z^{-1})} \quad (8)$$

The ISE is described as

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} E\,(j\omega)E(-j\omega)\; d\omega \quad (9)$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} |\,E(j\omega)\,|^2\; d\omega \; < \; \gamma \quad (10)$$

where $E = E_r$ or E_d and γ is positive constant. Because the $\mathcal{H}_2$-norm of $E(z^{-1})$ is defined as

$$\parallel E \parallel_2 = \left(\frac{1}{2\pi}\int_{-\pi}^{\pi} |\,E(j\omega)\,|^2\; d\omega\right)^{\frac{1}{2}} \quad (11)$$

the performance measure based on ISE results in the following two inequalities.

$$\parallel E_r \parallel_2 < \sqrt{\gamma_r} \quad (12)$$
$$\parallel E_d \parallel_2 < \sqrt{\gamma_d} \quad (13)$$

The purpose of this paper is to minimize γ_r in (12) for a given $\sqrt{\gamma_d}$.

In this paper, the error systems (7) and (8) are realized in the controllable canonical form as following equations.

$$E_r(z^{-1}) = C_{er}(zI - A_{er})^{-1}B_{er} + D_{er} \quad (14)$$
$$E_d(z^{-1}) = C_{ed}(zI - A_{ed})^{-1}B_{ed} + D_{ed} \quad (15)$$

where A_i, B_i, C_i and D_i ($i = er$ or ed) are given by the following matrices.

$$\begin{aligned} A_i &= A_{e0} + k_cA_{e1} + k_iA_{e2} + k_dA_{e3} \\ B_i &= [0 \quad 0 \quad 0 \quad 1]^T \\ C_{er} &= C_{er0} + k_cC_{er1} + k_iC_{er2} + k_dC_{er3} \\ &\qquad + k_\alpha C_{er4} + k_\beta C_{er5} \\ C_{ed} &= C_{ed0} + k_cC_{ed1} + k_iC_{ed2} + k_dC_{ed3} \\ D_{er} &= 1 \\ D_{ed} &= -1 \end{aligned} \quad (16)$$

where $A_i = A_{er} = A_{ed}$, $B_i = B_{er} = B_{ed}$ and where A_{e0} thru A_{e3}, C_{er0} thru C_{er5} and C_{ed0} thru C_{ed3} are given by constant matrices.

According to papers [3], the ISE criterions which are represented by $\mathcal{H}_2$ norm in (12) and (13) equal to following matrix inequality,

$$\Phi = \begin{bmatrix} \phi_{e0}(P^{-1},\mathbf{k}_1) & 0 & 0 \\ 0 & \phi_{ed}(P^{-1},\mathbf{k}_1,\gamma_d) & 0 \\ 0 & 0 & \phi_{er}(P^{-1},\mathbf{k}_2,\gamma_r) \end{bmatrix} \succ 0 \quad (17)$$

where '$\Phi \succ 0$' denotes that Φ is positive definite matrix, and where

$$\phi_{e0}(P^{-1},\mathbf{k}_1) = \begin{bmatrix} P^{-1} & P^{-1}B_{ed} & P^{-1}A_{ed} \\ B_{ed}^TP^{-T} & 1 & 0 \\ A_{ed}^TP^{-T} & 0 & P^{-1} \end{bmatrix} \quad (18)$$

and

$$\phi_{ed}(P^{-1},\mathbf{k}_1,\gamma_d) = \begin{bmatrix} \gamma_d & D_{ed} & C_{ed} \\ D_{ed}^T & 1 & 0 \\ C_{ed}^T & 0 & P^{-1} \end{bmatrix} \quad (19)$$

and

$$\phi_{er}(P^{-1},\mathbf{k}_2,\gamma_r) = \begin{bmatrix} \gamma_r & D_{er} & C_{er} \\ D_{er}^T & 1 & 0 \\ C_{er}^T & 0 & P^{-1} \end{bmatrix} \quad (20)$$

and where $\mathbf{k}_1 := [k_c, k_i, k_d]$ and $\mathbf{k}_2 := [k_c, k_i, k_d, k_\alpha, k_\beta]$ are the parameter vectors of the controller and P is a 4×4 positive symmetric matrix.

Since the continuous system (1) is perturbed, the four parameters a_1, a_2, b_0 and b_1 in realized systems have perturbations. In order to treat these perturbations of the control system, we assume here that 4 parameters belong to a perturbation set Ω, and the problem is formulated as

$$\begin{aligned} &\text{Minimize} \quad \gamma_r && (21)\\ &\text{subject to} \quad \gamma_d < \tilde{\gamma}_d && (21-\text{a})\\ &\mathbf{k}_1 \in Q_D && (21-\text{b})\\ &\Phi \succ 0 \ \text{ for all } \begin{bmatrix} a_1 \\ a_2 \\ b_0 \\ b_1 \end{bmatrix} \in \Omega && (21-\text{c}) \end{aligned}$$

where $\tilde{\gamma}_d$ is a constant given in advance, Q_D is a given hyper-rectangle in $\mathcal{R}^3$, and Ω denotes the set of perturbations as following equation.

$$\Omega := \left\{ \begin{bmatrix} a_1 \\ a_2 \\ b_0 \\ b_1 \end{bmatrix} \in \mathcal{R}^4 : \begin{matrix} a_{1min} \leq a_1 \leq a_{1max} \\ a_{2min} \leq a_2 \leq a_{2max} \\ b_{0min} \leq b_0 \leq b_{0max} \\ b_{1min} \leq b_1 \leq b_{1max} \end{matrix} \right\} \tag{22}$$

Because any $[a_1,\ a_2,\ b_0,\ b_1]^T$ in the set Ω can be described by linear combinations of 2^4 vertex vectors, the matrix inequalitiy (21-c) can be described by 2^4 BMIs. Although it is hard to solve BMI problems, which are NP hard in general, we can obtain the exact solution of BMI problem (21) via branch and bound algorithms discussing in Section 4 because it has only five parameters.

3. SWITCHING STRUCTURE BASED ON ADAPTIVE CONTROL METHOD

In order to reduce the conservativeness of the proposed controller, the switching structure for PD compensator is designed based on the adaptive control method in this section. This switching structure includes a system estimator, the switching criterion and a set of pre-specified PD parameters.

First, we construct the estimator in Fig1 base on recursive least square algorithms. To remove the influence of the stochastic noise $\xi(t)$ from system output $y(t)$, consider the low pass filter $H(z^{-1})$ which can effectively remove the high frequency noise. $H(z^{-1})$ is given by:

$$v(t) = H(z^{-1})y(t) \tag{23}$$

Similarly, consider the effective low pass filter $R(z^{-1})$ for the control input signal. This filter is added for more accurate estimating, that is given by the following equation.

$$w(t) = R(z^{-1})u(t) \tag{24}$$

Here, consider the following discrete-time model:

$$\tilde{A}(z^{-1})v(t) = z^{-1}\tilde{B}(z^{-1})u(t) \tag{25}$$

where

$$\begin{aligned} \tilde{A}(z^{-1}) &= 1 + \tilde{a}_1 z^{-1} + \tilde{a}_2 z^{-2} \\ \tilde{B}(z^{-1}) &= \tilde{b}_0 + \tilde{b}_1 z^{-1} \end{aligned} \tag{26}$$

Then, the following extended least squares estimation is employed:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\Gamma(t-1)\psi(t-1)\varepsilon(t)}{1+\psi^T(t-1)\Gamma(t-1)\psi(t-1)} \in \mathcal{R}^4 \tag{27}$$

$$\Gamma(t) = \Gamma(t-1) - \frac{\Gamma(t-1)\psi(t-1)\psi^T(t-1)\Gamma(t-1)}{1+\psi^T(t-1)\Gamma(t-1)\psi(t-1)} + \Gamma_0 \in \mathcal{R}^{4\times 4} \tag{28}$$

$$\varepsilon(t) := \Delta v(t) - \hat{\theta}^T(t-1)\psi(t-1) \quad \in \mathcal{R} \tag{29}$$

where $\varepsilon(t)$ denotes prediction errors. $\hat{\theta}(t)$ and $\psi(t-1)$ are the unknown parameter vector and the data vector of the form:

$$\hat{\theta}(t) := [\tilde{a}_1, \tilde{a}_2, \tilde{b}_0, \tilde{b}_1]^T \in \mathcal{R}^4 \tag{30}$$

$$\psi(t-1) : = [-\Delta v(t-1), -\Delta v(t-2), \Delta w(t-1), \Delta w(t-2)]^T \in \mathcal{R}^4 \tag{31}$$

By using (27) thru (31), The state of the system can be estimated recursively.

Next, we consider the switching criterion based on the estimated system condition. This paper deals with the stationary gain of the system (25) as the switching criterion. Let us define K_{sc}:

$$K_{sc} = \frac{\tilde{B}(1)}{\tilde{A}(1)} \tag{32}$$

and let PD compensator $C_2(z^{-1})$ be switched based on the following detection rule.

$$C_2(z^{-1}) = \begin{cases} C_2^{(1)}(z^{-1}) & K_{sc}^{(1)} < K_{sc} \leq K_{sc}^{(2)} \\ C_2^{(2)}(z^{-1}) & K_{sc}^{(2)} < K_{sc} \leq K_{sc}^{(3)} \\ \vdots & \vdots \\ C_2^{(p)}(z^{-1}) & K_{sc}^{(p)} < K_{sc} \leq K_{sc}^{(p+1)} \end{cases} \tag{33}$$

where $K_{sc}^{(j)}$ $(j = 1 \cdots p+1)$ are given and where $C_2^{(q)}(z^{-1})$ $(q = 1 \cdots p)$ are PD compensators defined for each sector.

Here we consider the design method of $C_2^{(q)}(z^{-1})$. From (32) and (33), the gain range $K_{sc}^{(q)} < K_{sc} \leq K_{sc}^{(q+1)}$ is expressed as the set of $[a_1,\ a_2,\ b_0,\ b_1]^T$ as follows.

$$\Lambda := \left\{ \begin{bmatrix} a_1 \\ a_2 \\ b_0 \\ b_1 \end{bmatrix} \in \mathcal{R}^4 : \begin{matrix} K_{sc}^{(q)}(1+a_1+a_2) \\ -(b_0+b_1) < 0 \\ -K_{sc}^{(q+1)}(1+a_1+a_2) \\ +b_0+b_1 \leq 0 \end{matrix} \right\} \tag{34}$$

Then the design problem of the PD compensator $C_2^{(q)}(z^{-1})$ corresponding to the pre-specified small-ranged system gain $K_{sc}^{(q)} < K_{sc} \leq K_{sc}^{(q+1)}$ can be formulated as

$$\text{Minimize} \quad \gamma_r \tag{35}$$

$$\text{subject to} \quad \Phi_{PD} \succ 0 \quad \text{for all} \begin{bmatrix} a_1 \\ a_2 \\ b_0 \\ b_1 \end{bmatrix} \in \Omega \cap \Lambda \tag{35 -a}$$

where Φ_{PD} is represented by the following matrix

$$\Phi_{PD} = \begin{bmatrix} \phi_{e0}(P^{-1}, \mathbf{k}_1) & 0 \\ 0 & \phi_{er}(P^{-1}, \mathbf{k}_2, \gamma_r) \end{bmatrix} \succ 0 \tag{36}$$

In the above problem, PID parameters k_c, k_i and k_d in parameter vecters of the controller $\mathbf{k}_1$ and $\mathbf{k}_2$ are given since they are already obtained by solving (21). Therefore, matrix inequality (36) can be represented by LMI, where variables are P^{-1}, k_α and k_β. The matrix inequality (35-a) can be expressed by two inequalities in (34) and 2^4 LMIs as well as the case of (21-c). Hence it is easy to obtain the optimal solution of the problem (35) because there exist polynomial algorithms based on the interior point method [4].

By using the estimator and the switching criterion as mentioned above, the most effective PD compensator which satisfies the good tracking property is selected out of the set of pre-specified PD parameters corresponding to the small divided perturbations. The switching algorithm for the proposed PID controller with two-degrees-of-freedom is summarized as follows.

【The switching algorithm for the PID controller with two-degrees-of-freedom】

[**Step 1**] Design the PID controller and the PD compensator by solving the BMI problem (21).
[**Step 2**] Design the set of PD parameters corresponding to the small divied perturbations by solving the LMI problem (35).
[**Step 3**] Estimate the system conditions using (27) thru (31).
[**Step 4**] Calculate K_{sc} from (32).
[**Step 5**] Choose the most effective PD parameter from the detection rule in (33).
[**Step 6**] Return to [**Step 3**].

4. BMI SOLUTION BY USING AN EXACT ALGORITHM

This section provides an exact algorithm for solving problem (21) based on branch and bound algorithms [5]. Branch and bound algorithms give us the lower bound Ψ_L and the upper bound Ψ_U satisfying $\Psi_L \leq \inf \gamma_r \leq \Psi_U$ and $(\Psi_U - \Psi_L)/\Psi_L \leq \varepsilon$ for any $\varepsilon > 0$. The lower bounds are obtained using the SDP relaxation [6,7].

Let us define the function $\Psi(\cdot)$, $\Psi_L(\cdot)$ and $\Psi_U(\cdot)$ as follows.

$$\Psi(Q) \equiv \inf_{\substack{\gamma_d < \tilde{\gamma}_d, [\mathbf{k}_1, \mathbf{k}_2]^T \in Q, \\ \Phi \succ 0 \text{ for all } [a_1, a_2, b_0, b_1]^T \in \Omega}} \gamma_r, \tag{37}$$

$$\Psi_L(Q) \equiv \inf_{\substack{\gamma_d < \tilde{\gamma}_d, [\mathbf{k}_1, \mathbf{k}_2]^T \in Q, \\ \hat{\Phi} \succ 0 \text{ for all } [a_1, a_2, b_0, b_1]^T \in \Omega}} \gamma_r, \tag{38}$$

$$\Psi_U(Q, \mathbf{k}_1^*, \mathbf{k}_2^*) \equiv \inf_{\substack{\gamma_d < \tilde{\gamma}_d, \\ \Phi^* \succ 0 \text{ for all } [a_1, a_2, b_0, b_1]^T \in \Omega}} \gamma_r, \tag{39}$$

where $\hat{\Phi}$ is the SDP relaxation of Φ obtained using the method in the papers [7,8],

$$\begin{bmatrix} \mathbf{k}_1^* \\ \mathbf{k}_2^* \end{bmatrix} = \arg \inf_{\substack{\gamma_d < \tilde{\gamma}_d, [\mathbf{k}_1, \mathbf{k}_2]^T \in Q, \\ \hat{\Phi} \succ 0 \text{ for all } [a_1, a_2, b_0, b_1]^T \in \Omega}} \gamma_r,$$

and Φ^* is obtained by substituting $[\mathbf{k}_1^*, \mathbf{k}_2^*]^T$ into Φ in (17). Then $\Psi_L(Q) \leq \Psi(Q) \leq \Psi_U(Q)$ holds for any Q. We can obtain Ψ_L^* and Ψ_U^* such that $\Psi_L \leq \inf \gamma_r \leq \Psi_U$ holds for any ε using the following algorithm.

【Branch and Bound Algorithm】

[**Step 1**] Set $k \leftarrow 0, Q_0 \leftarrow Q_D, S0 \leftarrow \{Q_0\}, L_0 \leftarrow \Psi(Q_0), U_0 \leftarrow_U (Q_0)$.
[**Step 2**] Select $\bar{Q}$ from S_k such that $L_k = \Psi_L(\bar{Q})$. $S_{k+1} \leftarrow S_k \setminus \{\bar{Q}\}$.
[**Step 3**] Split $\bar{Q}$ along its longest edge into $\bar{Q}_1$ and $\bar{Q}_2$.
[**Step 4**] For $i = 1, 2$ if $\Psi_L(\bar{Q}_i) \leq U_k$ then $S_{k+1} \leftarrow S_{k+1} \cup \{\bar{Q}_i\}$.
[**Step 5**] $U_{k+1} \leftarrow \min_{Q \in S_{k+1}} \Psi_U(Q)$.
[**Step 6**] Pruning: $S_{k+1} \leftarrow S_{k+1} \setminus \{Q : \Psi_L(Q) > U_{k+1}\}$.
[**Step 7**] $L_{k+1} \leftarrow \min_{Q \in S_{k+1}} \Psi_L(Q)$.
[**Step 8**] If $(U_k - L_k)/L_k \leq \varepsilon$ then end else $k \leftarrow k + 1$ and goto [**Step 2**].

5. NUMERICAL EXAMPLE

In order to investigate the behavior of the proposed control scheme, numerical simulation examples are illustrated in this section.

Let us consider the continuous-time model given by the following equation.

$$G(s) = \frac{K_0}{1+Ts}e^{-Ls} \quad \text{where} \begin{cases} 1.0 \le K_0 \le 1.5 \\ 8 \le T \le 12 \\ L = 3 \end{cases} \tag{40}$$

From (3) and (4), the system parameters of the descrete-time model which are transformed by using sampling time period $T_s = 1$ are obtained as follows:

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + a_2 z^{-2} \\ B(z^{-1}) &= b_0 + b_1 z^{-1} \end{aligned} \tag{41}$$

where

$$\begin{cases} -1.4335 \le a_1 \le -1.3959 \\ 0.4531 \le a_2 \le 0.4724 \\ -0.0528 \le b_0 \le -0.0362 \\ 0.0751 \le b_1 \le 0.1100 \end{cases} \tag{42}$$

By solving the BMI problem (21), parameters of the PID controller and the PD compensator are designed as follows:

$$\begin{array}{lll} k_c = 2.7525 & k_\alpha = 0.4496 & \gamma_r = 15.4295 \\ k_i = 0.3818 & k_\beta = 0.7565 & \gamma_d = 162.6861 \\ k_d = 4.3374 & & \end{array} \tag{43}$$

We designed the pre-specified PD compensators by solving the LMI problems (35), and they are obtained as follows:

$$C_2(z^{-1}) = \begin{cases} -0.4009 - 1.9002\Delta \\ \qquad 1.0 \ < K_{sc} \le 1.25 \\ \qquad (\gamma_r = 6.5864) \\ -0.7843 - 2.0138\Delta \\ \qquad 1.25 \ < K_{sc} \le 1.5 \\ \qquad (\gamma_r = 5.9511) \end{cases} \tag{44}$$

The system parameters of the control object are given as $K_0 = 1.0$ and $T = 11.5$ in the period from 0[step] to 400[step], and $K_0 = 1.4$ and $T = 12.0$ in the period from 401[step] to 1000[step]. The reference signal is given as $r(t) = 1$ in the period from 0[step] to 200[step], $r(t) = 2$ in the period from 201[step] to 700[step] and $r(t) = 2.5$ in the period 701[step] to 1000[step]. The stochastic noise $\xi(t)$ is given as a normal distribution with $\mathcal{N}(0, 0.001^2)$. Fig.2 shows the result. We can see that the influence by the stochastic noise can be reduced, and that the system can track the reference signal well.

6. CONCLUSIONS

In this paper, a BMI based design scheme for switched PID controllers with two-degrees-of-freedom has been proposed. According to the proposed scheme, two design specification based on $\mathcal{H}_2$ norm are formulated in BMIs, and PID parameters can be exactly obtained by solving the BMI problems via branch and bound algorithms. In order to reduce the conservativeness of the control system, the proposed PD compensators have switching structure based on adaptive control method. Numerical examples have shown the effectiveness of the proposed method.

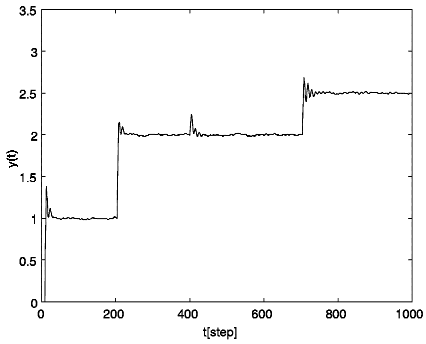

Fig. 2. Control result using the proposed PID control scheme.

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