A DISAGGREGATION TECHNIQUE FOR THE OPTIMAL PLANNING OF OFFSHORE PLATFORMS

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Abstract: There is a great incentive for developing systematic approaches that effectively identify strategies for planning oilfield complexes. This paper proposes an MILP that relies on a reformulation of the model proposed by Tsarbopoulou (UCL M.S. Dissertation, London, 2000). Moreover, a disaggregation technique is applied to the MILP. A master problem determines the assignment of platforms to wells and a planning subproblem calculates the timing for fixed assignments. Results show that the decomposition approach generates optimal solutions for instances of up to 145 wells and 64 platforms in 10 discrete time periods that otherwise could not be solved with a full-scale model.

Keywords: mathematical models, integer programming, discrete time, decomposition methods, optimization problems

1. INTRODUCTION

There is a great incentive for developing systematic approaches that effectively identify strategies for planning and designing oilfield complexes, due to the economic impact of the underlying decisions. On the other hand, the application of optimization techniques in problems that involve oilfield exploration represents a challenging and complex problem.

The literature presents models and solution techniques for solving problems in the design and planning of infrastructure in oilfields. This problem has been initially presented in the literature by Devine and Lesso (1972) that developed an optimization model for the development of offshore oilfields.

According to Van den Heever and Grossmann (2000), in the past decisions that concerned platform capacities, scheduling of perforations and production yields had been frequently made separately. Moreover, certain assumptions were made in order to reduce the required computational effort. Another approach was to assume a fixed perforation schedule and then to determine the production yield from an LP model. A third approach was to determine the perforation schedule for a fixed production yield from an LP and subsequently round the non integer solution to integer values or even to solve the MILP.

Frair (1973) proposed independent models for calculating the number of production platforms, their capacities and the scheduling of well perforation. However, this approach has lead to infeasible or sub-optimal decisions since these were not considered in an integrated model.

Iyer *et al.* (1998) proposed a multiperiod MINLP for the planning and scheduling of investment and operation in offshore oilfields. The formulation incorporates the nonlinear behavior of the reservoirs, pressure constraints in the well surface and equipment constraints. The formulation presents a general objective function that optimizes a given economic indicator, such as NPV. A sequential decomposition technique is proposed to solve the problem that relies on the aggregation of time periods followed by successive disaggregating steps.

In the case of the planning of infrastructure of petroleum fields, MINLP models have been avoided in favor of MILP or even LP models, because of the inherent difficulties of treating nonlinear constraints and in the latter case because of the combinatorial explosion that results from discrete decisions.

Iyer and Grossmann (1998) proposed a decomposition algorithm that solves a design problem in reduced space of binary variables to determine the assignment of wells to platforms. The planning model is then solved for fixed values determined in the design subproblem.

Tsarbopoulou (2000) proposed an MILP model for the optimization of the exploration of oil and gas in a petroleum platform. The proposed model relies on binary variables to determine the existence of a given platform and the potential connection between wells and platforms.

This paper proposes a reformulation of the MILP of Tsarbopoulou (2000) model that relies on a smaller number of binary variables that requires a smaller computational effort. Moreover, a disaggregation technique proposed by Iyer and Grossmann (1998) is applied to the reformulated model that is composed of assignment and planning sub problems. The master problem determines the assignment of platforms to wells and the planning sub problem that calculates the timing for fixed assignments.

2. PROBLEM DEFINITION

An offshore oil field consists of J wells that contain oil and gas. Platforms are required to extract these substances from one or more oilfields. The planning problem involves the selection of the number and types of units, such as platforms and wells, as well as the decision of assigning platforms to wells in a given time horizon.

3. MATHEMATICAL MODEL

The planning of infrastructure in offshore oilfields includes discrete and continuous decisions along the project lifetime, such as the selection of platforms and oilfields to invest as well as oil and gas production, respectively.

Based on these considerations, the model that represents the infrastructure is a Mixed Integer Programming (MIP) problem. The objective is to maximize the net present value (NPV).

3.1 Model Assumptions

The following are the main assumptions for the proposed model:

(A1) Only two substances are removed, which are oil and gas.
(A2) The productivity index is assumed constant throughout the planning horizon.
(A3) Whenever oil is removed from a reservoir, its pressure decreases linearly.
(A4) All wells in the reservoir were connected and therefore the pressure in each well is constant in a given time period.
(A5) There is no pressure loss along the pipelines between the wells and the platforms.
(A6) A linear model represents the gas-to-oil rate. This value is 0.7 when no oil is removed and reaches the maximum value of 1.0 when all the oil is removed.
(A7) The initial amounts of each substance are known for each well.
(A8) The production limit for each substance is known along the planning horizon.
(A9) The area of the field is known and it is divided into small rectangles. In the center of each rectangle it is possible to allocate a platform.
(A10) The wells are randomly distributed in the field.
(A12) The time horizon is discretized in intervals of equal length.
(A13) Production costs and yields all substances are known for each time period.
(A14) Interest and inflation rates are known and are constant along the planning horizon.

3.2 Notation

Indices:

g	gas
i	platform
j	well
o	oil
s	substance (gas or oil)
t	time period

Continuous variables:

CON	connection cost
CUM_{st}	cumulative substance production up to year t
DR	overall drilling cost
$FMAX_{sjt}$	maximum flow of substances from well at time period t
F_{sjt}	flow rate of substance s from well j during time period t
GOR_t	gas-to-oil ratio at time period t
P_t	pressure of all wells at time period t
ZI	objective function

Binary Variables

M_i	existence of platform i
x_{ijt}	connection of platform i to well j at time period t
X_{ij}	connection of platform i to well j

Parameters:

APG_t	annual gas price at time period t
APO_t	annual oil price at time period t
$COST_{ij}$	connection cost
D_t	depreciation rate at time period t
$INVAL_{sj}$	initial value for substance s in well j
Q_{st}	upper production limit for each component at time period t
PCG	production costs for gas
PCO	production costs for oil

PI_j productivity index for well j

Problem MR corresponds to a reformulation model from the one proposed by Tsarbopoulou (2000) denoted as model MO. The main difference between both models relies on the representation of the decision variables. Tsarbopoulou (2000) considered an extra binary variable that assigns wells to platforms besides the one that relates wells to platforms at every time period (x_{ijt}). The reformulated model contains only the last set of variables, which is sufficient to model the discrete decisions of the problem.

MR:

Max Zi=GAS+OIL-DR-CON (1)

s.t.

$$CUM_{s,t} = CUM_{s,t-1} + \sum_j F_{s,j,t} \quad \forall s,t \tag{2}$$

$$GOR_t = 0.7 + 3.10^{-8} \times CUM_{o,t} \quad \forall t \tag{3}$$

$$P_t = 100 - 0.000008 \times CUM_{o,t} \quad \forall t \tag{4}$$

$$FMAX_{o,j,t} = PI_j \times P_t \quad \forall j,t \tag{5}$$

$$FMAX_{g,j,t} = PI_j (60 - 2.6.10^{-6} CUM_{o,t}) \quad \forall j,t \tag{6}$$

$$F_{s,j,t} \le FMAX_{s,j,t} \quad \forall s, j, t \tag{7}$$

$$\sum_j F_{s,j,t} \le Q_{s,t} \quad \forall s, t \tag{8}$$

$$\sum_t F_{s,j,t} \le INVAL_{s,j} \quad \forall s, j \tag{9}$$

$$DR = \sum_i (100 M_i + 10 \times \sum_j \sum_t x_{i,j,t}) 10000 \tag{10}$$

$$CON = \sum_i \sum_j \sum_t COST_{i,j} \times x_{i,j,t} \tag{11}$$

$$A_{j,t} = A_{j,t-1} + \sum_i x_{i,j,t} \quad \forall j,t \tag{12}$$

$$F_{o,j,t} \le FOMAX \times A_{j,t} \quad \forall j,t \tag{13}$$

$$F_{g,j,t} \le FGMAX \times A_{j,t} \quad \forall j,t \tag{14}$$

$$\sum_i \sum_t x_{i,j,t} \le 1 \quad \forall j \tag{15}$$

$$\sum_t x_{ijt} \le M_i \quad \forall i, j \tag{16}$$

$$OIL = \sum_i \sum_j \sum_t \left[F_{o,j,t} \times (APO_t - PCO) \times D_t \right] \tag{17}$$

$$GAS = \sum_i \sum_j \sum_t \left[F_{g,j,t} \times (APG_t - PCG) \times D_t \right] \tag{18}$$

The objective function in eq. 1 is the expected net present value, which includes the revenues of oil and gas reduced by the drilling and connection costs. Equation 2 states that the cumulative production of each substance (oil/gas) is the same as the cumulative production in the previous time period increased by an amount equal to the flow from all wells at the present time. Equation 3 states that the gas-to-oil rate increases as oil is extracted. Equation 4 states that the initial pressure of the reservoir is 100 bar and that it decreases linearly with accumulated production. Equations 5 and 6 are related with the maximum flow of production of the oil and gas, respectively. Equation 7 states that the flows of each substance from each well should not exceed the maximum production limits. Equation 8 states that the flow any substance, from all the wells, should not exceed the upper production limits. Equation 9 states that the flow of all substances throughout the time horizon should not exceed their initial amounts. Equations 10 and 11 are related to the drilling and connection costs, respectively. The cost depends directly on the assignment of the well to the platform at time period t. Equation 12 states that a well is opened only once and remains open throughout the whole time period. Equations 13 and 14 state that the oil and gas flow should not exceed some specific limits. Equation 15 states that a well is connected to a platform once. Equation 16 states that a well is connected to a platform only if the same platform was allocated. Equations 17 and 18 are related to the revenues from oil and gas sales, respectively.

4. DISAGGREGATION APPROACH

Iyer and Grossmann (1998) proposed a two-level decomposition approach for the planning of process networks. Van der Heever and Grossmann (2000) then applied this approach to an oilfield infrastructure-planning model. In this section, a similar approach is applied to the reformulated model MR. The disaggregated model is denoted as MD that is decomposed into two subproblems: the master subproblem that solves a model that assigns platforms to wells (problem AP) and the timing subproblem (problem TP). The latter relies on the assignments that are obtained in the master subproblem and decides on when to install the platforms. The decomposition algorithm as applied to model MR can be seen in Figure 1. The proposed technique is similar to the one proposed by Van der Heever and Grossmann (2000), which however have considered non convex nonlinearities in the sub-problem and therefore could not guarantee global solutions.

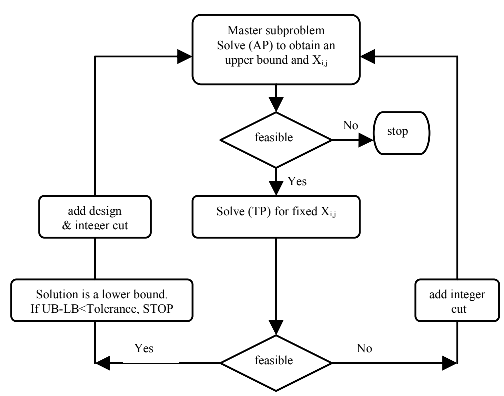

Fig. 1. Bilevel decomposition algorithm.

The assignment problem (AP) is defined as follows:

$$\max \quad ZI = GAS + OIL - DR - CON \tag{1}$$

s.t.

constraints (1) to (9), (17) and (18)

$$DR = \sum_i (100 M_i + 10 \sum_j X_{i,j}) 10000 \tag{19}$$

$$CON = \sum_i \sum_j \sum_t COST_{i,j} \times X_{i,j} \tag{20}$$

$$A_j = \sum_i X_{i,j} \quad \forall j \tag{21}$$

$$F_{o,j,t} \le FOMAX \times A_j \qquad \forall j,t \tag{22}$$

$$F_{g,j,t} \le FGMAX \times A_j \qquad \forall j,t \tag{23}$$

$$\sum_i X_{i,j} \le 1 \qquad \forall j \tag{24}$$

$$X_{ij} \le M_i \qquad \forall i,j \tag{25}$$

The solution of AP provides values for $X_{i,j}$. If this variable is fixed ($\overline{X}_{i,j}$), a feasible solution for TP is a feasible solution for MR and generates a lower bound for this problem, where TP is defined as follows:

Problem TP

$$\max \quad ZI = GAS + OIL - DR - CON \tag{1}$$

s.t. constraints (2) to (18)

$$x_{i,j,t} \le \overline{X}_{i,j} \qquad \forall i,j,t \tag{26}$$

$$A_{j,t} \le A_j \qquad \forall j,t \tag{27}$$

Similarly to Iyer and Grossmann (1998), constraints 26 and 27 select a subset of assignments for the planning problem.

The following are the constraints used in the algorithm to avoid subsets and supersets that would result in suboptimal solutions:

$$\sum_{n1 \in Z_1^r} \sum_{n2 \in Z_1^r} X_{n1,n2} + X_{i,j} \le \left| Z_1^r \right| \quad \forall i,j \in Z_0^r, r=1...R \tag{28}$$

$$\sum_{n1 \in Z_0^r} \sum_{n2 \in Z_0^r} X_{n1,n2} + X_{i,j} \ge 1 \quad \forall i,j \in Z_1^r, r=1...R \tag{29}$$

$$\sum_{i \in M_r} \sum_{j \in M_r} X_{i,j}^r - \sum_{i \in N_r} \sum_{j \in N_r} X_{i,j}^r \le \left| M_r \right| - 1 \tag{30}$$

where

$$M_r = \left\{ i / \overline{X}_{i,j}^r = 1 \text{ for configuration in iteration } r \right\}$$

$$N_r = \left\{ i / \overline{X}_{i,j}^r = 0 \text{ for configuration in iteration } r \right\}$$

$$Z_1^r = \left\{ i, j / X_{i,j} = 1 \right\}$$

$$Z_0^r = \left\{ i, j / X_{i,j} = 0 \right\}$$

Similarly to Iyer and Grossmann (1998), equation 28 states that if in any solution all the x variables in any set Z_1^r are 1, then all remaining variables must be zero in order to prevent a superset of Z_1^r from entering the solution of AP. Equation 29 shows cuts for precluding subsets of Z_1^r. Equation 30 has the effect of establishing the basis for deriving integer cuts on supersets and subsets of the configurations predicted by the assignment problem.

5. RESULTS

In this section, problems are solved to illustrate the performance of the model and of the solution strategy. The problems were modeled using GAMS (Brooke et al. (1998) and solved in the full space using the CPLEX solver (ILOG, 1999).

The reformulated model (MR) presented better computational performance with respect to the original model (MO) proposed by Tsarbopoulou (2000), as shown in Table 1 that presents he CPU times obtained for a problem with 16 platforms as a function of the number of wells (NW). Interestingly, the integrality gap is the same for both models and increases with problems size.

Note from Table 1 that none of the models is able to solve problems for more than 40 wells, despite a relatively small integrality gap verified for the smaller instances. Nevertheless, when MR is subject to the disaggregation strategy proposed in the previous section (denoted as MD), the computational gain is remarkable. The CPU times obtained for a problem with 16 platforms as a function of NW are compared to those from MO and MR in Figure 2.

Figures 3 and 4 illustrate the computational time for MD for different numbers of wells and for platforms, ranging from16 to 64.

Table 1 – Computational performance of the models

NW	CPU time (s) MO	CPU time (s) MR	gap (%)
05	0.8	0. 7	0.58
10	2.7	1.6	0.55
15	8.5	4.0	0.67
20	54.6	43.8	0.90
25	117.0	40.2	1.13
30	6.1	3.9	1.29
35	10.7	6.4	1.35
40	*	*	-

* No solution obtained after 18,000 CPU s.

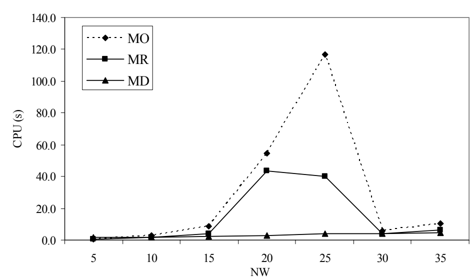

Fig. 2. CPU times for the proposed models

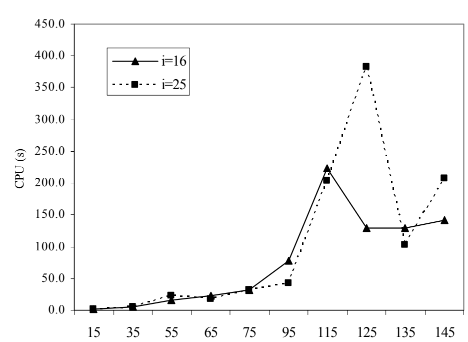

Fig. 3. Computational performance for MD in large instances

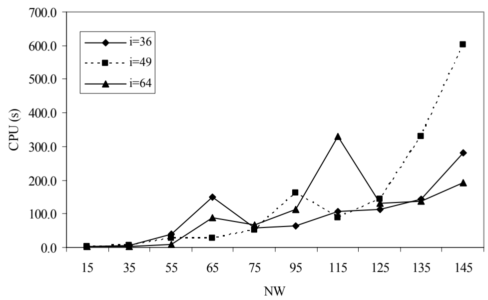

Fig. 4. Computational performance for MD in large instances

Table 2 presents the sizes of problems MO and MR, such as the number of equations (SE), number of continuous variables (SV) and number of discrete variables (DV) for several numbers of wells (NW) and 16 platforms.

Table 3 presents the corresponding sizes of problem MD, for several values of the number of wells. At each iteration, SV and DV are maintained, whereas there is an average increase de 20% in the number of equations from iteration 1 to 2, due to cut generation.

It can be seen from Table 3 that the reduction in the number of discrete values (DV) in MD is not significant with respect to MR. However the introduction of constraints (26) greatly reduces the search space and therefore the computational effort.

Table 2 – Dimensions of MO and MR

NW	MO			MR		
	SE	SV	DV	SE	SV	DV
05	1506	1351	1056	510	1111	816
10	2931	2481	1936	955	2161	1616
15	4356	3611	2816	1400	3211	2416
20	5781	4741	3696	1845	4261	3216
25	7206	5871	4576	2290	5311	4016
30	8631	7001	5456	2735	6361	4816
35	10056	8131	6336	3180	7411	5616

Table 3 – Size of problem MD

NW	Sub problem	1^{st} iteration SE	SV	DV
5	AP	465	346	101
	TP	1285	1111	866
10	AP	865	631	186
	TP	2505	2161	1716
15	AP	1265	916	271
	TP	3725	3211	2566
20	AP	1665	1201	356
	TP	4945	4261	3416
25	AP	2065	1486	441
	TP	6057	5311	4266
30	AP	2465	1771	526
	TP	7277	6361	5116
35	AP	2865	2056	611
	TP	8551	7411	5966

6. CASE STUDY

In this section we present in detail a case study as the one presented by Tsarbopoulou (2000) that provides a comparison between MO developed by the author and the proposed model MD. For this case 16 platforms and 30 wells are considered for a horizon of 10 years. In this problem, a rectangular oilfield of 10,000 ft by 15,000 ft was assumed. The interest rate was set to 15% and annual inflation rate to 3%. Upper production limits of oil and gas in each well are 1,250,000 and 875,000, respectively.

Data regarding productivity indexes (PI), initial amount of substances (oil and gas), the coordinates in the field, and depth (DP) in each well are given in Table 4. Cost and depreciation correlations, that depend on the well depth, as well as gas and oil prices are given in Tsarbopoulou (2000).

The optimal values obtained with MO, MR and MD are the same and reach $1.6464*10^9$. However, as can be noted from Table 1 and Figure 2, there is a reduction of approximately 60% in CPU time for MD. Only 3 subproblems are required for MD.

The well-platform assignments obtained for all iterations of MD are given in Table 5. Note that the sequence for the decision variable is (well, platform, time period).

Table 4 - Data for each well

j	X (ft)	Y (ft)	DP (ft)	PI $\left(\frac{ft^3}{yr.bar}\right)$	INVAL (10^5 ft^3/year) Oil	Gas
1	5336	1183	6.27	1840	8.5	5.95
2	6136	4283	5.26	2000	11.0	7.70
3	6338	6640	5.34	1760	12.0	8.40
4	12911	1082	5.61	1920	9.5	6.65
5	4528	8700	5.92	1980	10.0	7.00
6	10862	8990	5.16	1680	10.5	7.35
7	9683	4679	5.42	1620	8.0	5.60
8	2716	2677	5.11	1629	9.0	6.30
9	8808	4510	5.82	1740	10.0	7.00
10	6007	5702	5.66	1940	11.5	8.05
11	2999	6058	5.00	1840	8.5	5.95
12	13090	2313	6.22	2000	11.0	7.70
13	13855	5889	6.25	1760	12.0	8.40
14	7713	6440	4.90	1920	9.5	6.65
15	4369	2773	5.59	1980	10.0	7.00
16	10260	8099	5.26	1680	10.5	7.35
17	11416	4973	6.03	1620	8.0	5.60
18	6648	3866	5.17	1629	9.0	6.30
19	9834	3451	5.57	1740	10.0	7.00
20	8006	3679	5.73	1940	11.5	8.05
21	12096	2913	4.88	1840	8.5	5.95
22	7000	7869	4.58	2000	11.0	7.70
23	3477	1774	5.78	1760	12.0	8.40
24	9153	3104	6.08	1920	9.5	6.65
25	617	1034	4.76	1980	10.0	7.00
26	1071	3328	5.06	1680	10.5	7.35
27	4095	1249	5.06	1620	8.0	5.60
28	7440	9979	5.98	1629	9.0	6.30
29	7155	9232	6.29	1740	10.0	7.00
30	1095	7980	6.36	1940	11.5	8.05

Table 5 – Assignments for each iteration of MD

r=1		r=2
AP - $X_{i,j}^{(1)}$	TP - $x_{i,j,t}^{(1)}$	AP - $X_{i,j}^{(2)}$
2, 3	2, 3, 1	2, 3
2, 6	2, 6, 1	2, 6
2, 7	2, 7, 1	2, 7
2, 11	2, 11, 1	2, 11
2, 14	2, 14, 1	2, 14
2, 23	2, 23, 1	2, 23
3, 12	3, 12, 1	3, 12
3, 13	3, 13, 1	3, 13
3, 15	3, 15, 1	3, 15
3, 19	3, 19, 1	3, 19
4, 4	4, 4, 1	4, 4
4, 10	4, 10, 1	4, 10
9, 1	9, 1, 1	9, 1
9, 8	9, 8, 1	9, 8
9, 22	9, 22 ,1	9, 22
9, 24	9, 24, 1	9, 24
9, 29	9, 29, 1	9, 29
9, 30	9, 30, 1	9, 30
10, 2	10, 2, 1	10, 2
10, 5	10, 5, 1	10, 5
10, 9	10, 9, 1	10, 20
10, 20	10, 20, 1	11, 9
11, 25	11, 25, 1	11, 25
11, 28	11, 28, 1	11, 28
13, 18	13, 18, 1	13, 18
13, 27	13, 27, 1	13, 27
15, 17	15, 17, 1	15, 17
15, 21	15, 21, 1	15, 21

Note that constraints 28 to 30 do not allow the repetition of assignments neither the generation of sub and supersets. In this sense there is no significant change in the allocation obtained in AP in consecutive iterations. The only modification is the allocation of well 9 to platform 11 in iteration 2 in place of the assignment of well 20 to platform 10.

The cumulative productions of oil and gas as well as GOR, as functions of time are shown in Figures 5 and 6, respectively. Note that the cumulative flow rate of oil as well as the gas to oil ratio increase linearly with time up to time period 8. Afterwards, there is a reduction in this value due to the upper bound on the GOR that is set to one.

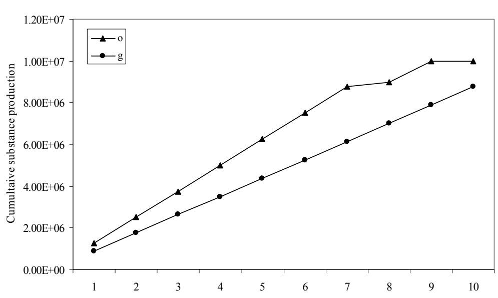

Fig. 5. Cumulative substance production up to year t

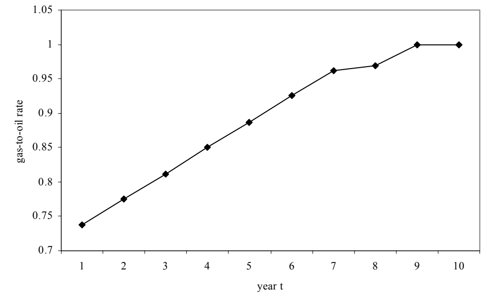

Fig. 6. Gas-to-oil ratio at year t

7. CONCLUSIONS

This paper presented a reformulated MILP for the planning of the oilfield infrastructure that presents a significant reduction in the number of discrete variables for the same relaxation gap with respect to the model developed by Tsarbopoulou (2000). Moreover, a decomposition approach that relies on the disaggregation of the assignment and timing decisions in analogy to the one proposed by Iyer and Grossmann (1998) has been presented. Results show that computational performance is greatly improved, whereas global optimality is guaranteed. Problems of 64 platforms and 145 wells are efficiently solved for a 10-year horizon.

ACKNOWLEDGMENT - The authors acknowledge financial support from CAPES (Brazil).

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