GENERALIZED PREDICTIVE CONTROL FOR A CLASS OF BILINEAR SYSTEMS

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Abstract: A new generalized predictive control algorithm for a kind of input-output bilinear system is proposed in the paper (BGPC). The algorithm combines bilinear and linear terms of I/O bilinear system, and constitutes an ARIMA model analogous to linear systems. Using optimization predictive information fully, the algorithm carries out multi-step predictions by recursive approximation. The heavy computation of generic nonlinear optimization is avoided with control law of analytical form being used to the non-minimum phase bilinear systems. Simulation results show the effectiveness of the algorithm and the performance of the algorithm is better than linear generalized predictive control (LGPC).

Key words: bilinear systems; bilinear generalized predictive control (BGPC); recursive approaches; non-minimum phase systems; analytical control laws

1. INTRODUCTION

Most of practical production processes are nonlinear systems. Nonlinear systems are usually described as I/O form with the expression of polynomial and rational fraction (Korenberg, et al., 1988). Until now, the research of nonlinear system control is very effective. Bilinear system is a kind of nonlinear system with simple structure. The practical processes, such as project, social economy, zoology and biology etc, can be widely described by bilinear systems, and it can include a large class of dynamic characteristics of strong nonlinear system within a bigger area of a steady operating point. Its approximation precision is still higher than that of traditional linear model. Bilinearization provides an effective approach for the analysis and design of nonlinear systems. Therefore, the research for bilinear system (Svoronos, Stephanopoulos and Aris, 1981;Eaton and Rawlings, 1990; Hua Xiangming, 1990; Akihiro and Toru, 2001) has been largely performed since the late of 1960s.

As a new computer control algorithm, model predictive control originated directly from industrial process control in the anaphase of 1970s.It has made quite great progress in the past twenty years. More attention has been given to GPC, since GPC algorithm (Clarke, et al., 1987) was proposed by Clarke etc in 1987. Predictive control technology of linear models has been widely developed (Doyle III, 1995) and predictive control research of nonlinear model has already made great progress. When a generic nonlinear model for model predictive control is adopted, nonlinear optimization will be involved, and on-line disposal is very difficult. While bigger error is brought using linear approximate model. Therefore predictive control with bilinear model describing original nonlinear system is meaningful to practical application and academic research. Model prediction is introduced to bilinear systems (Adhemar, et. al., 2002; Liu, 1996; Yao, 1997; Jiang, 1998,1999; He, 1999), and good effect is achieved. A new approach of bilinear generalized predictive control (Adhemar, et al., 2002) is presented. Bilinear model is handled, described as the time-step quasi-linearzed NARIMAX model and also improved, which overcomes the disadvantage that predictive error increases with the predictive horizon. Weighted adaptive predictive control is introduced to I/O discrete bilinear systems (Liu, 1996). The approach of point-by-point linearization approximation is introduced to I/O bilinear systems (Yao, et al., 1997). One-step and two-step predictive control (Jiang, 1998, 1999) are introduced to generic bilinear systems. Predictive model of generalized bilinear system based on Volterra series (Hemet al., 1999) is presented, and solving high order equation with one step prediction gains the optimal control law.

Apparently, the research on bilinear systems is inadequate by comparison to linear system predictive control. Even if there are some problems on the research mentioned above, such as it need try further to simplify Volterra series kernels identification. It needs the process's variety isn't very rapid in the approach of point-by-point linearization approximation. In conclusion, the existing result keep some distance with practicality and it need more perfect and develop. A multi-step GPC algorithm based on I/O discrete bilinear system is presented in this paper (BGPC). Bilinear and linear terms in the bilinear model are combined and the ARIMA model analogous to linear system is constituted. Making full use of optimal predictive control information, and carrying out multi-step prediction by recursive approximation, we obtain GPC algorithm with analytic form. The simulation results show the effectiveness of the algorithm.

2. REPRESENTATION OF BILINEAR SYSTEM

Consider a kind of SISO time-invariant bilinear systems

$$\begin{aligned} A(z^{-1})y(t) &= B(z^{-1})u(t-1) \\ &+ D(z^{-1})u(t-1)y(t-1) + C(z^{-1})e(t)/\Delta \end{aligned} \qquad 1$$

where $A(z^{-1}) = 1 + \sum_{i=1}^{n_a} a_i z^{-i}$,

$$B(z^{-1}) = \sum_{i=0}^{n_b} b_i z^{-i},$$

$$C(z^{-1}) = \sum_{i=0}^{n_c} c_i z^{-i},$$

$$D(z^{-1}) = \sum_{i=0}^{n_b}\sum_{j=0}^{n_a} d_{ij} z^{-i}\cdot z^{-j}.$$

bilinear term

$$D(z)u(t-1)y(t-1) = \sum_{i=0}^{n_b}\sum_{j=0}^{n_a} d_{ij}u(t-1-i)y(t-1-j)$$

for the sake of simplicity, this paper will mainly discuss under the condition of $i \neq j$, $d_{ij} = 0$, here

$$D(z^{-1}) = \sum_{i=0}^{n_d} d_{ii} z^{-i}\cdot z^{-i},$$

$$D(z)u(t-1)y(t-1) = \sum_{i=1}^{n_d} d_{ii}u(t-i)y(t-i)$$

while the common condition of $i \neq j, d_{ij} \neq 0$ may perform analogy. $\{u(t)\}$ and $\{y(t)\}$ are the input and output sequences respectively. $\Delta = 1 - z^{-1}$ is the difference operator, $\{e(t)\}$ is a zero-mean white noise sequence. The equation (1) can be written as

$$\begin{aligned} A(z^{-1})y(t) &= [B(z^{-1}) + D(z^{-1})y(t-1)] \\ &\quad u(t-1) + C(z^{-1})e(t)/\Delta \end{aligned} \qquad (2)$$

3. GPC ALGORITHM OF BILINEAR SYSTEM

The controlled object (2) is assumed to satisfy:

(i) n_a n_b n_c and n_d are known.

(ii) $C(z^{-1})$ is stable polynomial.

The cost function has the following form

$$\begin{aligned} J = E\{&\sum_{j=1}^{N}(y(t+j) - y_r(t+j))^2 + \\ &l\sum_{j=1}^{N_u}(\Delta u(t+j-1))^2 \end{aligned} \qquad (3)$$

Where N and N_u are the prediction and control horizon, whereas l is a weighting constant. In order to make the future outputs of system to track the set value y_0 as smooth as possible, the reference trajectory is:

$$y_r(t) = y(t)$$

$$y_r(t+j) = a y_r(t+j-1) + (1-a)y_0 \qquad (4)$$

where a is a smoothing factor.

To obtain j -step-ahead optimizing predictions, consider the following Diophantine equations:

$$C(z^{-1}) = E_j(z^{-1})A(z^{-1})\Delta + z^{-j}F_j(z^{-1}) \qquad (5)$$

$$\begin{aligned} B(z^{-1})E_j(z^{-1}) = C(z^{-1})G_{1j}(z^{-1}) + \\ z^{-j}L_j(z^{-1}) \end{aligned} \qquad (6)$$

$$\begin{aligned} D(z^{-1})E_j(z^{-1}) = C(z^{-1})N_j(z^{-1}) + \\ z^{-j}R_j(z^{-1}) \end{aligned} \qquad 7$$

where $j = 1, 2, \mathbf{L}, N$,

$$E_j = e_{j0} + e_{j1}z^{-1} + \mathbf{L} + e_{j(j-1)}z^{-(j-1)}$$

$$F_j = f_{j0} + f_{j1}z^{-1} + \mathbf{L} + f_{jn}z^{-n}$$

$$G_{1j} = g_{1j0} + g_{1j1}z^{-1} + \mathbf{L} + g_{1j(j-1)}z^{-(j-1)}$$

$$N_j = n_{j0} + n_{j1}z^{-1} + \mathbf{L} + n_{j(j-1)}z^{-(j-1)}$$

$$L_j = l_{j0} + l_{j1}z^{-1} + \mathbf{L} + l_{jm_1}z^{-m_1}$$

$$R_j = r_{j0} + r_{j1}z^{-1} + \mathbf{L} + r_{jm_2}z^{-m_2}$$

also $\deg E_j = j-1$, $\deg F_j = n$, $\deg G_{1j} = j-1$ $\deg N_j = j-1$, $\deg L_j = m_1$, $\deg R_j = m_2$, $n = \max(n_a, n_c - j)$, $m_1 = \max(n_b - 1, n_c - 1)$ $m_2 = \max(n_d - 1, n_c - 1)$ .For the purpose of simplicity, then $A(z^{-1})$ is written as A, and $B(z^{-1})$ is written as B. Others are the same. Furthermore, the lowercases express polynomial coefficients relative to their capital letters, for example: n_{ji} is the i th coefficient of N_j.

From equation 2 and equation 5 - 7 , the j-step model predictive output can be written as

$$y_m(t+j) = (G_{1j} + G_{2j})\Delta u(t+j-1) + \frac{F_j}{C}y(t) + \frac{L_j}{C}\Delta u(t-1) + \frac{P_j}{C}\Delta u(t-1) \quad (8)$$

i.e.

$$y_m(t+j) = G_j \Delta u(t+j-1) + M_j \quad (9)$$

where

$$G_{2j} = N_j y(t+j-1) = \sum_{i=0}^{j-1} g_{2ji} z^{-i} \quad (10)$$

$$P_j = R_j y(t-1) = \sum_{i=0}^{m_2} p_{ji} z^{-i} \quad (11)$$

In equation (10)and (11),

$$g_{2ji} = n_{ji} y(t+j-i-1) \quad (12)$$

$$p_{ji} = r_{ji} y(t-i-1) \quad (13)$$

where $y(t+j-1)$ in equation (12) is unknown, it is substituted by model predictive value $y_m(t+j-1)$ after time t. The value at time t and before time t can be substituted by its true value. The equation (9) can be written in the vector form

$$\boldsymbol{Y_m} = \boldsymbol{GU} + \boldsymbol{M} \quad (14)$$

$$\boldsymbol{M} = \frac{\boldsymbol{F}}{C}y(t) + \frac{\boldsymbol{L}}{C}\Delta u(t-1) + \frac{\boldsymbol{P}}{C}\Delta u(t-1) \quad (15)$$

where

$\boldsymbol{Y_m} = [y_m(t+1)\ \mathbf{L}\ y_m(t+N)]^T$,

$\boldsymbol{U} = [\Delta u(t)\ \mathbf{L}\ \Delta u(t+N_u-1)]^T$,

$\boldsymbol{M} = [M_1\ \mathbf{L}\ M_N]^T$,

$\boldsymbol{F} = [F_1\ \mathbf{L}\ F_N]^T$,

$\boldsymbol{L} = [L_1\ \mathbf{L}\ L_N]^T$,

$\boldsymbol{P} = [P_1\ \mathbf{L}\ P_N]^T$.

$$\boldsymbol{G} = \begin{bmatrix} g_0 & & & \\ g_1 & g_0 & & \\ \mathbf{L} & \mathbf{L} & \mathbf{L} & \mathbf{L} \\ g_{N-1} & g_{N-2} & \mathbf{L} & g_{N-N_u} \end{bmatrix}_{N\times N_u} \quad (16)$$

Define $\boldsymbol{y_r} = [y_r(t+1)\ \mathbf{L}\ y_r(t+N)]^T$

From the above definition, the cost function (3) can be written as

$$J = E\left\{(\boldsymbol{Y_m} - \boldsymbol{y_r})^T(\boldsymbol{Y_m} - \boldsymbol{y_r}) + l\boldsymbol{U}^T\boldsymbol{U}\right\} \quad (17)$$

Substituting equation (14) into equation (17), and minimize the cost function (17), we get

$$\boldsymbol{U} = (\boldsymbol{G}^T\boldsymbol{G} + l\boldsymbol{I})^{-1}\boldsymbol{G}^T(\boldsymbol{y_r} - \boldsymbol{M}) \quad (18)$$

The real-time optimal control law is given by

$$u(t) = u(t-1) + \boldsymbol{g}^T(\boldsymbol{y_r} - \boldsymbol{M}) \quad (19)$$

Where $\boldsymbol{g}^T$ is the first row of matrix $(\boldsymbol{G}^T\boldsymbol{G} + l\boldsymbol{I})^{-1}\boldsymbol{G}^T$.

4. SIMULATION RESEARCH

Consider the following bilinear system of non-minimum phase

$$y(t) - y(t-1) = u(t-1) + 1.3u(t-2) + 0.3u(t-1)y(t-1) + 0.5u(t-2)y(t-2) + e(t)/\Delta \quad (20)$$

Where $e(t)$ is normal school white noise signal with covariance 0.1. The each parameter of the paper's control algorithm (BGPC) is as follows:

The parameters of model: $n_a = n_b = n_d = 1$, $n_c = 0$

The parameters of controller: $N = 5$ $N_u = 5$ $a = 0.8$ $l = 1$.

Using linear GPC (LGPC) control system, we can make linearization to work point of the object $y(t) = 8$

$$y(t) - y(t-1) = 3.4u(t-1) + 5.3u(t-2) \quad (12)$$

The parameters of controller are the same as the every parameter of BGPC.

The simulation curve of the output and control are shown in the following figures.

The output response and control curve using BGPC is shown in figure 1 and figure 2, where the solid line is system output, and the dashed line is set point.

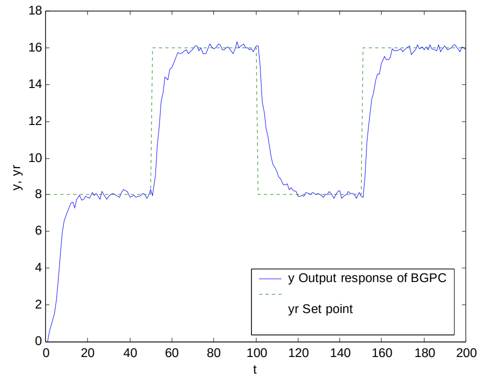

Fig. 1.Output response of the BGPC

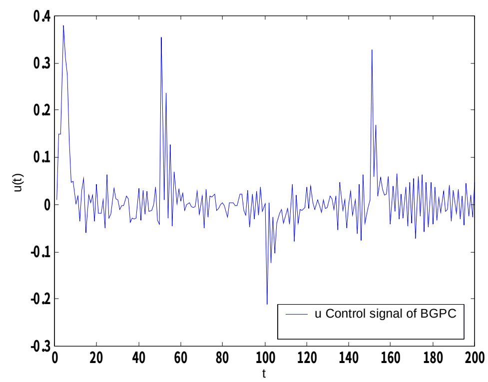

Fig. 2. Control signal of the BGPC

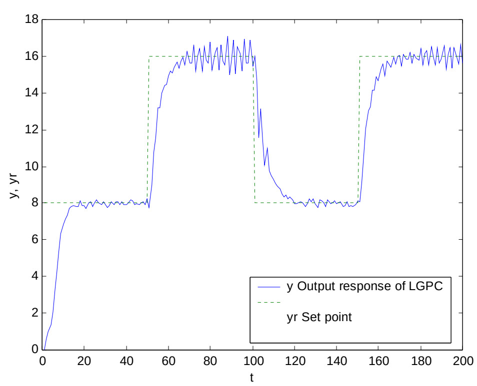

Fig.3.Output response of LGPC

The set point is changed by step amplitude 8. The output response and control curve using LGPC is shown in figure 3 and figure 4, comparing figure 1 with figure 3. It is obvious that the BGPC describes its dynamic characteristic in a biggish scope of set point because of the BGPC using nonlinear model

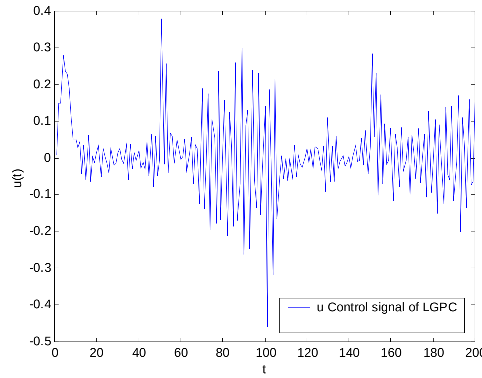

Fig.4. Control signal of LGPC

predictive, but the LGPC algorithm only makes linearization to nonlinear object's one set point, it just is stable in a baby-size scope of set point. The performance of LGPC algorithm is worse than the paper's BGPC algorithm, and the system's output can quickly track the variety of set point, BGPC algorithm's overflow in this paper is obviously more depressed than the LGPC algorithm's, and it can reject the noise well.

5. CONCLUSION

A GPC algorithm is applied to a kind of I/O bilinear systems. The analytic control law, being analogous to linear GPC, is obtained. It makes full use of optimal predictive information, and avoids the difficulty brought by generic nonlinear optimization. The simulation result proves that this algorithm is effective.

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