INPUT-OUTPUT PAIRING OF MULTIVARIABLE PREDICTIVE CONTROL

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Abstract: Regardless of what predictive control strategy is used, the predictive horizon is the main design parameter. The stability, control performance and robustness of predictive control system are mainly depended on it. For multivariable predictive controller, selection of predictive horizon is an input-output pairing problem. In this paper, Response Index Array, Dynamic Interaction Index Array and Relative Steady-State Index Array are proposed as the criteria for the selection of predictive horizon and pairing. The design procedure for multivariable predictive controller is summed up. As an example, the pairing of a heavy oil fractionator is given. The design has been successfully implemented on several industrial fractionators.

Keywords: Predictive control, Input-output pairing, MIMO System

1. INTRODUCTION

During the last two decades, model predictive control (MPC) has become an attractive control strategy within the area of process industries. MPC is a successful strategy for handling multivariable and/or constrained control problems (Garcia and Morari, 1989). Generally, the multivariable controller does not need input-output pairing, which is a main design problem in the multi-loop control, such as conventional PID control. If the predictive horizon and control horizon of MPC are determined, there is no input-output pairing problem. But, pairing problem will rise during MPC design to determine predictive horizon.

So far, the MPC presented in the literatures may be classified into two strategies:

1. MPC based on the input(manipulated variable, MV)-output (controlled variable, CV) model, such as MAC (Richalet, J et.al. 1978; Rouhani,R. and R.K. Mehra 1982), DMC (Cutler, C.R. and B.L. Remaker, 1980), GPC (Clarke, D.W. et.al. 1987,1989), IMC (Garcia and Morari ,1982,1985). Soeterboek (1992) proposed predictive control: a unified approach for such kind of MPC strategies.
2. MPC based on the state space model and state variable feedback (Yuan, 1993).

Sun and Yuan (1993, 1997) proposed Unified Predictive Control, which is based on Polynomial Matrix Description (PMD), for all kinds of the MPC strategies. Yu and Yuan (2002) proved theoretically that all kinds of MPC are equivalent, i.e., the same control performance, depends on prediction horizon P, will be achieved by different MPC strategies as long as there is no model mismatch and no disturbance. In real world, there are unknown disturbance and model mismatch. So different MPC are different in robustness and disturbance rejection. This topic will not be discussed in this paper.

For multivariable process, RGA (Bristol, 1966) is usually used to measure the interaction and the design of multi-loop control. RGA, based on steady-state gain of controlled process, is not suitable for the MPC design, which is based on the dynamic response. In the literatures, contributions on the design of MPC are presented as well as the different MPC strategies mentioned above. The main design issue is how to determine the predictive horizon. MPC has been widely used on multivariable systems, yet, by the author's knowledge, the discussion in literatures of how to determine the predictive horizon for multivariable systems is much less than that of SISO systems.

In this paper, the relationship between predictive horizon and stability, control performance and robustness of MPC system, as the basis of system design, are reviewed in second section. The design of multivariable MPC is an input-output pairing problem and dynamic response index, interaction index and relative steady-state index are proposed as pairing criteria in third section. MPC system design procedure was summed up in section IV. As an example, design of MPC for a heavy oil fractionator is illustrated.

2. PREDICTIVE HORIZON

For multivariable MPC, different CV has different control demand and different response to MV. A reasonable design is that every CV has its own predictive horizon p_i. The predictive horizon of the system **P** is a vector:

$$\mathbf{P} = \begin{bmatrix} p_1 & p_2 & \cdots & p_r \end{bmatrix}^T \qquad (2\text{-}1)$$

where: p_i is the predictive horizon (number of discrete interval) of i^{th} controlled variable.

For illustration and without loss of gernalization, MPC with single prediction algorithm (Yuan, 1992) is used in the following discussion.

The optimal control move was deduced as:

$$\Delta u(k) = S^{-1}(P)[Y_S(k) - Y_p(k)] \qquad (2\text{-}2)$$

where:

$u \in R^m$ Manipulated variable (MV);

$Y \in R^r$ Controlled variable (CV);

$\Delta u(k) = u(k) - u(k-1)$

$$\mathbf{S[(P)]} = \begin{bmatrix} S_{11}(p_1) & S_{12}(p_1) & \cdots & S_{1r}(p_1) \\ S_{21}(p_2) & \cdots & \cdots & S_{2r}(p_2) \\ \vdots & \vdots & \vdots & \vdots \\ S_{r1}(p_r) & \cdots & \cdots & S_{rr}(p_r) \end{bmatrix} \qquad (2\text{-}3)$$

$S_{ij}(p_i)$ is i^{th} CV response at p_i^{th} interval instant after j^{th} MV unit step.

$Y_S(k)$ = Set point of controlled variable;

$Y_p(k) = Y(k) + F_X(z^{-1})\Delta X(k) + F_u(z^{-1})\Delta u(k)$

(Prediction of CV while $\Delta u(k+i) = 0, \quad i \geq 0$)

$X \in R^n$ Measurable state variable (include CV);

$\Delta X(k) = X(k) - X(k-1)$

$F\ (z^{-1}) = F_0 + F_1 z^{-1} + \cdots + F_q z^{-q}$

(Feedback polynomial matrix)

Xi (1993), Yuan (1992, 1993, 1994, and 1997) and others proved some theoretical results (assuming no model mismatch and r=m) for stability and control performance of MPC system related to predictive horizon:

Theorem 1: $\det[\mathbf{S(P)}] \neq 0$ (2-4)

is a necessary stability condition for MPC system.

Theorem 2: If the controlled process is stable and functionally controllable, then:

$$\frac{\det[\mathbf{S(P)}]}{\det[\mathbf{S(\infty)}]} > 0 \qquad (2\text{-}5)$$

is a necessary stability condition for MPC system, where: $[\mathbf{S(\infty)}]$ is the steady-state gain matrix of controlled process.

Theorem 3: If the controlled process is stable and $p_i(i = 1, 2, \cdots, r)$ is tuned sufficiently large, then the MPC system is stable.

Theorem 4: If: $p_i = \delta_i + 1;$ and Theorem1 and Theorem2 are satisfied, then: the i^{th} CV reaches to perfect control.

If $p_i = \delta_i + 1; \quad i = 1, 2, \cdots, r$ (2-6)

And both Theorem 1 and Theorem 2 are satisfied; then: the MPC system reaches to perfect control (all CV reach to perfect control), where: $\delta_i = \delta_i^d - \delta_i^n - 1$, δ_i^d and δ_i^n are the orders of denominator and nominator of i^{th} row in impulse transfer function matrix, respectively.

Perfect Control is defined as: if CV reaches to its set-point at every control (sampling) instant after minimum time delay of set-point or disturbance step change. It is obvious that perfect control is decoupled between CV and CV to disturbance.

In real world, perfect control can be reached only for a class of controlled process with special dynamic property. In most cases, it is difficult to reach, not only limited by the above condition, but also limited by model mismatch and robustness. The control (MV) move is usually another limit. For same CV's deviation, large control move usually lead to fast response and weaker robustness. If increasing prediction horizon p_i makes smaller control move, then, the sluggish response and the better robustness; otherwise, if increasing predictive horizon leads to larger control move (may be constrained by limit), then, the contrary.

According to above analysis, Yuan (1992) proposed to use Relative Predictive Horizon (RPH) β for SISO system to select predictive horizon and trade-off the control performance and control move constraints. RPH is defined as:

$$\beta = \frac{S(p)}{S(\infty)} \qquad (2\text{-}7)$$

Where: $S(p)$ is the value of step response at predictive horizon; $S(\infty)$ is the steady-state value of step response.

$\beta = 0.3 \sim 0.8$ is recommended. Large β leads to a stronger robustness, less control move and sluggish response.

If β is specified, predictive horizon P can be calculated from eq.(2-7). Since β is a float variable and P is an integer,

If $S(\infty) \neq 0$, $\dfrac{S(n-1)}{S(\infty)} < \beta \leq \dfrac{S(n)}{S(\infty)}$, then: $p = n$;

If $S(\infty) = 0$, then: $p = \infty$. (2-8)

This result is extended to multivariable system in this paper.

3. INPUT-OUTPUT PAIRING CRITERIA

For MIMO system, every CV is related to m

manipulated variables, and different MV has different dynamic response. If β is specified, different MV has different predictive horizon. Which MV should be used to determine the corresponding predictive horizon? In this point, the input-output pairing is still a problem for multivariable predictive control system as well as multi-loop control system, but in different content.

For MIMO system, better control performance is desired as well as SISO system and fast response MV should be selected. The distinction is the interaction between CV and MV, and decoupling or less interaction is always required. More MV than CV or more CV than MV made the system more complicated.

The starting point of MPC design is to satisfy the required control performance, which is related to the Relative Predictive Horizon RPH as mentioned above. For MIMO system, the required control performance of i^{th} CV and corresponding RPH β_i can be specified previously. But, the predictive horizon p_i is different for different MV. If β_i is specified, to determine p_i is a problem of input (MV)-output (CV) pairing.

For input-output paring, three Index Arrays are defined.

Definition 1: Response Index Array r_{ij} ***(RIA)***

For i^{th} CV, if β_i is specified, corresponding predictive horizon for j^{th} MV is $p_{ij}\,(j=1,2,\cdots,m)$,

Let: $p_{i\min}=\underset{j}{Min}\{p_{ij}\},\ \ \gamma_{ij}=\dfrac{p_{i\min}}{p_{ij}}$.

$$RIA=\{\gamma_{ij}\}=\begin{bmatrix}\gamma_{11} & \gamma_{12} & \cdots & \gamma_{1m}\\ \gamma_{21} & \gamma_{22} & \ddots & \vdots\\ \vdots & \ddots & \ddots & \vdots\\ \gamma_{r1} & \cdots & \cdots & \gamma_{rm}\end{bmatrix} \qquad (3\text{-}1)$$

is defined as Response Index Array (RIA).

RIA is a criterion of response speed of different MV. The larger the p_{ij}, the faster the response of i^{th} CV to j^{th} MV. In order to make i^{th} CV has better control performance, by the knowledge of SISO system mentioned in Section 2, the prediction horizon P_i may be selected as $p_i=Min\{p_{ij}\}\ \ (j=1,2,\cdots,m)$, and correspondingly $\gamma_{ij}=1$. However, for multivariable system, the interaction must be taken into account.

Definition 2: Dynamic Interaction Index Array μ_{ij} ***(DIA)***

For i^{th} CV, if β_i is specified, it has m possible CV-MV pairing with corresponding predictive horizon p_{ij}. For every possible pairing, the corresponding Dynamic Interaction Index is defined as:

$$\mu_{ij}=\frac{\left|S_{ij}(p_{ij})\right|}{\sum_{l=1}^{m}\left|S_{il}(p_{ij})\right|} \qquad (3\text{-}2)$$

The larger the $\mu_{ij,}$ the weaker the interaction for i^{th} CV-j^{th} MV pairing. It is a possible pairing candidate.

If $\mu_{ij}=1$, it implies that i^{th} CV is affected only by the j^{th} MV and has no interaction with other MV in dynamic. It is a prior pairing candidate. However, the steady-state property must be considered also.

Definition 3: Steady-State Index Array λ_{ij} ***(SIA)***

$$\lambda_{ij}=\frac{\left|S_{ij}(\infty)\right|}{\sum_{l=1}^{m}\left|S_{il}(\infty)\right|} \qquad (3\text{-}3)$$

If $\lambda_{ij}=1$, it implies that i^{th} CV is affected only by the j^{th} MV and has no interaction with other MV in steady-state.

Model predictive control, as showed in eq.(2-2), is a non-steady-state error control strategy for step input and decoupled in steady-state, but the control move may be too large, so, the main consideration of the SIA is the effectiveness and limit of MV.

The larger the λ_{ij}, the smaller the control move in steady-state. If λ_{ij} is near to zero, it means that this MV is ineffective.

RIA, DIA and SIA should be considered in MIMO system design. In addition, the optimization, safety and other requirements of MV should be also considered. The following pairing index $\{a_{ij}\}$ is suggested.

Definition 4: Pairing Index

$$A=\{a_{ij}\}=\begin{bmatrix}\xi_{11} & \cdots & \xi_{1m}\\ \vdots & \ddots & \vdots\\ \xi_{r1} & \cdots & \xi_{rm}\end{bmatrix}\begin{bmatrix}\delta_1 & 0 & \cdots & 0\\ 0 & \delta_2 & \ddots & \vdots\\ \vdots & \ddots & \ddots & 0\\ 0 & \cdots & 0 & \delta_m\end{bmatrix} \qquad (3\text{-}4)$$

Where: $\xi_{ij}=\gamma_{ij}+q_i\mu_{ij}+w_i\lambda_{ij}$ (3-5)

q_i = interaction weighting factor for i^{th} CV.
w_j = control move weighting factor for i^{th} CV.
δ_j = weighting factor for j^{th} MV.

For i^{th} CV, pairing MV is:

$$MV(j):\ \ \{Max[a_{ij}],(j=1,2,\cdots,m)\} \qquad (3\text{-}6)$$

4. MPC DESIGN PROCEDURE

According to the above results, the design procedures for predictive horizon and input-output pairing are summed up as:

1. Give the priority of each CV and corresponding β_i according to the requirement of control performance.

β_i =0.3~0.8 is recommended. Large β leads to a stronger robustness, less control move and sluggish response. Usually, higher priority CV may have smaller β_i.

2. If the controlled process has more MV than CV, give the control priority, optimum priority and target for each MV. If the controlled process has more CV than MV, give the weighting factor of each CV. These two cases, which are beyond the scope of this paper, will not be discussed in detail.

3. Calculate p_{ij}, r_{ij}, μ_{ij}, λ_{ij}, ξ_{ij}.

4. From higher to lower priority of CV, the MV who made least value in ξ_{ij} should be selected as the pairing for control. If the selected MV has been used by higher priority CV, then in the remaining MVs, the one who made ξ_{ij} the least value is recommended in order to have stronger robustness. This procedure results a predictive horizon for each CV and predictive horizon vector $P=[p_1 \quad p_2 \quad \cdots \quad p_r]^T$ for MPC.

4. Check stability by Theorem 1, 2. If unsatisfied, tune β_i or p_i and return to step 1. According to Theorem 3, larger β_i or p_i may usually lead to a stable MPC system.

5. Check control move: MPC design should meet the requirement of control move limit. However, the control move depends on the set-point change, disturbance and status of controlled process. In order to evaluate the control move in design phase, assume all set-point has unit step and initial state equal to zero, check the control move at first sampling instant and steady-state.

The control move at first sampling instant after set-point unit step is:

$$\Delta u = S^{-1}(P) \tag{4-1}$$

The control move at steady-state after unit step is:

$$\Delta u = S^{-1}(\infty) \tag{4-2}$$

So the maximum control move is:

$$\Delta u_{j\max} = \max_i \{\bar{S}_1^j(P), \bar{S}_2^j(P), \cdots, \bar{S}_m^j(P)\} \qquad (4\text{-}2)$$

$$(j=1,2,\cdots,m)$$

Where:

$\bar{S}_i^j(P)$ is the i^{th} element of j^{th} row of $S^{-1}(P)$ or $S^{-1}(\infty)$;

$S(P)$ = step response matrix [eq.(2-3)]

If $\Delta u_{j\max}$ violates the limit, then tune β_i or p_i and return to step 1. Large β_i or p_i usually lead to smaller control move.

6. Simulation. If unsatisfied, choose P again and return to step 1.

The design procedure may be extended to the case of more MV than CV or more CV than MV.

5. EXAMPLE

For illustration, consider the pairing of a heavy oil fractionator, shown in Fig.1. The fractionator has top and two side-draw products. In order to keep the product specification, top and two side-draw temperatures are main controlled variables, as CV1, CV2 and CV3 in Fig.1. Usually, it has three PID controllers TC to keep the temperatures at their set-points.

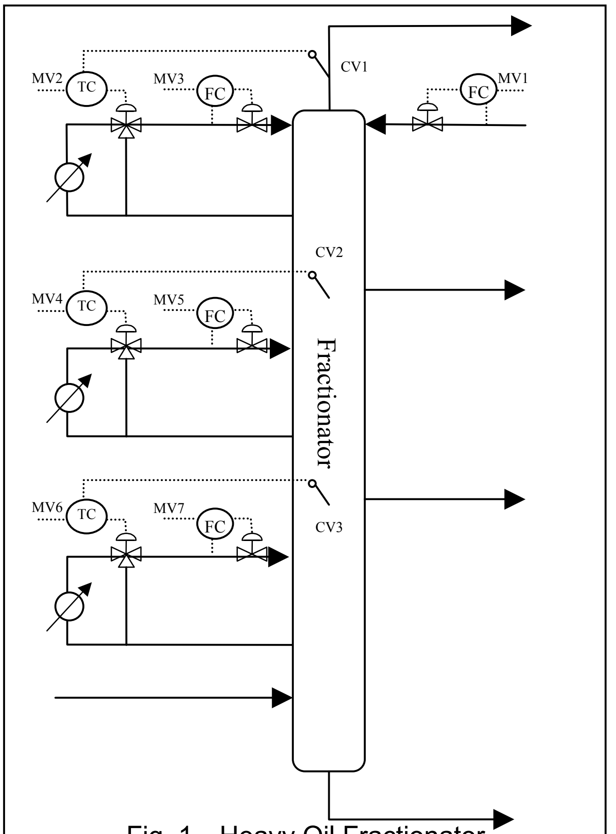

Fig. 1 Heavy Oil Fractionator

The fractionator may have seven manipulated variables:
MV1: Top Reflux Flow rate (PID set point)
MV2: Top Heat Remove Circulation Flow rate (PID set point)
MV3: Set Point of Top Temperature PID Controller (Three-way valve)
MV4: First Heat Remove Circulation Flow rate (PID set point)
MV5: Set Point of first draw Temperature PID Controller (Three-way valve)
MV6: Second Heat Remove Circulation Flow rate (PID set point)
MV7: Set Point of second draw Temperature PID Controller (Three-way valve)

All of the MV has high and low limit as well as corresponding valve opening. If one MV is limited, the controller will select other unlimited MV. So, all of the possible CV-MV pairing and corresponding predictive horizon should be given. For the 3 CV and 7 MV of a fractionator, it has 21 possible pairings. But, if the pairing has too small value of pairing index a_{ij}, it is not suitable for control, which will be illustrated below. If a CV has more suitable MV, the priority of MV should be specified according to the value of a_{ij} and optimization requirement.

Since fractionator has more MV than CV, it is able to push some MV to its optimum value while keep the control performance by other suitable MVs. Usually the optimization targets are minimum heat remove flowrate or minimum open of by-pass (three-way) valve of heat exchanger or steam generator.

The step responses of CV1, CV2 and CV3 to the 7 MVs are given in Fig.2, Fig.3 and Fig.4 respectively.

The priority of CV is specified as: CV1, CV2 and CV3 from higher to lower. The relative predictive horizon is specified as:

$$\beta = [\beta_1\ \beta_2\ \beta_3] = [0.6 \quad 0.6 \quad 0.6]$$

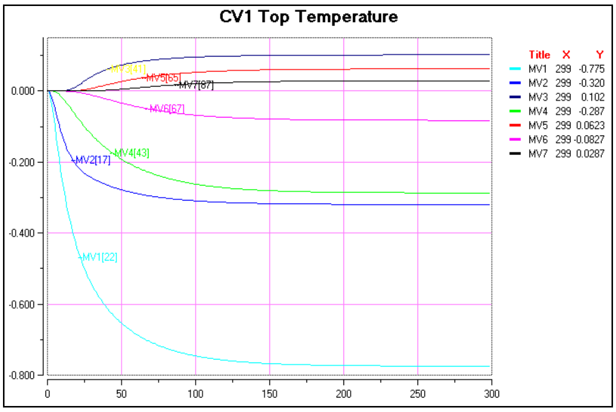

Fig. 2 CV1 Unit Step Response

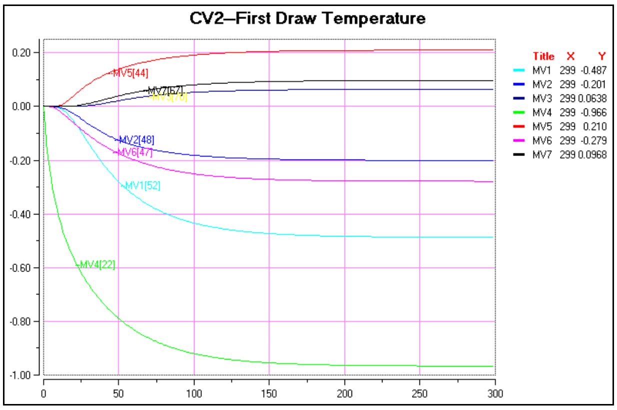

Fig. 3 CV2 Unit Step Response

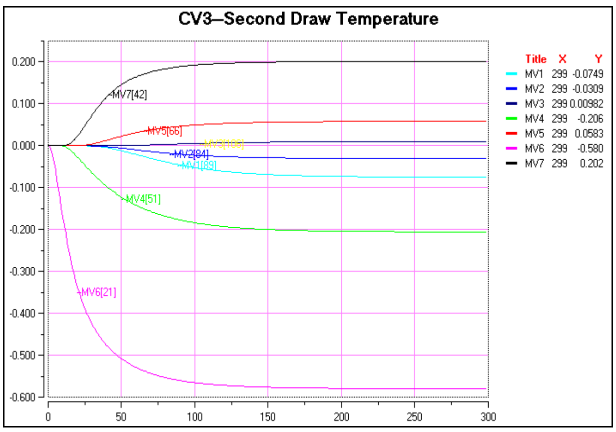

Fig. 4 CV3 Unit Step Response

According to the unit step responses, the predictive horizon, RIA, DIA and SIA are calculated as:

$$\{p_{ij}\} = \begin{bmatrix} 22 & 17 & 41 & 43 & 64 & 67 & 85 \\ 52 & 48 & 70 & 22 & 44 & 47 & 66 \\ 87 & 83 & 104 & 50 & 66 & 21 & 42 \end{bmatrix}$$

$$\{\gamma_{ij}\} = \begin{bmatrix} 0.775 & 1.0 & 0.415 & 0.395 & 0.266 & 0.254 & 0.2 \\ 0.424 & 0.459 & 0.315 & 1.0 & 0.5 & 0.467 & 0.333 \\ 0.241 & 0.253 & 0.202 & 0.42 & 0.318 & 1.00 & 0.5 \end{bmatrix}$$

$$\{\mu_{ij}\} = \begin{bmatrix} 0.568 & 0.296 & 0.06 & 0.147 & 0.027 & 0.035 & 0.009 \\ 0.181 & 0.077 & 0.02 & 0.715 & 0.085 & 0.112 & 0.032 \\ 0.044 & 0.018 & 0.005 & 0.151 & 0.038 & 0.855 & 0.169 \end{bmatrix}$$

$$\{\lambda_{ij}\} = \begin{bmatrix} 0.467 & 0.193 & 0.061 & 0.173 & 0.037 & 0.05 & 0.017 \\ 0.211 & 0.087 & 0.028 & 0.42 & 0.091 & 0.121 & 0.032 \\ 0.064 & 0.027 & 0.008 & 0.185 & 0.05 & 0.5 & 0.174 \end{bmatrix}$$

Assuming: $Q = W = diag[1] = I$

$$\delta_1 = 0.3,\ \delta_2 = \cdots = \delta_7 = 1.0$$

the pairing index a_{ij} is:

$$\{a_{ij}\} = \begin{bmatrix} 0.548 & 1.489 & 0.637 & 0.712 & 0.33 & 0.339 & 0.226 \\ 0.288 & 0.623 & 0.363 & 2.135 & 0.676 & 0.7 & 0.407 \\ 0.105 & 0.298 & 0.316 & 0.856 & 0.406 & 2.355 & 0.843 \end{bmatrix}$$

According to the value of ξ_{1j}, pairing is determined.

For CV1: MV1, MV2, MV3, MV4 are suitable pairings. MV5, MV6, MV7 have smaller pairing index, so they are not suitable pairings. But MV4 is a better pairing candidate to CV2, so the final pairings for CV1 are MV1, MV2 and MV3. The priority is: MV2, MV3, and MV1 from higher to lower. (MV1 has lower value of pairing index, however it is mainly required to reach its optimum value.)

For CV2: MV4 and MV5 are suitable pairings, and the priority is MV4, MV5 from higher to lower.

For CV3: MV6 and MV7 are suitable pairings, and the priority is MV4, MV5 from higher to lower.

These results show that among the 21 possible pairings only seven pairings are suitable. Each CV has fewer pairings than whole MV. Nevertheless, the control system is multivariable according to the eq.(2-2). These pairings have been applied to several industrial heavy oil fractionators.

For heavy oil fractionator, Final Boiling Point (FBP) of top product and 95% ASTM of first draw product are more important controlled variables. They are depended on the top temperature and first draw temperature respectively. They have the same step responses and use same manipulated variables of temperature control, and, the same predictive horizon as well as pairings.

FBP and 95%ASTM should be keep on specified setpoint since they are designed as set point controlled variable. Top and first draw temperatures are designed as zone controlled variables. If the predicted temperatures do not violate their high or low limits, no control is required. The number of CV need to control and the number of available MV are depended on the operation situation. So, the structure of the fractionator as a controlled process is varied. A varied structure predictive coordinated control system based on above design and control requirements for the fractionator was implemented in several industrial plants.

The application shows that the pairing design is suitable for the multivariable control. Fig.5 is a real-time trend acquired from the industrial plant. Set-point of 95% ASTM (D) has been decreased at 9:17 and first heat remove exchanger (steam generator) by-pass valve (F) has been gradually closed to its optimum value. FBP is nearly decoupled to the set-point change of 95% ASTM. Both FBP and 95% ASTM are running with less deviation to their setpoints. MV1 is kept at its optimum value (not showed in Fig.5).

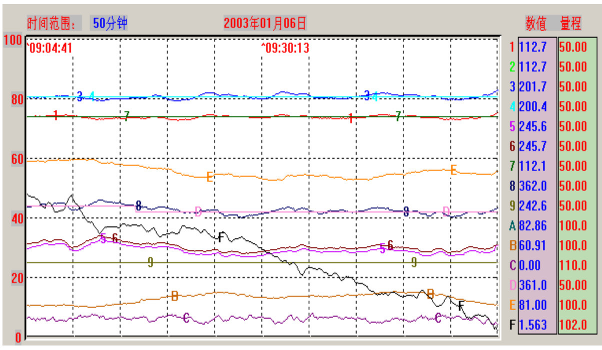

Fig.5 Real time trend of fractionatotr
1,2,7: top temperature and its set-point(CV1)
3,4: Final Boiling Point and its setpoint
5,6,9: first draw temperature and its set-point(CV2)
8,D: 95% ASTM of first draw its set-point
B: first heat remove flowrate(MV4)
C: top heat remover exchanger by-pass valve(MV3)
E: top heat remover circulation flowrate(MV2)
F: first heat remover exchanger by-pass valve(MV5)

6. CONCLUSION

Input-output pairing is a basic problem for multivariable control system design as well as the model predictive control regardless of multivariable or multi-loop structure. Pairing based on dynamics of controlled process is better than that based on steady-state gain. Response index and interaction index proposed in this paper catch on the dynamics and main control system design problems. They are effective criteria for the design of multivariable predictive control systems. The pairing problem should be developed comprehensively.

REFERENCES

Bristol, E.H. (1966). On a New Measure of Interaction for Multivariable Process Control, *IEEE. Trans. Automatic Control*, **AC-11, No.1**, pp133-134.

Clarke, D.W. , C. Mohtadi and P. S. Tuffs (1987). Generalized Predictive Control, *Automatica,* **23(2)**, pp137-160.

Clarke, D.W. and C. Mohtadi (1989). Properties of Generalized Predictive Control, *Automatica*, **25(6)**, pp859-875.

Cutler, C.R. and B.L. Remaker (1980). Dynamic Matrix Control – A Computer Control Algorithm. *Joint ACC Preprints,* Paper **WP5-B**, San Francisco.

Garcia, C.E. and M. Morari (1989). Model predictive control: theory and practice – a survey. *Automatica,* **25**, pp355-348.

Garcia C.E. and M. Morari (1982). Internal Model Control – A Unifying Review and Some New Results, *Ind. Eng. Chem. Process Des. Dev.* **21**, pp308.

Garcia C.E. and M. Morari (1985). Internal Model Control 2 – Design procedure for multivariable systems. *Ind. Eng. Chem. Process Des. Dev.* **24**, pp472-484.

Richalet, J et.al.(1978). Model predictive heuristic control.: Application to industrial processes, *Autimatica*, **14**, pp413-428.

Rouhani,R. and R.K. Mehra (1982). Model Algorithmic Control (MAC): Basic Theoretical Properties. *Automatica,* **18(4)**, pp401-414.

Soeterboek, Ronald (1992). *Predictive Control: A Unified Approach.* Prentice Hall.

Sun, D.X. and P. Yuan (1993). A Unified Predictive Control Algorithm, *Industrial Process Modeling and Control.* **6,** pp105-111.

Sun, Y.H. and P. Yuan (1994). An approach of state feedback predictive control and decoupling, *J. Da-Qing Petroleun Institute*, **18(3)**, pp73-76. (in Chinese)

Xi, Yu Geng (1993). *Predictive Control.* National Defence Publisher.

Yu Z.J. and P. Yuan (2002). Decoupling Features of Model Predictive Control. *J. University of Petroleum*, **26(3)**, pp108-112. (in Chinese)

Yuan P. (1992). Single Prediction Predictive Control, *J. of University of Petroleum,* **16(5)**, pp100-109. (in Chinese)

Yuan P., H.T. Zheng and X. Zuo (1993). State Feedback Predictive Control, *Sinica Automatica,* **19(5)**, pp569-577. (in Chinese)
Journal of Chinese Automation, **6(3)**, pp113-121, 1994 (Allerton Press Inc.)

Yuan P. (1994). *Process Dynamic Model and Its On-line Applications*. China Petro-Chemical Publisher.

Yuan P., D. X. Sun and T. Bai (1997). Unified Predictive Control, *Proceedings 2nd ASCC,* **III**, pp155-159. Seoul, Korea.