A RECEDING OPTIMIZATION CONTROL POLICY FOR PRODUCTION SYSTEMS WITH QUADRATIC INVENTORY COSTS

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Abstract: For stochastic disturbance, such as stochastic demands and breakdown of the system, the production systems is presented as a piecewise deterministic process model. At any given time only one type of product can be produced by the system. A setup (with setup time and cost) is required if production is to be switched from one type of product to another. Preventive maintenance activity is performed for reducing the aging of the system, and the jump rates of the system state depend on the aging of the system. The objective of the problem is to minimize the costs of setup, production, maintenance and the quadratic costs of inventory. The decision variables are a sequence of setups and the production and preventive maintenance plan. According to two time horizons, the original problem is decomposed into two sub-problems. The asymptotic optimality solution of the original problem is constructed via receding optimize the sub-problems. Simulation results show the feasibility of the proposed approach in practice.

Keywords: Optimal production, Equipment aging, Dynamic programming, Receding optimization

1. INTRODUCTION*

Many works focused on the optimal production of manufacturing systems, and the outstanding efforts have been made in the past years (see, e.g., Yan and Zhang, 1997; Sethi and Zhang, 1994, 1995; Gershwin, 1989 and the references therein). However, there are very few lectures casting light on the production systems in processing industry such as a miner water production line. In the system, it is needed to wash pipeline when the production process of a type of mine water product is finished and switched to another type of product. Furthermore, to avoid deteriorating, the duration time of the products being stocked will not be long and their value reduces with time. On the other hand, the failure-prone equipment will be aging during its using. Recently there have been some scholars focusing on the field. Shu and Perkins (2001), and Boukas and Liu (2001) discussed manufacturing systems with deteriorating items. But in their lectures, setup and aging were not discussed. Boukas (1987) and G. Liberopoulos and M. Caramanis (1994) discussed the aging of the machine that affects the frequency of its failure. Boukas and Haurie (1990) and Boukas et al. (1994) discussed preventive maintenance of flexible manufacturing systems considering the machine age function.

In most stochastic production systems with unreliable machines, the optimal production planning is an extremely difficult problem, both theoretically and computationally. The nature of the production system provides it with hybrid dynamic systems characteristics, i.e., possessing both continuous variable dynamical systems characteristics and discrete event dynamical systems characteristics. Based on above, the optimal production of these systems is far more difficult than that of manufacturing systems. And some conclusions about the optimal production of manufacturing systems could not be directly applied to that of the production systems. In the paper, the production system is presented as a piecewise deterministic process model with controlled Markov disturbance. The aging of system and preventive maintenance is taken into consideration and the optimal production of the production systems is discussed. Here, according to two time horizons, the original problem is decomposed into two sub-problems. And the asymptotic optimality solution of the original problem is constructed via receding optimized the sub-problems.

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The paper is organized as the follows. Section 2 presents production system description, dynamic model and objective function. Section 3 simplifies the original problem into two sub-problems, and a receding algorithm framework is also given. Section 4 presents an example exposing basic advantages of the method, and the last Section concludes.

2. DESCRIPTION OF THE PROBLEM

The production system consisting of a set of unreliable equipments can produce n different types of product P_i, i=1, … , n with only one at any given time. Moreover, a setup (with setup duration and setup cost) is required if production is to be switched from one type of product to another. The equipment is subject to random failure and repairs. For reducing the aging of the equipment, the maintenance activity involving lubrication, routine adjustments, etc, will be preformed when the equipment is being used. It is assumed for i, j=1, … , n and $i≠j$, constants $\theta_{ij}\geq 0$ and $K_{ij}\geq 0$, which denote the setup duration and cost of switching from production of P_i to P_j, respectively. Moreover, for any i, j, k=1, … , n, $i≠j$ and $j≠k$, $max\{\theta_{ij}, K_{ij}\}>0$, $\theta_{ij}+\theta_{jk}-\theta_{ik}\geq 0$ and $K_{ij}+K_{jk}e^{-\rho\theta ij}-K_{ik}>0$. If $i=j$, then $\theta_{ij}=K_{ij}=0$. Here $0<\rho<1$ denotes the discount rate.

2.1 *THE DYNAMIC MODEL OF THE SYSTEM*

For $t\geq 0$, let $x_i(t)\in R^1=(-\infty,\infty)$, $u_i(t)\in R^+=[0,\infty]$, and $z_i(t)\in R^+$ denote the surplus, production rate, and the rate of demand for product P_i at time t, i=1, … , n. X, U, and Z are used to denote vectors $[x_1(t), x_2(t), \ldots, x_n(t)]^T\in R^n$, $[u_1(t), u_2(t), \ldots, u_n(t)]^T\in R^{+n}$, and $[z_1(t), z_1(t), \ldots, z_n(t)]^T\in R^{+n}$, respectively, where A^T denotes the transpose of a vector (or a matrix) A. $h(t)$ is used to represent the age of the equipment at time t, $h(t)\in R^+$. The inventory/shortage levels and equipment age of the system are described by the following dynamic differential equations:

$$\begin{cases}\dot{X}(t)=F(\alpha,U(t),Z(t))=U^{\alpha}(t)-Z(t)\\ \dot{h}(t)=f(u^{\alpha}(t),v^{\alpha}(t)) \qquad (X(0),h(0))=(X_0,h_0)\end{cases} \quad (1)$$

Where $F(\cdot,\cdot)=[F_1, F_2, \ldots, F_n]^T$, $U^{\alpha}=[u^{\alpha}_1, \ldots, u^{\alpha}_n]^T$ and $u^{\alpha}(t)=\sum_{i=1}^{n}u_i^{\alpha}(t)$. $v_i^{\alpha}(t)\in R^+$, $u_i^{\alpha}(t)$ is the maintenance rate of the system, the instantaneous production rate of type of product P_i, respectively, at time t with the equipment state $\zeta(t)=\alpha$(defined later). The function f in Eq.(1) represents the effect of the production rate $u^{\alpha}(t)$ on the equipment age and $f(u^{\alpha}(t))=0$ when the equipment is under repair (Boukas and Haurie, 1990). The unreliable equipment states can be classified as (i) breakdown, denoted by state 0; (ii) maintenance, denoted by state 1; (iii) operational or setup, denoted by state 2. Under operational or maintenance state, any type of product can be produced; under breakdown state, nothing is produced. Let $\zeta(t)$ denote the state process of the equipment, and let $E=\{0,1,2\}$ be the state space of the process $\zeta(t)$, $\zeta(t)\in E$.

Let $q_{\alpha\beta}(h(t))$ be the jump rate of the process $\zeta(t)$ from state α to state β at time t. These jump rates are defined by

$$P[\zeta(t+dt)=\beta|\zeta(t)=\alpha] =q_{\alpha\beta}(h(t))dt+o(dt) \quad (2)$$
$$P[\zeta(t+dt)=\alpha|\zeta(t)=\alpha] =1+q_{\alpha\alpha}(h(t))dt+o(dt) \quad (3)$$

Where $\lim_{dt\to 0} o(dt)/dt=0$, $q_{\alpha\alpha}(\cdot)=-\sum_{\beta\neq\alpha} q_{\alpha\beta}(\cdot)$. It is assumed that the jump rate $q_{\alpha\beta}(h(t))$ are bounded and satisfy the following conditions: $|q_{\alpha\beta}(h(t))-q_{\alpha\beta}(h'(t))|\leq C|h(t)-h'(t)|$, $\forall\, h(t), h'(t)\in R$, for some constant C and $|q_{\alpha\alpha}|\geq c_0>0$, $q_{\alpha\beta}(h(t))\geq 0$.

When the equipment has a breakdown, it goes through a repair process. The repair time is usually random and described by the repair rates. The equipment repaired is considered renewed, i.e., the age of the equipment is reset to 0. Since f=0 when the equipment is under repair, for convenience, the age $h(t)$ is reset to 0 at the beginning instead of the end of the repair process. Since inventory control is considered, the outcome will not be influenced, as during the repair process the equipment age remains a constant and its value does not influence the inventory level. Thus, according to our notation, if there is a jump from state α to state β, then the age function $h(t)$ jumps to $h'(t)=\beta h(t)$. According to the above, the following holds

$$h(t+\Delta t)=\begin{cases}0, & \text{If } \zeta(t+\Delta t)=1 \text{ and } \zeta(t-\Delta t)\neq 1;\\ a(t), & \text{Other.}\end{cases} \quad (4)$$

where $\Delta t>0$ is small enough.

2.2 *THE COST FUNCTION AND CONSTRAINTS*

Over the infinite horizon, we are concerned with the optimality problem of finding a production control policy that minimizes the following cost function:

$$J(i,X,s,\Xi,U(\cdot),h(\cdot),v(\cdot),\alpha)=\int_0^s e^{-\rho t}G(X(t),0,2)dt$$
$$+E\left(\int_s^{\infty} e^{-\rho t}G(X(t),U(t),\zeta(t))dt+\sum_{l=0}^{\infty}e^{-\rho\tau_l}K_{i_l i_{l+1}}\right) \quad (5)$$

Where s denotes the remaining setup time, $0\leq s\leq\theta_{ij}$. The decision variables are the rates of production $U(\cdot)$ over time and a sequence of setups denoted by $\Xi=\{(\tau_0, i_0i_1), (\tau_1, i_1i_2), \ldots\}$, where a setup (τ, ij) is defined by the starting time τ and a pair ij denoting that the equipment was already set up to produce P_i and is being switched to be able to produce P_j. Let

G($X(t)$,$U(t)$,$\zeta(t)$) denote the instantaneous cost function of the surplus, repair and maintenance. We denote

$$G(X(t), U(t),\zeta(t))=\sum_{i=1}^{n} c^{+}_{i}(x_i^{+})^2+c^{-}_{i}x_i^{-}$$
$$+c_r ind\{\zeta(t)=0\}+c_m ind\{\zeta(t)=1\} \qquad (6)$$

Positive surplus is supposed to incur a holding cost of c_i^+ per unit commodity per unit time, while the negative a cost of c_i^-, with $c_i^+>0$, $c_i^->0$. $x_i^+:=max(x_i,0)$, $x_i^-:=max(-x_i,0)$. Where c_r and c_m denote cost parameter of repair and maintenance respectively, and $c_r >> c_m$. They are nonnegative constants. $ind\{\zeta(t)=\alpha\}$ is the indicator function of set $\{\zeta(t)=\alpha\}$. The quadratic instantaneous cost function is a useful cost approximation for systems where products are perishable or may become obsolete, as well as systems with storage-space competition.

For $t\geq 0$, the production constraints are given as follows:

$$\begin{cases} 0 \le u_i(t) \le \zeta(t) r_i, \quad i=1,2,\cdots,n \\ u_j(t)=0, \quad j \ne i \\ 0 \le v(t) \le v_{\max,} \end{cases} \qquad (7)$$

Where r_i denotes the maximum production rate of P_i and v_{max} is a constant. Let $U(\alpha)$, a close subset of R^{+n}, denote the production rate control constraints, $\forall\alpha\in E$. Any measurable function $U(t)$ defined on $U(\alpha)$, for each $\alpha\in E$, is called an admissible control. The set $\Theta=\{U(t):t\geq 0\}$ is an admissible policy. The admissible control function $U(t)$ is supposed to be piecewise continuous in t and continuously differentiable with bounded partial derivatives in X. $U(t)$ is a feedback admissible control which can react to the current state. Feedback controls are of practical importance because they will adjust any unfavorable deviation of the state from the targeted position at any time and hence render a better performance, especially when uncertainties or disturbances are presented in the system.

Let ($X(t)$,α,ij) denote the system state at time t, and the space of the system state is $R^n\times E\times\{ij|i, j=1, 2, \ldots, n, i\neq j\}$. The problem is to find an admissible decision $(\Xi,U(\cdot))\in\Omega=(\Xi,\Theta)$ that minimizes $J(i, X, s, \Xi, U(\cdot), h(\cdot), v(\cdot), \zeta(t))$ which is subject to Eq.(1), (7).

3. SIMPLIFIED MODEL FOR THE PRODUCTION SYSTEM

For the large-sized production systems and the presence of some stochastic events, it may be quite difficult to obtain exact optimal feedback policies to run these systems, both theoretically and computationally. One way to cope with these complexities is to develop methods of hierarchical control of these systems. Gershwin, Sethi and Zhang reached many significant conclusions in the direction. Here, the original problem is decomposed into two sub-problems according to the occur frequency of events. Based on the nature of production systems the setup is treated as a typical controllable event. In detail, the setup series and production rate are gotten in the static problem without considering unreliability of the system, and real-time production rate and maintenance rate are solved in the dynamic problem according to the aging of the system. Both static and dynamic problems are discussed on receding horizon. And the asymptotic control policy is composed of the solutions of static and dynamic problems.

3.1 *THE STATIC PROBLEM*

Without losing generality, let s=0, and P_i denote the initial product being produced. Over the finite horizon $[0,T]$ the objective function can be written as the following without considering the dynamic properties of the system

$$J(i,X,0,\Xi,U(\cdot),2)=\int_0^T e^{-\rho t}G(X(t),U(t),2)dt+\sum_{l=0}^{m} e^{-\rho\tau_l}K_{i_l i_{l+1}}$$
$$=\int_0^{T_1} e^{-\rho t}G(X(t),U(t),2)dt+\int_{T_1}^{T_1+\theta_{12}} e^{-\rho t}G(X(t),0,2)dt+e^{-\rho T_1}K_1$$
$$+\int_{T_1+\theta_{12}}^{T_1+\theta_{12}+T_2} e^{-\rho t}G(X(t),U(t),2)dt+\cdots$$
$$+\int_{\sum_{i=0}^{m-1}\theta_{ii+1}+\sum_{i=0}^{m-1}T_i}^{\sum_{i=0}^{m-1}\theta_{ii+1}+\sum_{i=0}^{m}T_i} e^{-\rho t}G(X(t),U(t),2)dt$$
$$+\int_{\sum_{i=0}^{m-1}\theta_{ii+1}+\sum_{i=0}^{m}T_i}^{\sum_{i=0}^{m}\theta_{ii+1}+\sum_{i=0}^{m}T_i} e^{-\rho t}G(X(t),0,2)dt+e^{-\rho(\sum_{i=0}^{m-1}\theta_{ii+1}+\sum_{i=0}^{m}T_i)}K_i \qquad (8)$$

Where $T_0=K_0=\theta_{01}=0$, and T denotes the terminate time when the whole production ends, and T_i denotes the terminate time when the ith type of production is over, for i=1, 2, …, m. It is obvious that $T\to\infty$ as $m\to\infty$.

In any optimal policy, there is always some nonzero time for producing the intended product after the completion of each setup (Sethi and Zhang, 1995), i.e., $T_i>\Delta>0$. Since T_i responds to the inventory $X(T_i)$, T_i as a new state variable is incurred. Let $И_i=[T_1, T_2, \ldots, T_i]$, i=1, 2, …, k, $T_i\in R^+$, then $И_i$ denotes the production time series before i+1th setup. Let the optimal decision of Eq.(8) be $V_{k-i}[j, X(i), И_i]$ when the initial state is $(j, X(i), И_i)$, then a Bellman equation of Eq.(8) can be gotten by dynamic programming

$$V_{k-i}[j,X(i),И_i]=\min_{u_j(i),\Xi}\{J(j,X(i),И_i)+V_{k-(i+1)}[l,X(i+1),И_{i+1}]\} \qquad (9)$$

The solution of Eq.(9) is the optimal production of the system over the finite horizon $[0,T]$ when unreliability is not considered. And the setup series Ξ^*, $x^*_i(T^*_1)$ and T^*_1 will be conveyed to the dynamic problem as an expected value.

3.2 *THE DYNAMIC PROBLEM*

Since the optimal setup times, production rate and the optimal inventory have been gotten by the static planning level, furthermore, the optimal production duration of the initial product being produced is also determined, this paragraph is focused on how to get the real-time production rate and maintenance rate when considering the unreliability and the aging of the system. For using receding algorithm, only given type of product is discussed in the following. Most of papers on optimal control of (non)flexible manufacturing systems consider the demand a constant, which is in fact assumed constant over a short term but not accurate over a long term. It is a piecewise function at least. Demand rate can be gotten by analyzing orders for commodity or the historical data with predictive method. Demand rate is treated as a variable here. According to dynamic property and aging of the equipment, the real time control is carried out using receding horizon control policy.

Without losing generality, let s=0, and the discrete model of the dynamic system on the horizon can be written as follows

$$X_i(k+1)=X_i(k)+U_i(k)-Z_i(k) \tag{10}$$

$$h(k+1)=h(k)+f(u(k), v(k)),\quad u(k)=\sum_{i=1}^{n} u_i(k) \tag{11}$$

$$J(i,X,0,\Xi^*,U(\cdot),h(\cdot),v(\cdot),\alpha)=E\left(\sum_{k=0}^{T/t_p} e^{-\rho k} G(X(k),U(k),\zeta(k))+\sum_{l=0}^{m} e^{-\rho\tau_l} K_{i_l i_{l+1}}\right) \tag{12}$$

Where t_p denote receding horizon. To get the optimal policy of Eq.(12), a Hamilton-Jacobi- Bellman equation must be solved, and Sethi and Zhang reached many significant conclusions in the direction via viscosity solution. The asymptotic optimality solution of the original problem can be gotten by receding optimized the two sub-problems. And the receding algorithm framework is described as follows:

Step1: initialization: assume product P_i is to be produced. $z_i(0)=z_0$, $x_i(0)=x_0$;
Step2: use the given θ_{ij}, K_{ij} to compute: Ξ^*, $x^*_i(T^*_1)$ and T^*_1 by s.(1), (7), (9);
Step3: solve Eq.(10), (11), (12) according to Ξ^* to get u^*_i and v^*_i;
Setp4: if T^*_1 is reached, go to step 2, else go to step 3;

4. SIMULATION OF EXAMPLES

The performance of the policy is shown with examples including following specifications: n=2, and ρ =0.9, z_1=z_2=0.4, $\alpha \in E$={0,1,2}. The other parameter is shown in Tab.1. It is assumed that f is linear. For various initial conditions $X(0)$, the optimal production control is listed in Tab.2. Only optimal production durations of the initial product being produced are listed. Simulation shows that the optimal production control policy is of region switching structure, and of hedging point policy when f is linear.

The trend of the objective function $J(\cdot,\cdot)$ changing with T_1 is illustrated in Fig.1 as T_i is optimal, i=2, 3, …, k. And T_1 is the optimal production duration when $J(\cdot,\cdot)$ is its minimum. Five curves in Fig.1 agree with those five examples in Tab. 2. The simulation results show that different initial conditions respond to different optimal production durations of the initial product being produced. In examples 1, 2, 4 (the sold line), since product P_2 is not sufficient, sometimes even deficient, the policy shortens the optimal production duration of P_1, which are different from Ex. 3(the dotted line). In Ex. 3, P_2 is sufficient, which prolongs the optimal production duration of P_1, but each product is sufficient in Ex. 5(the dashed line), which makes the system produce nothing. The results also agree with hedging point policy.

Table.1 Parameters of the system

θ_{12}	θ_{21}	K_{12}	K_{21}	C_r	C_m
0.65	0.75	1.25	1.15	1.86	0.23

Table.2 Results of simulation

Ex.	$x_1(0)$	$x_2(0)$	C_1^+	C_1^-	C_2^+	C_2^-	T_1	min $J(\cdot,\cdot)$
1	-2.5	-2.0	0.5	3.0	0.6	3.0	1.40	14.2745
2	-1.5	0.0	0.5	3.0	1.0	3.0	3.20	4.5565
3	-2.5	1.5	0.5	3.0	1.0	3.0	5.40	7.5554
4	0.0	0.0	1.0	3.0	1.0	3.0	0.90	1.7482
5	2.0	2.5	1.0	3.0	1.0	3.0	0.0	5.1151

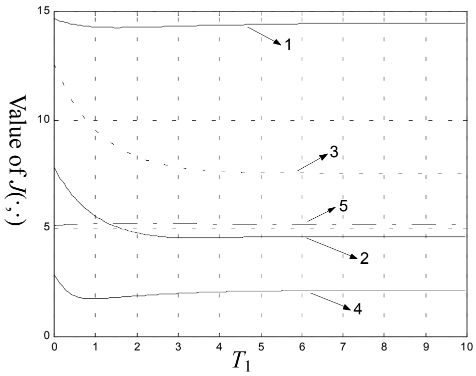

Fig. 1 Tendency of value $J(\cdot,\cdot)$ to T_1

Over a finite time, the policy not only keeps the system run at the least cost but perfectly satisfies the demand. Moreover the policy makes the production satisfy the customers in sum and balances all types of the products, keeping the inventory in low level.

5. CONCLUSIONS

In the paper, the age function, which affects the occur frequency of the failure, is incurred to the objective function with quadratic inventory costs. Based on the nature of the production systems, setup is treated as a typically controllable event in the static sub-problem without considering the details of the setup. Furthermore, the production duration T_i of one type of product as a new state variable is introduced to the problem. In the dynamic sub-problem, based on the age function of the equipment, the production rate and maintenance rate in real-time are gotten by receding algorithm.

The policy decreases the complexities of the original problem, i.e., reduces the stochastic optimal production control problem of multi-dimension vector to the determinist optimal production control problem of multi-dimension vector, and keeps near to the stochastic problem by sliding on one dimension, which renders the receding algorithm feasible, more accurate and real-time. Simulation results show the merits. The age function of the equipment and quadratic inventory costs make the objective near to the practice. However, the optimized solution is not optimal over all globe, but an asymptotic solution. And the receding control policy can decrease the drawback.

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