A METHOD OF CONTROLLING UNSTABLE, NON-MINIMUM-PHASE, NONLINEAR PROCESSES

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Abstract: A method of controlling general nonlinear processes is presented. It is applicable to stable and unstable processes, whether non-minimum- or minimum-phase. The control system includes a nonlinear state feedback and a reduced-order nonlinear state observer. The state feedback induces an approximately linear response. The application and performance of the control method are shown by implementing it on a chemical reactor with multiple steady states. The control system is used to operate the reactor at one of the steady states, which is unstable and non-minimum-phase. The simulation results show that the closed-loop system is globally asymptotically stable.

Keywords: nonlinear control; unstable systems; feedback linearization; non-minimum-phase systems; model-based control

1. INTRODUCTION

During the past 20 years, many advances have been made in nonlinear model-based control, mainly in the frameworks of model-predictive and differential-geometric control. In model-predictive control, the controller action is the solution to a constrained optimization problem that is solved on-line. In contrast, differential-geometric control is a direct synthesis approach in which the controller is derived by requesting a desired closed-loop response in the absence of input constraints. In other words, model-predictive control involves numerical model inversion, while differential-geometric control involves analytical model inversion. In model-predictive control, non-minimum-phase behavior is handled simply by increasing prediction horizons, but in differential geometric control, special treatment is needed.

Differential-geometric controllers were initially developed for unconstrained, minimum-phase (MP) processes. During the past two decades, these controllers were extended to unconstrained, non-minimum-phase (NMP), nonlinear processes. The resulting controllers include those developed by (Kravaris and Daoutidis, 1990; Isidori and Byrnes, 1990; Isidori and Astolfi, 1992; Wright and Kravaris, 1992; van der Schaft, 1992; Isidori, 1995; Chen and Paden, 1996; Devasia et al., 1996; Doyle III et al., 1996; McLain et al., 1996; Hunt and Meyer, 1997; Niemiec and Kravaris, 1998; Kravaris et al., 1998; Devasia, 1999). Most of these controllers are applicable only to single-input single-output, NMP processes. Although controllers of Niemiec and Kravaris (1998), Isidori and Byrnes (1990),

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Isidori and Astolfi (1992), van der Schaft (1992), Chen and Paden (1996), Hunt and Meyer (1997), Devasia et al., (1996), Devasia (1999), and Isidori (1995) are applicable to multi-input multi-output (MIMO), NMP processes, either sets of partial differential equations must be solved (Isidori and Byrnes, 1990; Isidori and Astolfi, 1992; van der Schaft, 1992), or the controllers are applicable to a very limited class of processes (Chen and Paden, 1996; Hunt and Meyer, 1997; Devasia et al., 1996; Devasia, 1999; Isidori, 1995). Recently a differential-geometric control law was developed by Kanter et al. (2002) for stable, nonlinear processes with input constraints and deadtimes, whether the delay-free part of the process is non-minimum- or minimum-phase. This control law cannot be used to operate a process at an unstable operating point.

This paper presents a control method that is applicable to stable or unstable nonlinear processes, whether minimum- or non-minimum-phase. The control system includes a nonlinear state feedback and a reduced-order nonlinear state observer. The state feedback induces an approximately linear response. The application and performance of the control method are shown by implementing it on a chemical reactor with multiple steady states.

This paper is organized as follows. The scope of the study and some mathematical preliminaries are given in Section 2. Section 3 presents the nonlinear feedback control method. The application and performance of the control method are illustrated by numerical simulation of a chemical reactor with multiple steady states in Section 4.

2. SCOPE AND MATHEMATICAL PRELIMINARIES

Consider the general class of multivariable processes with a mathematical model in the form:

$$\left.\begin{aligned} \frac{dx}{dt} &= f(x, u), \qquad x(0) = x_0 \\ y &= h(x) \end{aligned}\right\} \tag{1}$$

where $x = [x_1 \cdots x_n]^T \in \Re^n$ is the vector of state variables, $u = [u_1 \cdots u_m]^T \in \Re^m$ is the vector of manipulated inputs, $y = [y_1 \cdots y_m]^T \in \Re^m$ is the vector of controlled outputs, $f(x, x) = [f_1(x, u) \cdots f_n(x, u)]^T$ and $h(x) = [h_1(x) \cdots h_m(x)]^T$ are smooth. The relative order (degree) of a state x_i, is denoted by r_i, where r_i is the smallest integer for which $\partial[d^{r_i}x_i/dt^{r_i}]/\partial u \neq 0$.

For a given setpoint value, y_{sp}, the corresponing steady state values of the state variables and manipulated inputs satisfy:

$$\begin{aligned} 0 &= f(x_{ss}, u_{ss}) \\ y_{sp} &= h(x_{ss}) \end{aligned}$$

These relations are used to describe the dependence of a nominal steady state, x_{ss_N}, on the setpoint: $x_{ss_N} = F(y_{sp})$.

Let $H(x) = x$ and define the following notation:

$$\begin{aligned} H_i^1(x) &= \frac{dx_i}{dt} \\ &\;\vdots \\ H_i^{r_i-1}(x) &= \frac{d^{r_i-1}x_i}{dt^{r_i-1}} \\ H_i^{r_i}(x, u) &= \frac{d^{r_i}x_i}{dt^{r_i}} \\ H_i^{r_i+1}(x, u^{(0)}, u^{(1)}) &= \frac{d^{r_i+1}x_i}{dt^{r_i+1}} \\ &\;\vdots \\ H_i^{p_i}(x, u^{(0)}, u^{(1)}, \ldots, u^{(p_i-r_i)}) &= \frac{d^{p_i}x_i}{dt^{p_i}} \end{aligned} \tag{2}$$

where $p_i \geq r_i$ and $u^{(\ell)} = d^\ell u/dt^\ell$.

3. NONLINEAR CONTROL METHOD

A state feedback that induces approximately linear responses to the state variables, is first derived. A reduced-order state observer is then designed to reconstruct unmeasured state variables from the output measurements. To add intgeral action to the state feedback-state observer system, a dynamic system is finally added.

3.1 State Feedback Design

Let us request a linear response of the following form for each of the state variable:

$$\begin{aligned} (\epsilon_1 D + 1)^{p_1} x_1 &= x_{ss_{N_1}} \\ &\;\vdots \\ (\epsilon_n D + 1)^{p_n} x_n &= x_{ss_{N_n}} \end{aligned} \tag{3}$$

where $D = d/dt$, and $\epsilon_1, \cdots, \epsilon_n$ are positive constants that set the speed of the state responses. The state responses in (3) can be obtained only when $n = m$. However, since in many processes $m < n$ (there are more state variables than manipulated inputs), the state responses in (3) cannot be achieved. We relax the request for the linear responses by trying to obtain state responses that are as close as possible to the the linear ones described by (3). To this end, we solve the following moving-horizon optimization problem:

$$\min_{u(t)} \sum_{i=1}^{m} w_i \left\| x_{d_i}(\tau) - \hat{x}_i(\tau) \right\|^2_{q_i, [t,\, t+T_{h_i}]} \tag{4}$$

subject to:

$$u^{(\ell)}(t) = 0, \quad \ell \geq 1,$$

where t represents the present time, and $\hat{x}_i(\tau)$ and $x_{d_i}(\tau)$ are predicted values of the state variable x_i and the desired (reference) trajectory of the state variable, respectivey. $||x_i(\tau)||_{q_i,[t,\, t+T_{h_i}]}$ denotes the q_i-function norm of the scalar function $x_i(\tau)$ over the finite time interval $[t, t+T_{h_i}]$ with $T_{h_i} > 0$:

$$||x_i(\tau)||_{q_i,[t,t+T_{h_i}]} = \left[\int_t^{t+T_{h_i}} |x_i(\tau)|^{q_i} d\tau\right]^{\frac{1}{q_i}},\ q_i \geq 1$$

and $w_1, \cdots, w_m$ are adjustable positive scalar weights whose values are set according to the relative importance of the state variables: the higher the value of w_i, the smaller the mismatch between x_{d_i} and x_i.

3.1.1. Output Prediction Equation The future value of the ith state variable over the time interval $[t, t+T_{h_i}]$ is predicted using a truncated Taylor series:

$$\hat{x}_i(\tau) = x_i(t) + \frac{dx_i(t)}{dt}[\tau - t] + \cdots + \frac{d^{p_i}x_i(t)}{dt^{p_i}}\frac{[\tau - t]^{p_i}}{p_i!},\quad i = 1, \cdots, m \tag{5}$$

where

$$\begin{aligned} x_i(t) &= H_i(x(t)) \\ \frac{dx_i(t)}{dt} &= H_i^1(x(t)) \\ &\vdots \\ \frac{d^{r_i-1}x_i(t)}{dt^{r_i-1}} &= H_i^{r_i-1}(x(t)) \\ \frac{d^{r_i}x_i(t)}{dt^{r_i}} &= H_i^{r_i}(x(t), u^{(0)}(t)) \\ &\vdots \\ \frac{d^{p_i}x_i(t)}{dt^{p_i}} &= H_i^{p_i}(x(t), u^0(t), \cdots, u^{(p_i-r_i)}(t)) \end{aligned}$$

3.1.2. Reference Trajectory The reference trajectory of the ith state variable, x_{d_i}, describes the path that the ith state variable, x_i, is forced to follow at time t. The reference trajectory is trackable when the following conditions are satisfied:

$$\begin{aligned} x_{d_i}(t) &= x_i(t) = H_i(x(t)) \\ \frac{dx_{d_i}(t)}{dt} &= \frac{dx_i(t)}{dt} = H_i^1(x(t)) \\ &\vdots \\ \frac{d^{r_i-1}x_{d_i}(t)}{dt^{r_i-1}} &= \frac{d^{r_i-1}x_i(t)}{dt^{r_i-1}} = H_i^{r_i-1}x(t)). \end{aligned}$$

Furthermore, every reference trajectory, x_{d_i}, should take its corresponding state variable, x_i, to its set-point value, $x_{ss_{N_i}}$ as $t \to \infty$. A class of reference trajectories that has these properties is described by

$$\begin{bmatrix} (\epsilon_1 D + 1)^{p_1} x_{d_1}(\tau) \\ \vdots \\ (\epsilon_m D + 1)^{p_m} x_{d_m}(\tau) \end{bmatrix} = x_{ss_N}$$

subject to the "initial" conditions:

$$\begin{aligned} x_{d_i}(t) &= H_i(x(t)) \\ &\vdots \\ \frac{d^{r_i-1}x_{d_i}(t)}{dt^{r_i-1}} &= H_i^{r_i-1}(x(t)) \\ \frac{d^{r_i}x_{d_i}(t)}{dt^{r_i}} &= H_i^{r_i}(x(t), u^{(0)}(t)) \qquad i = 1, \cdots, m \\ &\vdots \\ \frac{d^{p_i-1}x_{d_i}(t)}{dt^{p_i-1}} &= H_i^{p_i-1}(x(t), u^{(0)}(t), \cdots, u^{(p_i-1-r_i)}(t)) \end{aligned}$$

A series solution for the reference trajectory, x_{d_i}, has the following form:

$$\begin{aligned} \hat{x}_{d_i}(\tau) &= H_i(x(t)) + \sum_{\ell=1}^{r_i-1} H_i^{\ell}(x(t))\frac{[\tau - t]^{\ell}}{\ell!} \\ &+ \sum_{\ell=r_i}^{p_i-1} H_i^{\ell}(x(t), u^{(0)}(t), \cdots, u^{(\ell-r_i)}(t))\frac{[\tau - t]^{\ell}}{\ell!} \\ &+ \left[\frac{x_{ss_{N_i}} - H_i(x(t)) - \sum_{\ell=1}^{r_i-1}\epsilon_i^{\ell}\binom{p_i}{\ell}H_i^{\ell}(x(t))}{\epsilon_i^{p_i}} - \frac{\sum_{\ell=r_i}^{p_i-1}\epsilon_i^{\ell}\binom{p_i}{\ell}H_i^{\ell}(x(t), u^{(0)}(t), \cdots, u^{(\ell-r_i)}(t))}{\epsilon_i^{p_i}}\right] \\ &\times \frac{[\tau - t]^{p_i}}{p_i!} + \text{higher order terms} \end{aligned} \tag{6}$$

3.1.3. State Feedback For a process in the from of (1), by using the series forms of the output prediction and reference trajectory equations in (5) and (6), the optimization problem in Eq.4 is:

$$\min_u \sum_{i=1}^{m} w_i \left[\frac{x_{ss_{N_i}} - H_i(x) - \sum_{\ell=1}^{r_i-1} \epsilon_i^\ell \binom{p_i}{\ell} H_i^\ell(x)}{\epsilon_i^{p_i}} \right.$$

$$\left. - \frac{\sum_{\ell=r_i}^{p_i} \epsilon_i^\ell \binom{p_i}{\ell} H_i^\ell(x, u, 0, \cdots, 0)}{\epsilon_i^{p_i}} \right]^2$$

$$\times \left\| \frac{[\tau - t]^{p_i}}{p_i!} \right\|^2_{q_i, [t,\, t+T_{h_i}]} \qquad (7)$$

In the case that $n = m$, the performance index in (4) takes the value of zero and thus, the linear closed-loop state responses of (3) are achieved.

The preceding state feedback is represented in a compact form by:

$$u = \Psi(x, x_{ss_N}) \qquad (8)$$

3.2 Reduced-Order State Observer

In general, measurements of all state variables are not available. In such cases, estimates of the unmeasured state variables can be obtained from the output measurements. Here, we use a reduced-order nonlinear state observer to reconstruct the unmeasured state variables. The details and properties of this estimator can be found in (Soroush, 1997).

For a nonlinear process in the form of (1), the non-redundancy of the controlled outputs ensures the existence of a locally invertible state transformation of the form

$$\begin{bmatrix} \eta \\ y \end{bmatrix} = \mathcal{T}(x) = \begin{bmatrix} Px \\ h(x) \end{bmatrix}$$

where $\eta = [\eta_1, \cdots, \eta_{n-q}]^T$, and P is a constant $(n-q) \times n$ matrix which for the sake of simplicity, is chosen such that (i) each row of P has only one nonzero term equal to one, and (ii) locally

$$rank \left\{ \frac{\partial}{\partial x} \begin{bmatrix} Px \\ h(x) \end{bmatrix} \right\} = n$$

The new variables $\eta_1, \cdots, \eta_{n-q}$ are simply $(n - q)$ state variables of the original model of (1), which satisfy the preceding rank condition, and thus the state transformation $[\eta \;\; y]^T = \mathcal{T}(x)$ is at least locally invertible. In many cases such as the process example considered in this article, the measurable outputs are some of the state variables. In such cases, the state transformation is linear and globally invertible.

The system of (1), in terms of the new state variables $\eta_1, \cdots, \eta_{n-q}, y$, takes the form

$$\begin{cases} \dot{\eta} = F_\eta(\eta, y, u) \\ \dot{y} = F_y(\eta, y, u) \end{cases} \qquad (9)$$

where

$$F_\eta(\eta, y, u) = Pf\left[\mathcal{T}^{-1}(\eta, y), u\right];$$

$$F_y(\eta, y, u) = \left.\frac{\partial h(x)}{\partial x}\right|_{x=\mathcal{T}^{-1}(\eta,y)} f\left[\mathcal{T}^{-1}(\eta, y), u\right]$$

One can then design a closed-loop, reduced-order observer of the form:

$$\begin{aligned} \dot{z} &= F_\eta(z + Ly, y, u) - LF_y(z + Ly, y, u) \\ \hat{x} &= \mathcal{T}^{-1}(z + Ly, y) \end{aligned} \qquad (10)$$

where the constant $[(n - q) \times q]$ matrix L is the observer gain. The observer gain should be set such that the observer error dynamics are asymptotically stable (Soroush, 1997).

3.3 Integral Action

To ensure offset-free response of the closed-loop system in the presence of constant disturbances and model errors, the final control system should have integral action. The integral action can be added by using the dynamic system:

$$\begin{aligned} (\epsilon_1 D + 1)^{p_1} \xi_1 &= \phi_1(x, u) \\ &\vdots \\ (\epsilon_n D + 1)^{p_n} \xi_n &= \phi_n(x, u) \end{aligned} \qquad (11)$$

where

$$\phi_i(x, u) = \sum_{\ell=0}^{r_i-1} \epsilon_i^\ell \binom{p_i}{\ell} H_i^\ell(x) +$$

$$\sum_{\ell=r_i}^{p_i-1} \epsilon_i^\ell \binom{p_i}{\ell} H_i^\ell(x, u^{(0)}, \cdots, u^{(\ell-r_i)}), \quad i = 1, \cdots, m$$

3.4 Control System

Combing the equations in (8), (10) and (11) leads to the following control system that has intgeral action:

$$\begin{aligned} \dot{z} &= F_\eta(z + Ly, y, u) - LF_y(z + Ly, y, u) \\ \hat{x} &= \mathcal{T}^{-1}(z + Ly, y) \\ (\epsilon_1 D + 1)^{p_1} \xi_1 &= \phi_1(x, u) \\ &\vdots \\ (\epsilon_n D + 1)^{p_n} \xi_n &= \phi_n(x, u) \\ v &= F(y_{sp}) - \hat{x} + \xi \\ u &= \Psi(\hat{x}, v) \end{aligned} \qquad (12)$$

The control system parameters $\epsilon_1, \cdots, \epsilon_n$ set the speed of the closed-loop state responses; the smaller the value ϵ_i, the faster the x_i response. The parameters $p_1, \cdots, p_n$ should be chosen such that $p_1 = r_1, \cdots, p_n = r_n$ when the process is minimum-phase, and $p_1 > r_1, \cdots, p_n > r_n$ when the process is non-minimum-phase.

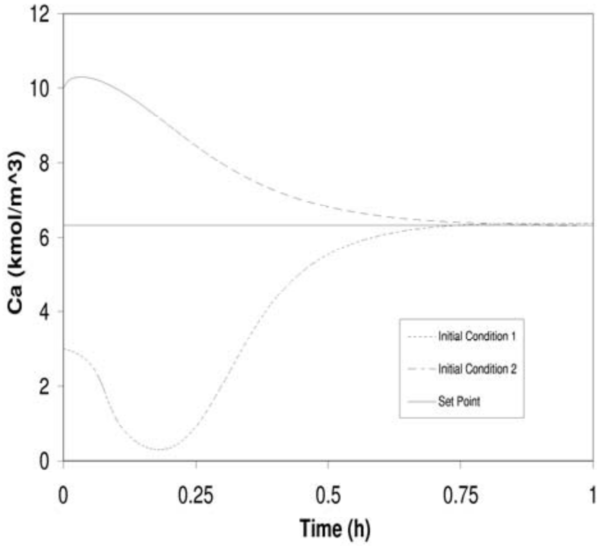

Fig. 1. Closed-loop response of the reactant outlet concentration for different initial conditions.

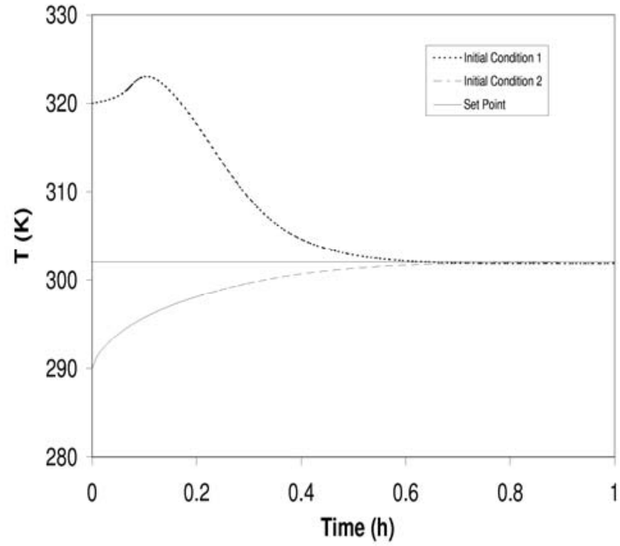

Fig. 2. Closed-loop response of the outlet stream temperature corresponding to Figure 1.

4. APPLICATION TO A CHEMICAL REACTOR

Consider a constant-volume, non-isothermal, continuous-stirred-tank reactor, in which the reaction $A \rightarrow B$ takes place in liquid phase. The reactor dynamics are represented by the following model:

$$\begin{aligned} \frac{dC_A}{dt} &= -kC_A + (C_{A_i} - C_A)u/V \\ \frac{dT}{dt} &= \gamma k C_A + (T_i - T)u/V + q \\ y &= T \end{aligned} \qquad (13)$$

where $k = 5.0 \times 10^8 \exp(-8100/T) \quad s^{-1}$, $\gamma = 3.9 \ m^3 \ K \ kmol^{-1}$, $q = -2.519 \times 10^{-2} \ K.s^{-1}$, $C_{A_i} = 12 \ kmol \ m^{-3}$, $T_i = 300 \ K$, and $V = 0.1 \ m^3$.

The control method of (13) is applied to the reactor, and the resulting controller is used to operate the reactor at the unstable, non-minimum-phase steady state ($6.319 \ kmol.m^{-3}$, $302.0 \ K$). The following controller parameter values are used: $\epsilon_1 = 360 \ s$, $\epsilon_2 = 360 \ s$, $p1 = 2$, $p_2 = 2$, and $L = 0.5$

For the two sets of initial conditions, $[C_A(0), \ T(0)]$= [3.0, 320] and [10.0, 290], the peformance of the controller is shown in Figures 1–3. As can be seen from these figures, the controller is capable of operating the process at the desired steady state, regardless of the initial conditions of the process.

NOTATION

A= Reactant
B = Product
C_{A_i}= Inlet concentration of the reactant, $kmol \ m^{-3}$.
C_A= Outlet concentration of the reactant, $kmol \ m^{-3}$.
D = Differential operator, $D = d/dt$.
k = Reaction rate constant, s^{-1}.
m= Number of manipulated inputs and controlled outputs.
n = Process order.
r_i = Relative order of state variable x_i.
t = Time, s.
T = Reactor outlet temperature, K.
T_i = Reactor inlet temperature, K.
u = Process input vector.
V = Reactor volume, m^3.
x = Vector of state variables.
y = Vector of controlled outputs.
y_{sp} = Vector of set-points.

Greek

$\epsilon_1, \cdots, \epsilon_n$ = adjustable parameters of controller.
$\xi_1, \cdots, \xi_n$ = State variables of the controller.
γ = Reactor model parameter, $K \ m^3 \ kmol^{-1}$.

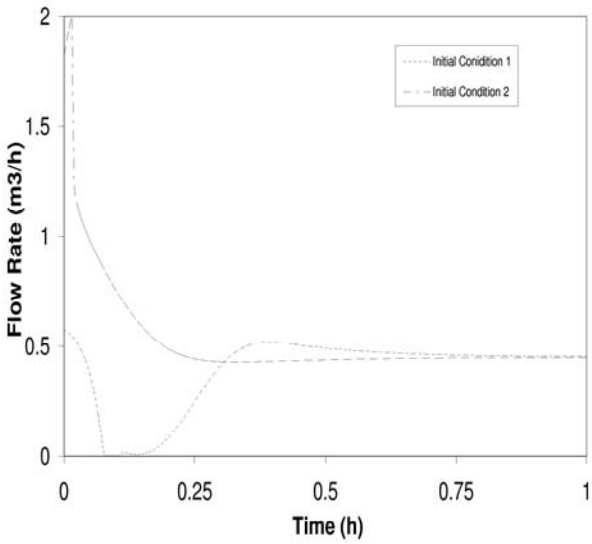

Fig. 3. Manipulated input profiles corresponding to Figures 2 and 3.

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