NON-FRAGILE PID STABILIZING CONTROLLER ON SECOND-ORDER SYSTEMS WITH TIME DELAY

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Abstract: Based on an extension of the Hermite-Biehler Theorem to the quasipolynomial stability problem, this paper studies the problem of stabilizing a second-order plant with dead time via a PID controller. The region in PID parameters space for the closed-loop stability is given. For a feasible proportional gain (k_p), the region of all the admissible integral gains (k_i) and derivative gains (k_d) is a convex polygon. The PID controller design is formulated as a convex optimization problem of load disturbance rejection with constraints on stability and non-fragility, which can be solved by using existing linear programming techniques.

Key words: PID control; stability; non-fragile; quasipolynomial; linear programming

1. INTRODUCTION

In today's process industry it is still PID controllers that are the most frequently used controllers. Estimates indicate that more than 90% of all controllers used are of the PID type. The main reason is its relatively simple structure, which can be easily understood and implemented in practice (Åström & Hägglund, 1995). In order to satisfy the increasing requirements for control systems performance, knowing all stabilizing PID controllers and using this information in controller design can be extremely useful. To this extent, Ho, Datta, and Bhattacharyya (1996) obtained a characterization of all stabilizing gains using a generalized Hermite-Biehler Theorem. They (1997a,b) have then extended this result to characterize stabilizing PID controllers. Recently, Silva et al (2001) solved the problem of stabilizing a first-order plant with time delay via a PI controller. On the other hand, in practice, controllers do have a certain degree of errors due to finite word length in any digital systems, the imprecise inherent in analog systems and need for additional tuning of parameters in the final controller implementation. It is shown that relatively small perturbations in controller parameters could even destabilize the close-loop system (Kell and Bhattacharyya 1997, Dorato 1998). This brings a new issue: how to design a controller for a given plant such that the controller is insensitive to some amount of errors with respect to its parameters, i.e., the controller is non-fragile.

In this paper, the problem of designing a non- fragile PID controller is studied for a class of second-order systems with time delay. First the region in PID parameters space for the closed-loop stability is derived based on a suitable extension of the Hermite-Biehler Theorem. Then the primary goal of the design problem is to achieve good disturbance rejection, which in mathematical terms corresponds to minimizing the integrated error. According to Åström *et al*. (1998), this is equivalent to maximizing the integral gain k_i for a step change in the load disturbance. Finally the PID controller design is to maximize the integral gain k_i with constraints on stability and non-fragility.

This paper is organized as follows. In Section 2, some preliminary results due to Pontryagin and others are presented for the stability of systems with time delay. These results are used in Section 3 to study the stabilization problem via a PID controller. The procedure for determining the PID parameters is presented in Section 4. The simulation and experiment examples are given in Section 5 and Section 6 to demonstrate the usefulness of the proposed results.

2. PROBLEM STATEMENT AND PRELIMINARY RESULTS

Consider the feedback control system shown in Fig.1,

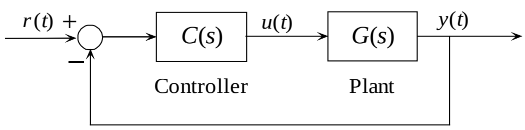


Fig.1. Feedback control system.

where the transfer function $G(s)$ and the PID controller $C(s)$ are in the form of

$$G(s) = \frac{k}{as^2 + bs + 1} e^{-Ls} \tag{1}$$

$$C(s) = \frac{k_d s^2 + k_p s + k_i}{s} \tag{2}$$

where k, a, b, L are known, k_d, k_p, k_i are the PID parameters.

When the time delay L of the plant model is zero, the characteristic equation of the closed-loop system is given by

$$\delta(s) = as^3 + (b + kk_d)s^2 + (1 + kk_p)s + kk_i \quad . \qquad (3)$$

It can be concluded from the Routh-Hurwitz stability criterion that the closed-loop system is stable if

$$a > 0\,, \quad b + kk_d > 0\,,$$
$$k_p > \frac{ak_i}{b + kk_d} - \frac{1}{k}\,, \quad kk_i > 0. \qquad (4)$$

or

$$a < 0\,, \quad b + kk_d < 0\,,$$
$$k_p < \frac{ak_i}{b + kk_d} - \frac{1}{k}\,, \quad kk_i < 0. \qquad (5)$$

When the delay of the model is nonzero, the closed-loop characteristic equation of the system is given by

$$\delta(s) = k(k_d s^2 + k_p s + k_i)e^{-Ls} + s(as^2 + bs + 1) \qquad (6)$$

that includes an exponent term. So the region of parameters k_d, k_p, k_i can't be determined directly by Routh-Hurwitz stability criterion for closed-loop stability. To overcome the difficulty, a new method is put forward based on the Hermite-Biehler Theorem and its extension.

Consider the closed-loop characteristic equation of the system with time delay

$$\delta(s) = d(s) + e^{-sT_1} n_1(s) + \cdots + e^{-sT_m} n_m(s) \qquad (7)$$

where $d(s), n_i(s)\ (i = 1,2,\cdots,m)$ are polynomials with real coefficients. The characteristic equations of this form are known as quasipolynomials. To study the stability of certain classes of quasiplynomials, we first introduce the extension of the Hermite-Biehler Theorem, which was developed by Bhattacharyya et al (1995). In (7), assuming

A1. $\deg[d(s)] = n$ and $\deg[n_i(s)] < n$,
for $i = 1,2,\cdots,m$;

A2. $0 < T_1 < T_2 < \cdots < T_m$.

Instead of (7), we consider

$$\delta^*(s) = e^{sT_m}\delta(s)$$
$$= e^{sT_m} d(s) + e^{s(T_m - T_1)} n_1(s) + e^{s(T_m - T_2)} n_2(s) + \cdots + n_m(s) \qquad (8)$$

Since e^{sT_m} does not have any finite zeros, the Hurwitz stability of $\delta(s)$ is equivalent to that of $\delta^*(s)$. The following Lemma presents a necessary and sufficient condition for the Hurwitz stability of $\delta(s)$.

Lemma 1. (**Extended Hermite-Biehler Theorem**) Let $\delta^*(s)$ be given by (8), and write

$$\delta^*(j\omega) = \delta_r(\omega) + j\delta_i(\omega)$$

where $\delta_r(\omega)$ and $\delta_i(\omega)$ represent, respectively, the real and imaginary parts of $\delta^*(j\omega)$. Under assumptions (A1) and (A2), $\delta^*(s)$ is Hurwitz stable if and only if

(1) $\delta_r(\omega)$ and $\delta_i(\omega)$ have only single real roots and these interlace;

(2) $\delta_i'(\omega_0)\delta_r(\omega_0) - \delta_i(\omega_0)\delta_r'(\omega_0) > 0$, for some ω_0 in $(-\infty, \infty)$.

where $\delta_r'(\omega)$ and $\delta_i'(\omega)$ denote the first derivative with respect to ω of $\delta_r(\omega)$ and $\delta_i(\omega)$, respectively.

A crucial step in applying the above theorem to check stability is to ensure first that $\delta_r(\omega)$ and $\delta_i(\omega)$ have only real roots. Such a property can be ensured by using the following result (Bellman & Cooke, 1963).

Lemma 2. Let M and N denote the highest powers of s and e^s, respectively, in $\delta^*(j\omega)$, and η be an appropriate constant such that the coefficients of terms of highest degree in $\delta_r(\omega)$ and $\delta_i(\omega)$ do not vanish at $\omega = \eta$. Then for the equations $\delta_r(\omega) = 0$ or $\delta_i(\omega) = 0$ to have only real roots, it is necessary and sufficient that in the interval $\omega \in [-2l\pi + \eta,\ 2l\pi + \eta]$ $\delta_r(\omega)$ or $\delta_i(\omega)$ has exactly $4lN + M$ real roots starting with a sufficiently large number l.

3. STABILIZATION USING A PID CONTROLLER

In this section, a stabilizing region in PID parameters space is given based on the extended Hermite-Biehler Theorem. Obviously, the equation (6) satisfies the assumptions (A1) and (A2). A quasipolynomial is constructed as follows:

$$\delta^*(s) = e^{Ls}\delta(s)$$
$$= k(k_d s^2 + k_p s + k_i) + s(as^2 + bs + 1)e^{Ls}$$

Substituting $s = j\omega$ in the above yields

$$\delta^*(j\omega) = \delta_r(\omega) + j\delta_i(\omega)\,.$$

where

$$\delta_i(\omega) = \omega[kk_p - (a\omega^2 - 1)\cos(L\omega) - b\omega\sin(L\omega)]\,;$$
$$\delta_r(\omega) = kk_i - kk_d\omega^2 + \omega(a\omega^2 - 1)\sin(L\omega) - b\omega^2\cos(L\omega).$$

The controller parameter k_p only affects the imaginary part of $\delta^*(j\omega)$. Whereas k_i and k_d affect the real part $\delta^*(j\omega)$. Let $z = L\omega$, then

$$\delta_r(z) = k\left[k_i - k_d\frac{z^2}{L^2} - h(z)\right] \qquad (9)$$

$$\delta_i(z) = \frac{z}{L}\left[kk_p - \left(a\frac{z^2}{L^2} - 1\right)\cos(z) - b\frac{z}{L}\sin(z)\right] \qquad (10)$$

where

$$h(z) = \frac{z}{kL}\left[b\frac{z}{L}\cos(z) - \left(a\frac{z^2}{L^2} - 1\right)\sin(z)\right]. \qquad (11)$$

A general assumption on $k > 0, a > 0, b > 0$, $L > 0$ is suitable for a second-order model with time delay. The following theorem gives a stabilizing region in PID parameters space.

Theorem 1. Under the assumption on $k>0$, $a>0$, $b>0$ and $L>0$, the closed-loop system with transfer function $G(s)$ as in (1) is stable if and only if

$$k_p \in (\max(-1/k, k_{plow}), k_{pup});$$

$$k_i>0,\quad k_d>-\frac{b}{k},\quad k_p>\frac{ak_i}{b+kk_d}-\frac{1}{k};$$

$$k_i-k_d\frac{z_1^2}{L^2}<h_1,\ \ldots,\quad k_i-k_d\frac{z_{e-2}^2}{L^2}<h_{e-2};$$

$$k_i-k_d\frac{z_2^2}{L^2}>h_2,\ \ldots,\quad k_i-k_d\frac{z_{f-2}^2}{L^2}>h_{f-2}. \qquad (12)$$

Where

(1). k_{plow} and k_{pup} denote the upper bound of all minimum values and lower bound of all maximum values, respectively, for

$$k_p(z)=\frac{1}{k}\left[\left(a\frac{z^2}{L^2}-1\right)\cos(z)+\frac{b}{L}z\sin(z)\right];$$

(2). $z_j>0$ ($j=1,2,3,\cdots$) denote the roots of $\delta_i(z)$ associated with a given parameter k_p;

(3). When j is an odd number, k_{dj} denote k_d in the joints of $k_i-k_d(z_j^2/L^2)=h_j$ and $k_i=0$. Then e is the minimum odd number satisfying $k_{de}<k_{d1}$;

(4). When j is an even number, k_{dj} denotes k_d in the joints of $k_i-k_d(z_j^2/L^2)=h_j$ and $k_i-k_d(z_1^2/L^2)=h_1$. Then f is the minimum even number satisfying $k_{df}>k_{d2}$.

(5).Where

$$h_j=h(z_j)=\frac{z_j}{kL}\left[b\frac{z_j}{L}\cos(z_j)-\left(a\frac{z_j^2}{L^2}-1\right)\sin(z_j)\right].$$

Proof:

Step 1: Check the condition 2 of Lemma 1. Let $\omega_0=z_0=0$, thus

$$\delta_i'(z_0)\delta_r(z_0)-\delta_i(z_0)\delta_r'(z_0)=kk_i(kk_p+1)/L.$$

From the above assumption and (4), then $\delta_i'(z_0)\delta_r(z_0)-\delta_i(z_0)\delta_r'(z_0)>0$ if $k_i>0$ and $k_p>-1/k$.

Step 2: Check the condition 1 of Lemma 1. From (10) the roots of the imaginary part can be computed, i.e.,

$$\delta_i(z)=\frac{z}{L}\left[kk_p-\left(a\frac{z^2}{L^2}-1\right)\cos(z)-b\frac{z}{L}\sin(z)\right]=0.$$

The solution are $z=0$ and

$$k_p=\frac{1}{k}\left[\left(a\frac{z^2}{L^2}-1\right)\cos(z)+\frac{b}{L}z\sin(z)\right]. \qquad (13)$$

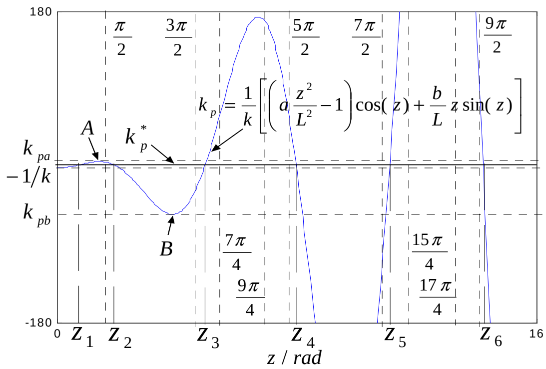
For $j=1,2,3,\cdots$; the derivatives of k_p versus z

$$k_p'\left[\left(j-\frac{1}{2}\right)\pi\right]=\frac{1}{k}\left[(-1)^j\left(a\frac{(2j-1)^2\pi^2}{4L^2}-1\right)+(-1)^{j-1}\frac{b}{L}\right], \qquad (14)$$

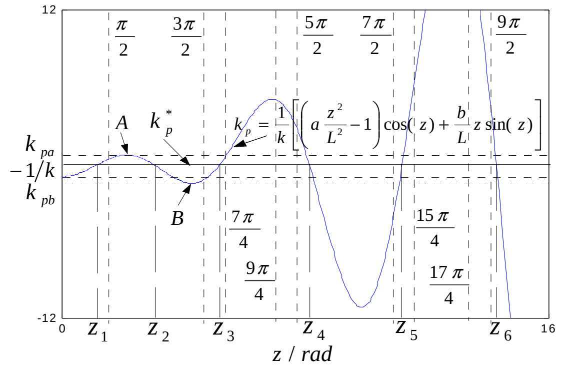
$$k_p'(j\pi)=(-1)^j\frac{j}{k}\left(\frac{2a}{L^2}+\frac{b}{L}\right)\pi. \qquad (15)$$

From (14), (15) and the assumption on $k>0$, $a>0$, $b>0$, $L>0$, it can be seen that k_p is strictly monotonously increasing in $(2j-1)\pi$, while it is strictly monotonously decreasing in $2j\pi$. This means that k_p versus z depicted by (13) is oscillatory and nonconvergent, and its oscillatory period is gradually to tend towards 2π. The curve of k_p versus z depicted by (13) is shown in figure 2, where A, B, C and D represent extremums of the curve, respectively.

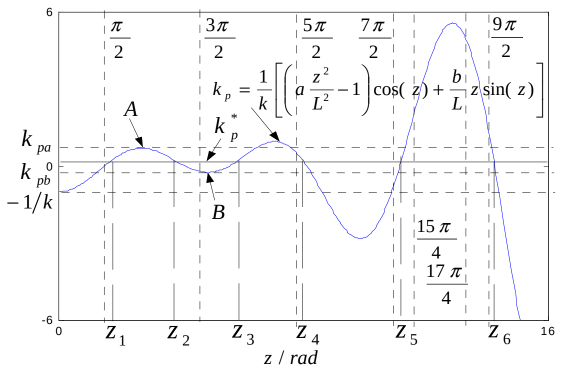
Now check if $\delta_i(z)$ has only real roots using Lemma 2. Substituting $s_1=Ls$ in the expression for $\delta^*(s)$, it can be seen that for the new quasi-polynomial in s_1, $M=3$ and $N=1$. Select $\eta=\pi/4$ to satisfied the requirements that $\sin(\eta)\neq 0$ and $\cos(\eta)\neq 0$. Now from Figs 2(a) and 2(b), it is seen that for a given $k_p^*\in(-1/k, k_{pa})$, $\delta_i(z)$ has four real roots in the interval $[0, 2\pi-\pi/4]=[0, 7\pi/4]$, including a root at $z=0$. Since $\delta_i(z)$ is an even function of z, it follows that in the interval $[-7\pi/4, 7\pi/4]$, $\delta_i(z)$ will have seven roots, whereas $\delta_i(z)$ has no root in the interval $[7\pi/4, 9\pi/4]$. Thus, $\delta_i(z)$ has $4N+M=7$ real roots in the interval $[-2\pi+\pi/4, 2\pi+\pi/4]$. Moreover, it is clear that $\delta_i(z)$ has two real roots in each of the intervals $[2l\pi+\pi/4, 2(l+1)\pi+\pi/4]$ and $[-2(l+1)\pi+\pi/4, -2l\pi+\pi/4]$ for $l=1,2,\cdots$. Hence, it follows that $\delta_i(z)$ has exactly $4lN+M$ real roots in $[-2l\pi+\pi/4, 2l\pi+\pi/4]$ starting from $l=1$ for any given $k_p^*\in(-1/k, k_{pa})$. At the same time, starting from $l=2$, $\delta_i(z)$ has $4lN+M$ real roots in the interval $[-2l\pi+\pi/4, 2l\pi+\pi/4]$ for any given $[-2l\pi+\pi/4, 2l\pi+\pi/4]$ shown in Fig.2(c); while in Fig. 2(d), starting from $l=3$, $\delta_i(z)$ has $4lN+M$ real roots in the interval $[-2l\pi+\pi/4, 2l\pi+\pi/4]$. Hence from Lemma 2, it can be concluded that $\delta_i(z)$ has $4lN+M$ roots in the interval $[-2l\pi+\pi/4, 2l\pi+\pi/4]$ starting from a large enough value of l, for $k_p\in(\max(-1/k, k_{plow}), k_{pup})$.



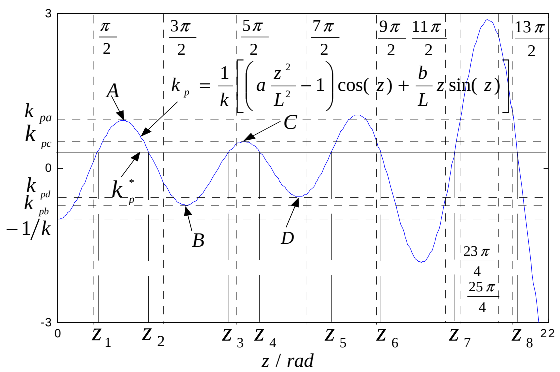
(a) for $a\frac{\pi^2}{4L^2}-1>\frac{b}{L}$



(b) for $a\frac{\pi^2}{4L^2}-1<\frac{b}{L}<a\frac{9\pi^2}{4L^2}-1$



(c) for $a\frac{9\pi^2}{4L^2}-1<\frac{b}{L}<a\frac{25\pi^2}{4L^2}-1$



(d) for $a\frac{25\pi^2}{4L^2}-1<\frac{b}{L}<a\frac{49\pi^2}{4L^2}-1$

Fig.2. The curve of k_p versus z by equation (13)

Let z_j denote roots of $\delta_i(z)$, then interlacing of the roots of $\delta_i(z)$ and $\delta_r(z)$ is equivalent to $\delta_r(z_0)>0$ (since $k_i>0$ as derived in step 1), $\delta_r(z_1)<0$, $\delta_r(z_2)>0$, $\delta_r(z_3)<0$, $\delta_r(z_4)>0$, and so on. Using this fact and (9), (11) it is obtained

$$\begin{aligned}
\delta_r(z_0)>0 &\quad\Rightarrow\quad k_i>0\\
\delta_r(z_1)<0 &\quad\Rightarrow\quad k_i-k_d\left(z_1^2/L^2\right)<h_1\\
\delta_r(z_2)>0 &\quad\Rightarrow\quad k_i-k_d\left(z_2^2/L^2\right)>h_2\\
\delta_r(z_3)<0 &\quad\Rightarrow\quad k_i-k_d\left(z_3^2/L^2\right)<h_3\\
\delta_r(z_4)>0 &\quad\Rightarrow\quad k_i-k_d\left(z_4^2/L^2\right)>h_4\\
&\quad\vdots
\end{aligned} \tag{16}$$

where $h_j=h(z_j)$ for $j=1,2,3,\ldots$.

Eq. (16) should be simplified since it includes infinite inequalities. As shown in Fig.2, z_j is approaching $(j-3/2)\pi$ as j increases.

For odd number j, $\lim_{j\to\infty}\cos(z_j)=0$ and $\lim_{j\to\infty}\sin(z_j)=-1$. Let k_{dj} denotes k_d in joints of $k_i-k_d\left(z_j^2/L^2\right)=h_j$ and $k_i=0$. Then

$$k_{dj}=-\frac{L^2h_j}{z_j^2}=-\frac{L}{k}\left[\frac{b}{L}\cos(z_j)-\left(a\frac{z_j}{L^2}-\frac{1}{z_j}\right)\sin(z_j)\right].$$

Using this fact, if $k_{de}<k_{d1}$ (e is an odd number), then $k_{dj}<k_{d1}$ when $j>e$.

For even number j, we have $\lim_{j\to\infty}\cos(z_j)=0$ and $\lim_{j\to\infty}\sin(z_j)=1$. Let k_{dj} denotes k_d in joints of $k_i-k_d\left(z_j^2/L^2\right)=h_j$ and $k_i-k_d\left(z_1^2/L^2\right)=h_1$. Then

$$k_{dj}=L^2\frac{h_1-h_j}{z_j^2-z_1^2}$$

$$=\frac{\frac{L^2h_1}{z_j^2}+\frac{L}{k}\left[\left(a\frac{z_j}{L^2}-\frac{1}{z_j}\right)\sin(z_j)-\frac{b}{L}\cos(z_j)\right]}{1-\left(z_1^2/z_j^2\right)}$$

where $z_j\neq 0$.

Using this fact, if $k_{df}>k_{d2}$ (f is an even number), then $k_{dj}>k_{d2}$ when $j>f$.

In a word, for a controlled plant $G(s)$ described by (1), the closed-loop system is stable if and only if (12) is satisfied. □

4. PID CONTROLLER DESIGN

Using Theorem 1, a region in (k_i,k_d), which is a convex polygon, can be determined to stabilize a second-order system with time delay for a feasible k_p. By linear programming, the extremum can be computed in this region with maximum k_i, which is also a vertex of this convex polygon. Thus the closed-loop system will possibly be unstable if there are small perturbations in controller parameters, i.e., this controller is fragile. In order to overcome the drawback problem, a non-fragile PID controller will be presented. It is given by solving the following optimization problem

Maximize k_i

subject to

$k_p\in\left(\max\left(-1/k,k_{plow}\right)+d\ ,\ k_{pup}-d\right)$;

$k_i-r>0$, $k_d-r>-b/k$,

$k_i+r\sqrt{1+(1+kk_p)^2/a^2}-k_d(1+kk_p)/a<b(1+kk_p)/ak$,

$k_i+r\sqrt{1+(z_1/L)^4}-k_d(z_1/L)^2<h_1$,

$\vdots$

$$k_i + r\sqrt{1+(z_{e-2}/L)^4} - k_d\left(z_{e-2}/L\right)^2 < h_{e-2}\,,$$
$$k_i - r\sqrt{1+(z_2/L)^4} - k_d\left(z_2/L\right)^2 > h_2\,,$$
$$\vdots$$
$$k_i - r\sqrt{1+(z_{f-2}/L)^4} - k_d\left(z_{f-2}/L\right)^2 > h_{f-2}\,. \qquad (17)$$

Where d denotes an acceptable perturbation size of k_p. r denotes an acceptable perturbation size of k_i and k_d, also is the distance between both borders of the (k_i, k_d) regions given by (12) and (17), for a feasible k_p. Both regions are two similar convex polygons each other. As a result, the closed-loop system will be guaranteed to be stable as long as perturbations in the controller parameters are smaller than r and d.

5. SIMULATION EXAMPLE

Consider a high-order and heavily oscillatory process

$$G(s) = \frac{1}{(s^2+s+1)(s+2)^2} e^{-0.1s}\,.$$

Its second-order model (Wang, 1999) is given by

$$\hat{G}(s) = \frac{0.222}{1.256s^2 + 1.101s + 1} e^{-0.837s}\,.$$

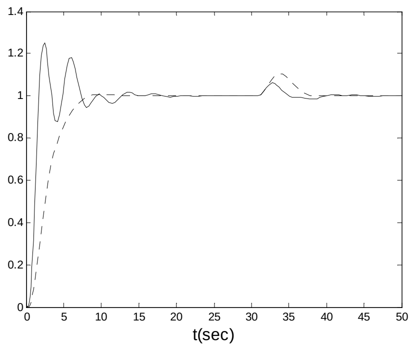
With the proposed PID controller design procedure, $k_{plow} = -4.5045$, $k_{pup} = 10.0995$, then when $d = r = 4$, the PID controller designed is

$$C(s) = 4.4485 + \frac{5.107}{s} + 8.3013s\,.$$

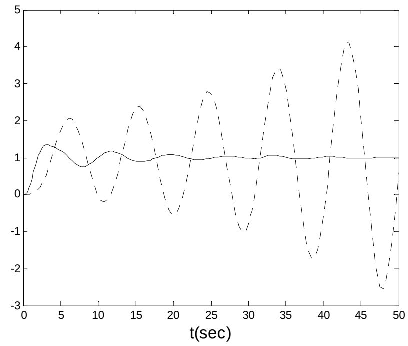
Wang's method (1999) gives rise to

$$C'(s) = 1.503 + \frac{1.366}{s} + 1.715s\,.$$

The closed–loop performances of the proposed PID controller (solid line) and Wang's PID controller (dot line) are shown in Fig.3, where a step load disturbance is introduced to at $t = 30$ sec. Both controllers parameters k_p, k_i and k_d in Fig. 3(a) are not perturbed, and in Fig. 3(b)-(d) are perturbed, i. e., they are deviated $-1.5, 1.5, 1.5$ in Fig. 3(b), $1.5, -1.366, 1.5$ in Fig. 3(c), $1.5, 1.5, -1.5$ in Fig. 3(d) from their design values, respectively. It can be seen from the results of simulation that the proposed method is superior to Wang's method in the rejection of load disturbances and non-fragility.



(a)



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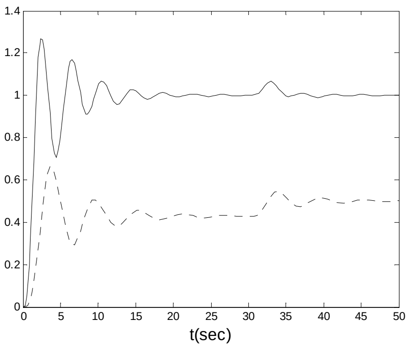

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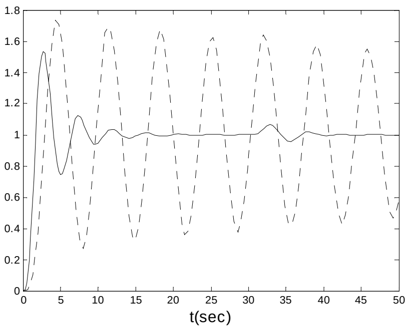

(d)

Fig 3. Step responses of the process

6. EXPERIMENT EXAMPLE

The above approach of PID controller design will be tested on a water level control plant with three tanks. The plant is described as

$$G(s) = \frac{1.39}{3136s^2 + 137.6s + 1} e^{-30s}\,.$$

With the proposed PID controller design procedure for this model, $k_{plow} = -0.7194$, $k_{pup} = 3.89$, then when $r = 0.05$, the PID controller designed is

$$C(s) = 2.738 + \frac{0.0513}{s} + 125.6s\,.$$

Åström's method (1984) gives

$$C'(s) = 2.09 + \frac{0.012}{s} + 92s\,.$$

The step responses with the above two PID controllers: the proposed PID (up) and Åström's PID (down) are shown in Fig.4, a step load disturbance is introduced to at $t = 900$ sec. There is a higher overshoot in the step responses with the proposed method , but it is superior to Åström's method in the rejection of load disturbances.

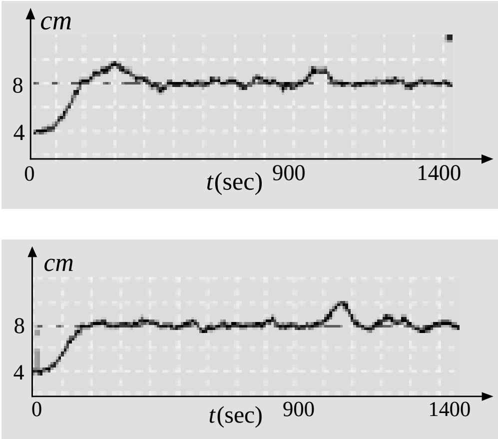

Fig 4. Step responses of the water level control plant

The acceptable perturbation size (as $r = 0.0513$) of the proposed PID parameters is larger than that (as $r = 0.012$) of Åström's PID parameters. For instance, when the integral gains (k_i) of both controllers are deviated -0.045 and -0.01 from their design values, respectively, the step responses with the proposed PID (up) and Åström's PID (down) are shown in Fig.5. It is obvious that the proposed controller can tolerate a larger perturbation extent compare with Åström's controller.

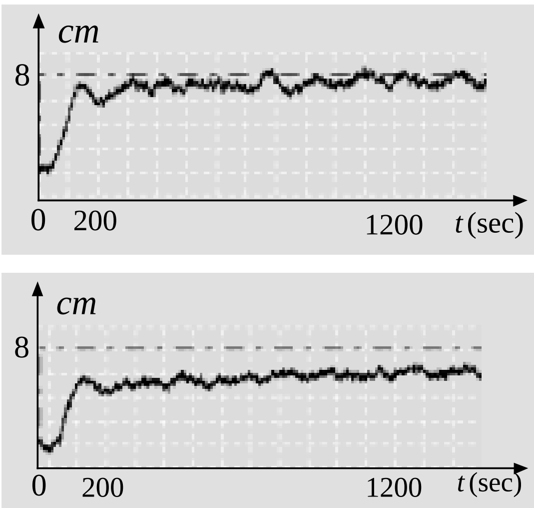

Fig 5. Step responses of the process for k_i is deviated from the its design value

7. CONCLUSIONS

Based on an extension of the Hermite-Biehler Theorem to the quasipolynomial stability problem, a characterization of the complete set of stabilizing PID controller have been obtained for a given second-order plant with dead time. This result opens up the possibility of designing PID controller to optimize a given performance criteria. The main reason to optimize the load disturbance response instead of the set point response is that load disturbances are more likely to change during operation compared to set points, which are usually kept fixed. A good set point tracking can be achieved by using the feed forward term of two degrees of freedom PID controller (Panagopoulos, 1999). The non-fragile PID controller can tolerate a larger parameter perturbation extent. Consistent and satisfactory responses are obtained as shown in simulation and experiment example results.

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