Zone Model Predictive Control Algorithm Using Soft Constraint Method

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Abstract: A zone model predictive control algorithm is proposed and developed through the soft constraint method. The estimation of zone violation is avoided; as a consequence the selection of the approximate setpoint when the control variable violates its zone constraint is skipped. To further improve control performance, zone trajectory method is proposed and a parameter is provided to trade off the response performance and model accuracy. The effective performance is proved by the simulation results. The stability of the algorithm is also analyzed. *Copyright © 2002 IFAC*

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1. INTRODUCTION

Although in industrial control applications, the controlled variable usually has a specific set point. It is common that many of the controlled variables have range limits rather than set point. This kind of process variable is treated as zone variable in most industrial MPC controller such as RMPCT, DMCPlus and HIECON, which all provide zone and setpoint options for CVs to meet industrial need (Richalet, *et al.*, 1978; Qin and Badgwell, 1997; Morari and Lee, 1999).

Zone control is also necessary for over-specified processes, whose process model can be cast at steady state by the following form (Muske and Rawlings, 1993)

$$\begin{bmatrix} y_1 \\ \vdots \\ y_s \end{bmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{1r} \\ \vdots & & \vdots \\ a_{s1} & \cdots & a_{sr} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_r \end{bmatrix} + \begin{bmatrix} d_1 \\ \vdots \\ d_s \end{bmatrix} \qquad (1)$$

where a_{ij} is steady gain , d_i is disturbance. When the number of outputs exceeds the number of inputs, all the set points cannot be met at the same time. If one of the set points is changed into a zone specification, the outputs specifications are relaxed slightly. The probability that the process will meet all of its specifications increases. Moreover, because the output's change within zone is ignored, the need to coordinate the movement of inputs is largely eliminated, which decreases its sensitivity to model mismatch and improves its robust performance, especially for the process whose outputs and inputs variables are interacted with each other strongly.

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In conventional dynamic model control, zone control cannot be solved directly. But the receding optimization formulation of model predictive control provides the possibility to realize zone control. Zhou (2001) used setpoint approximation method to implement zone control, but the limit was that it still needed estimation of zone violation concomitant with the selection of the approximate setpoint value.

In this paper, a zone model predictive control algorithm using the soft constraint method is proposed to achieve better control performance and to avoid the mentioned problem. To further improve control performance, zone trajectory method is proposed which provides a tuning parameter to trade off the response performance and model accuracy. The stability of the algorithm is analyzed finally.

2. ZONE CONTROL ALGORITHM

Consider a stable multi-input multi-output system represented by the following model (Garcia, *et al.*, 1989)

$$y(k+j|k) = \sum_{i=1}^{N-1} H_i \Delta u(k+j-i) + H_N u(k+j-N) + d(k+j|k)$$

$$d(k+j|k) = d(k|k) = y(k) - \sum_{i=1}^{N-1} H_i \Delta u(k-i) - H_N u(k-N) \qquad (2)$$

$$H_i = \begin{bmatrix} a_{11}(i) & \cdots & a_{1r}(i) \\ \vdots & & \vdots \\ a_{s1}(i) & \cdots & a_{sr}(i) \end{bmatrix}$$

where

$y(k+j|k)$=Predicted output vector at time $k+j$

$y(k)$=Actual output vector at time k

$u(k)$=Actual input vector at time k

$d(k+j|k)$=Predicted disturbance vector at time $k+j$
N = model horizon length
r = number of inputs
s = number of outputs

For setpoint control, the optimization problem at every sampling time is solved (Cutler and Ramaker, 1979; Garcia and Prett, 1986; Garcia, *et al.*, 1989): Find the a optimal sequence of M future manipulated variable moves $\Delta u(k),\cdots,\Delta u(k+M-1)$ so that the prediction of the manipulated variables and controlled outputs satisfy the criteria which minimizes the sum of squared deviations of the predicted CV values from a time varying reference trajectory over P future time steps. The formulation of optimization problem is:

$$\min_{\Delta u(k),\cdots \Delta u(k+M-1)} J_k = \sum_{j=1}^{P} \left\| y(k+j|k) - w(k+j|k) \right\|_Q^2 + \sum_{j=0}^{M-1} \left\| \Delta u(k+j) \right\|_S^2 \tag{3}$$
$$s.t. \quad u^- \le u(k+j) \le u^+$$
$$\Delta u^- \le \Delta u(k+j) \le \Delta u^+, \ \forall j = 1, M$$

where
$w(k+j|k)$=reference trajectory value at time $k+j$

A zone region is defined by the minimum and maximum values of a controlled variable's desired range of values. One way to simply implement zone control is to use setpoint approximation method: when the CV is predicted to lie within its zone, its weight coefficient of matrix Q is set to zero so the controller will ignore it; when the CV is predicted to violate its zone limits, its weight is non-zero and a point within zone is defined as the approximate setpoint and is chose to drive the output back into the zone. The simple way to estimation the zone violation of output is by examining the initial predictive value of outputs.

Even though the initial predictive value of outputs meets its zone limits, some of output predictive value still may violate its limits when correcting other outputs error during calculating the optimal inputs moves sequences. The controller will transiently move the output farther outside its zone limit, because the controller ignores the output's error when the predictive initial value of outputs lie within its zone. The solution of set point approximation method is generally sub-optimal. Moreover, the selection of the approximate setpoint when the control variable violates its zone constraint lacks rigorous analysis rules, because distinct response performance can be achieved by selecting different approximate setpoint values.

For zone control, the deviation between the output predictive value and zone limits $[\ y_c^- \quad y_c^+\ ]$ is defined as

$$e(k+j|k) = \begin{cases} y(k+j|k) - y_c^+ , \text{if } \ y(k+j|k) \ge y_c^+ \\ y_c^- - y(k+j|k), \text{ if } \ y(k+j|k) \le y^- \\ 0 \ , \text{ if } y(k+j|k) \le y_c^+ \text{ and } \ y(k+j|k) \ge y_c^- \end{cases} \tag{4}$$

The optimization problem of zone control can be formulated as

$$\min_{\Delta u(k),\cdots \Delta u(k+M-1)} J_k = \sum_{j=1}^{P} \left\| e(k+j|k) \right\|_Q^2 + \sum_{j=0}^{M-1} \left\| \Delta u(k+j) \right\|_S^2$$
$$\text{s.t} \quad u^- \le u(k+j) \le u^+ \tag{5}$$
$$\Delta u^- \le \Delta u(k+j) \le \Delta u^+, \ \forall j = 1, M$$

Apparently, $e(k+j|k)$ is the optimal value $\varepsilon^*(k+j|k)$ of following optimization problem

$$\min_{\varepsilon(k+j|k)} \varepsilon(k+j|k)$$
$$s.t \quad y_c^- - \varepsilon(k+j|k) \le y(k+j|k) \le y_c^+ + \varepsilon(k+j|k)$$
$$\varepsilon(k+j|k) \ge 0$$

Therefore, optimization problem (5) can be further transformed as

$$\min_{\substack{\Delta u(k),\cdots,\Delta u(k+M-1) \\ \varepsilon(k+1|k),\cdots,\varepsilon(k+P|k)}} J_k = \sum_{j=1}^{P} \left\| \varepsilon(k+j|k) \right\|_Q^2 + \sum_{j=0}^{M-1} \left\| \Delta u(k+j) \right\|_S^2$$
$$s.t \quad u^- \le u(k+j) \le u^+ \tag{6}$$
$$\Delta u^- \le \Delta u(k+j) \le \Delta u^+, \ \forall j = 1, M$$
$$y_c^- - \varepsilon(k+j|k) \le y(k+j|k) \le y_c^+ + \varepsilon(k+j|k)$$
$$\varepsilon(k+j|k) \ge 0 \ , \ \forall j = 1, P$$

In the above problem formulation, the zone limits is treated as soft constraints by adding a slack variable. At the same time the slack variables are also included in the objective function to be minimized.

Soft Constraints are used to prevent the controller from introducing transient errors by defining soft constraints on the controlled outputs at intervals from the current interval to predictive horizon. When the controlled variable has a set point instead of a zone region, both the upper and lower limits of the zone are set equal to the set point.Through soft constraint method, the estimation on the zone violation is avoided; as a consequence the selection of the approximate setpoint when the control variable violates its zone constraint is skipped.

In order to drive the outputs back into its zone region more slowly to avoid overshoot consequently, zone trajectory is introduced for each controlled output as follows

$$\min_{\substack{\Delta u(k),\cdots,\Delta u(k+M-1) \\ \varepsilon(k+1|k),\cdots,\varepsilon(k+P|k)}} J_k = \sum_{j=1}^{P} \left\| \varepsilon(k+j|k) \right\|_Q^2 + \sum_{j=0}^{M-1} \left\| \Delta u(k+j) \right\|_S^2$$
$$s.t \quad u^- \le u(k+j) \le u^+$$
$$\Delta u^- \le \Delta u(k+j) \le \Delta u^+ \tag{7}$$
$$y_r^-(k+j) - \varepsilon(k+j|k) \le y(k+j|k) \le y_r^+(k+j) + \varepsilon(k+j|k)$$

where $y_r^-(k+j)$ $y_r^+(k+j)$ is determined as follows:
If $y(k)$ within $[\ y_c^- \quad y_c^+\ ]$, then
$y_r^-(k+j) = y_c^-$ and $y_r^+(k+j) = y_c^+$
If $y(k) \ge y_c^+$, then
$y_r^-(k+j) = y_c^-$ and
$y_r^+(k+j) = \alpha^j y(k) + (1-\alpha^j) y_c^+$
If $y(k) \le y_c^-$, then
$y_r^+(k+j) = y_c^+$ and
$y_r^-(k+j) = \alpha^j y(k) + (1-\alpha^j) y_c^-$
where α is the time constant, which is determined

by the trade-offs that inherently exist between speed of response and model accuracy or inputs movement. A smaller value gives faster response and consequently large MV movement, which requires a more accurate model for stable control. A larger value, on the contrary gives slower response with smaller MV movement and works well with a less accurate model.

The controller is obliged to keep the CV within the constraints defined by the zone trajectory, but it is allowed to follow any figure within these constraints. The sensitivity to model error is decreased and the robustness is improved

3. STABILITY ANALYSIS

Alex Zheng and Manfred Morari(1995) analyzed the closed-loop stability for constrained MPC with setpoint control. Zone Control also has the similar property when using soft constraint method.

Assume:

a) There is no model mismatch
b) Predictive horizon is infinite
c) Steady-state gain matrix of the model has full row rank.

then the closed-loop system is asymptotically stable if and only if the optimization problem (7) is feasible at the first sampling time.

Proof:

If the optimization problem is not feasible, then the controller is not defined.

At sampling time k, the optimal solution is

$$\Delta u^*(k+i|k), i = 0,\cdots,m-1$$
$$\varepsilon^*(k+i|k), i = 1,\cdots,\infty$$

At sampling time k+1,the solution (18) is a feasible solution but may not be the optimal solution.

$$\Delta u(k+i|k+1) = \Delta u^*(k+i|k), i = 1,\cdots,m-1$$
$$\Delta u(k+m|k+1) = 0$$
$$\varepsilon(k+i|k+1) = \varepsilon^*(k+i|k), i = 2,\cdots,\infty$$

Define $\Delta u_k = \Delta u^*(k|k)$ $\varepsilon_k = \varepsilon^*(k+1|k)$

The above feasible control input yields:

$$J_{k+1}^* \le J_{k+1} = J_k^* - \varepsilon_k^T Q\varepsilon_k - \Delta u_k^T S\Delta u_k$$

$$J_{k+1}^* \le J_k^* - \varepsilon_k^T Q\varepsilon_k - \Delta u_k^T S\Delta u_k$$

Therefore, the sequence $\{J_k^*\}$ is non-increasing, its low boundary is zero. Consequently, the sequence $\{J_k^*\}$ converges. So

$$\lim_{k\to\infty}\left(\varepsilon_k^T Q\varepsilon_k + \Delta u_k^T S\Delta u_k\right)$$
$$\le \lim_{k\to\infty}\left(J_k^*\right) - \lim_{k\to\infty}\left(J_{k+1}^*\right) = 0$$

This together with Q、$S > 0$ implies that $\varepsilon_k \to 0$ and $\Delta u_k \to 0$ as $k \to \infty$. Since the steady-state gain matrix of the model is bounded, $y(k)$ approaches the steady-state value asymptotically.

4. SIMULATION

(1) Consider the two-input three-output system:

$$G(s) = \begin{bmatrix} \dfrac{1.77e^{-28s}}{60s+1} & \dfrac{5.88e^{-27s}}{50s+1} \\ \dfrac{5.72e^{-14s}}{60s+1} & \dfrac{6.9e^{-15s}}{40s+1} \\ \dfrac{4.42}{44s+1} & \dfrac{7.2}{19s+1} \end{bmatrix}$$

with the following input constraints

$$-0.5 \le u_1, u_2 \le 0.5 \qquad |\Delta u_1|,|\Delta u_2| \le 0.03$$

and the following initial conditions

$$y_1 = y_2 = y_3 = 0 \qquad u_1 = u_2 = 0$$

Choose T=5s,N=100,M=4,P=30,Q=I,S=I,α= 0.95

If all of the controlled outputs have set points

$$y_1 = 0.59 \quad y_2 = 0.64 \quad y_3 = 0.67$$

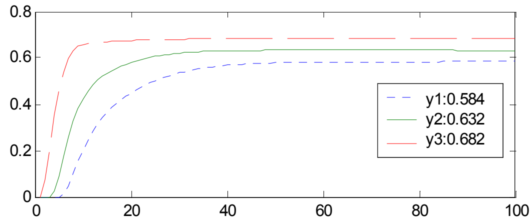


Fig. 1. Responses of setpoint control

Because the degree of freedom is insufficient, it is physically impossible to keep all output at setpoint or within range. When the set point for y_3 is replaced by zone limit $[0.65 \quad 0.7]$, all output specification would be met.

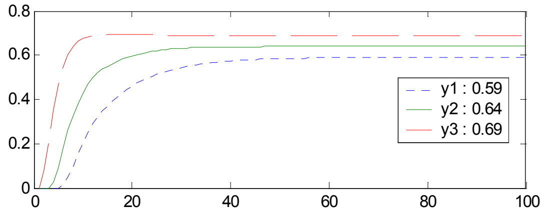


Fig.2. Responses of zone Control

(2) Consider the system:

$$G(s) = \begin{bmatrix} \dfrac{1.77e^{-28s}}{60s+1} & \dfrac{5.88e^{-27s}}{50s+1} & \dfrac{4.05e^{-27s}}{50s+1} \\ \dfrac{5.72e^{-14s}}{60s+1} & \dfrac{6.9e^{-15s}}{40s+1} & \dfrac{5.39e^{-18s}}{50s+1} \\ \dfrac{4.42e^{-22s}}{44s+1} & \dfrac{7.2}{19s+1} & \dfrac{4.38e^{-20s}}{33s+1} \end{bmatrix}$$

with the following input constraints

$$-1 \le u_1, u_2, u_3 \le 1 \qquad |\Delta u_1|,|\Delta u_2|,|\Delta u_3| \le 0.03$$

and the following output regulatory objective

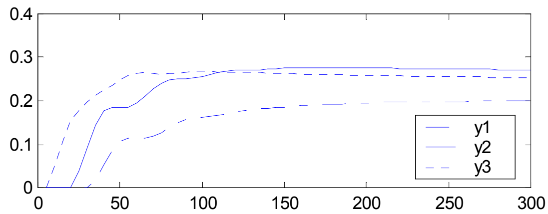
$$y_1 = 0.2 \qquad -0.5 \le y_2, y_3 \le 0.5$$

and the following initial conditions

$$y_1 = y_2 = y_3 = 0 \qquad u_1 = u_2 = u_3 = 0$$

Choose T=5,N=100,M=4,P=30,Q=I,S=I,α= 0.95

When using set point approximation, the result is shown as follows:



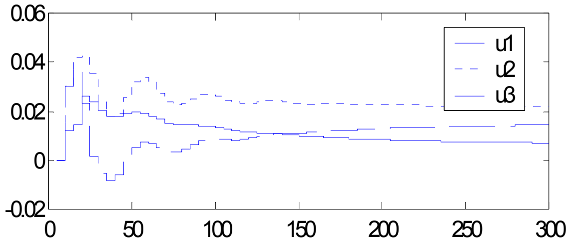


Fig.3. Responses of set point approximation method

When using soft constraint method, the result is shown as follows:

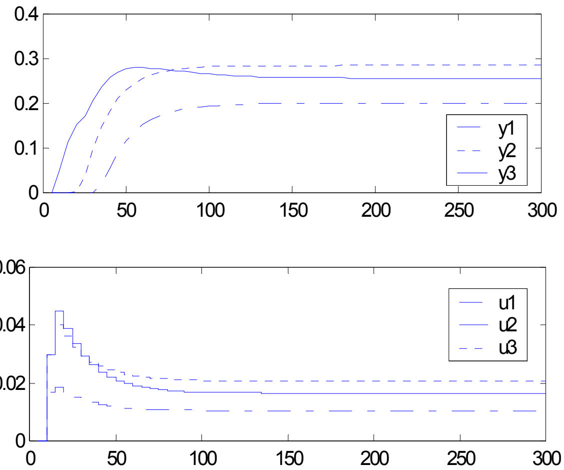


Fig. 4. Responses of soft constraint method

From the simulation result, the soft constraint method prevent the controller from moving a CV farther outside zone while correcting other CV errors by defining constraints on the CVs that are imposed at intervals from the current interval out to the predictive horizon. In setpoint approximation method, the controller will ignore the CV when the CV is predicted to be within its zone, so its performance is worse than that with soft constraint method.

5. CONCLUSION

Estimating the violation of zone output limits in the setpoint approximation method is simply through examining its output predictive initial value, but it can not always keep zone output in its zone limit while correcting other outputs errors. Using the soft constraint method, zone specification is directly imposed as constraints in optimization formulation, while correcting other CV errors, it will not violate zone output limits, but its computing burden is larger than the setpoint approximation method. The tuning parameter provided by zone trajectory method enables a flexible way to achieve better performance and reach a tradeoff between performance and model accuracy.

REFERENCES

Zheng, A. and Morari, M. (1995). Stability of model predictive control with mixed constraints. *IEEE Trans. Aut Control*, 40(10),pp.1818-1823.

Cutler, C.R. and B.L.Ramaker (1979). Dynamic matrix control – a computer algorithm .*In AIChE 86th National Meeting*,Houston,TX,

Gracia, C.E.,and A.M.Morshedi(1986).Quadratic programming solution of dynamic matrix control (QDMC). *Chem.Eng.Commun*.,46,pp.73-87

Gracia, C.E.,D.M.Prett, M.Morari(1989). Model predictive control: theory and practice-a survey. *Automatica*,25(3),pp.335-348

Gracia, C.E.and D.M.Prett (1986). Advances in industrial model predictive control. *CPCIII*, pp.249-294, Cache and Elsevier, Amsterdam.

Richalet,J., J.Rault, J.L.Testutud, J.Papon(1978). Model predictive heuristic control: applications to industrial processes. *Automatica*, 14(5), pp. 413-428

Muske, K R. and J. B. Rawlings (1993). Model predictive control with linear models. *AIChE* J., 39(2), pp.262-287

Morari, M. and J.H. Lee (1999). Model predictive control: past, present and future. *Comp. Chem. Eng.*, 23,pp.667-682

Qin, S.J.and T. A.Badgwell(1997). An overview of industrial predictive control technology. In (J.C.Kantor, C.E.Garcia, B.Carnahan, editors), *Chemical Process Control-V*, 232-256, AICHE.

Zhou, Lifang, and JiXin, Qian, (2002). Study on predictive control algorithm for horizon control. In *Proceedings of the 4th World Congress on Intelligent Control and Automation*, pp. 717-721