A FAULT ACCOMMODATION CONTROL FOR NONLINEAR PROCESSES

Yingwei Zhang, Fuli Wang, Ge Yu
School of Information Science & Engineering, P.O.Box 135,
Northeastern University, Shenyang, Liaoning, 110004
P. R. China
Email address: zhang_yingwei2001@yahoo.com.cn
and
Furong Gao
Department of Chemical Engineering
Hong Kong University of Science & Technology
Clear Water Bay, Kowloon, Hong Kong

Abstract: An active fault accommodation control law is developed for a class of nonlinear processes to guarantee the closed-loop stability in the presence of a fault, based on a neural network representation of the dynamics due to faults. Applications of the proposed design indicate that the fault accommodation control law is effective for a typical nonlinear fermentation process.

Keyword: Neural networks, fault-accommodation, corrective control law

1. INTRODUCTION

The study of fault diagnosis and fault-tolerant control has attracted much attention recently [1-8], due to the industrial demands for safety and efficiency. For certain processes, it is important not only to detect (and identify) but also to accommodate any faults quickly. Fault-tolerant controls have been developed to keep such processes in control, in spite of the occurrence of a fault. Based on the nature of its design, a fault-tolerant control can be categorized into the passive or active two types. A passive fault-tolerant control uses the same control scheme before and after fault, without specific accommodating parameters, typically by introducing a conservative law. For an active fault-tolerant control, a control reconfiguration takes place, following the diagnosis of a fault, to counteract any dynamic changes caused by this fault.

Within the category of the passive fault-tolerant controls, reliable control is widely used. Results and scheme details can be found in references [3-5]. Robust control design is often adopted for reliable control to have the guaranteed closed-loop stability and H_∞ performance. This type control is typically conservative, without controller adjustment after detection of a fault; the tolerance comes at the cost to the control performance.

In an active fault-tolerant control, faults are accommodated, typically by a reconfiguration of the feedback control law. An excellent overview on the subject has been given by Patton [6]. Faults are typically associated with sensors and actuators failures; in correspondence, respective accommodation strategies can be so designed. For examples, sensor fault accommodations for MIMO systems have been discussed by Tortora [7]; actuator fault accommodations are given by Michael [8]. Adaptive approaches have also been used in fault tolerant controls. For examples, an adaptive compensation method for actuator fault with known plant dynamics has been formulated by Boskovic [9]; and a nonlinear adaptive fault accommodation controller has been designed by Idan [10] to make use of redundancy.

In this paper, a new fault accommodation control design is presented for a class of uncertain nonlinear processes. The dynamic changes due to faults are represented by a neural network, based on which an adaptive corrective control law is formulated to ensure the system stability.

The remainder of the paper is organized as follows. The problem statement and its assumptions are given in section 2, followed by the formulation of our controller and its relevant proofs in section 3. An illustrative example is given in section 4 to demonstrate the effectiveness of the proposed method. Finally, conclusions are drawn in section 5.

2. PROBLEM STATEMENTS

Consider a system described as:

$$\dot{x} = \zeta(x) + \Delta\zeta(x) + G(x)[u + \Delta g(x)] + \beta(t-T) f(x) \qquad (1)$$

where $x \in R^n, u \in R^m$ are the state and input of the system, respectively, $\Delta\zeta(x)$ and $\Delta g(x)$ are the model uncertainty in the normal operation,

The paper was supported in part by Liaoning Natural Science Foundation of China(002013).

f characterizes the changes in the dynamics due to a failure. The normal system, in the absence of any faults, is described by

$$\dot{x} = \zeta(x) + \Delta\zeta(x) + G(x)[u + \Delta g(x)] \quad (2)$$

The nonlinear fault function f is multiplied by a switching function $\beta(t-T)$,

$$\beta(t-T) = diag\ (\beta_1(t-T), \beta_2(t-T), \cdots, \beta_n(t-T)) \quad (3)$$

where $\beta_i(t-T) = \begin{cases} 0 & if \quad t < T \\ 1 & if \quad t \geq T \end{cases}$, $i = 1, 2, \cdots, n$.

where T is the fault occurrence time. The problem considered is as follows:

Fault accommodation (FA) problem: Given system (1), design a control u_N for the normal system, and an additional control u_F for fault compensation, so that $u = u_N + u_F$ as the new control after the occurrence of a fault can guarantee the resulted closed-loop nonlinear system to be stable.

The following assumptions are used.

Assumption 1: There exists $u = u^a(x)$ and Lyapunov function $\bar{V}(x)$, such that

$$k_1|x|^2 \leq \bar{V}(x) \leq k_2|x|^2, \quad (4)$$

$$\frac{\partial \bar{V}(x)}{\partial x}\left(\zeta(x) + G(x)u^a(x)\right) \leq -k_3\left|\frac{\partial \bar{V}(x)}{\partial x}\right|^2 \leq -k_4\bar{V}(x), \quad (5)$$

where $k_1, k_2, k_3, and\ k_4$ are positive constants.

Assumption 2: For system (1)

$$\|\Delta g(x)\| \leq \xi(x) \quad (6)$$

$$\left\|\left(\frac{\partial \bar{V}(x)}{\partial x}\right)^T \Delta\zeta(x)\right\| \leq \rho(x)$$

where $\dfrac{\rho(x)}{\left\|G^T(x)\left(\frac{\partial \bar{V}(x)}{\partial x}\right)\right\|}$ is continuous,

$\xi(\bullet)$ and $\rho(\bullet)$ are known and continuous.

Remark 1: From assumption 2, we have $\rho(x) = 0$, if $G^T(x)\left(\frac{\partial \bar{V}(x)}{\partial x}\right) = 0$.

3. FAULT ACCOMMODATION

Firstly, let's use a neural network to represent fault function $f(x)$. Where, x is the input vector to the neural network. It can be shown that there exists an optimized matrix W^* such that $\left|f(x) - W^*S(x)\right| \leq \varepsilon$ is satisfied for any given $\varepsilon > 0$. $S(x)$ is the sigmoid function. $W^*S(x)$ can approximate $f(x)$ to any degree of accuracy, with bounded W^*, $\|W^*\| \leq M_W$. With the above, system (1) can be rewritten as:

$$\dot{x} = \zeta(x) + \Delta\zeta(x) + G(x)[u + \Delta g(x)] + W^*S(x) + \varepsilon(x) \quad (7)$$

where, $\varepsilon(x) = \left|f(x) - W^*S(x)\right| \leq \varepsilon$ is the estimation error. If we denote W as the estimate of the uncertain weight matrix W^*, then

$$\dot{x} = \zeta(x) + \Delta\zeta(x) + G(x)[u + \Delta g(x)] - \widetilde{W}S(x) + WS(x) + \varepsilon(x) \quad (8)$$

where $\widetilde{W} = W - W^*$ and it has the appropriate dimension..

Theorem 1: Under assumptions 1 and 2, we can design a controller in the form of the following:

$$u = u_N + u_F \quad (9)$$

$$u_N = u^a + u^b + u^c$$

where u^a is given by assumption 1, and let

$$E = \left\{ x \,\middle|\, G(x)^T \frac{\partial \bar{V}(x)}{\partial x} = 0 \right\},$$

$$u^b = \begin{cases} -\dfrac{G^T(x)\frac{\partial \bar{V}(x)}{\partial x}}{\left\|G^T(x)\frac{\partial \bar{V}(x)}{\partial x}\right\|}\xi(x), & x \notin E \\ 0 & x \in E \end{cases}, \quad (10)$$

$$u^c = \begin{cases} -\dfrac{G^T(x)\frac{\partial \bar{V}(x)}{\partial x}}{\left\|G^T(x)\frac{\partial \bar{V}(x)}{\partial x}\right\|^2}\rho(x), & x \notin E \\ 0 & x \in E \end{cases}, \quad (11)$$

$$u_F = \frac{G^T(x)WS(x)}{\lambda[1 + \|G(x)\|^2]} + \frac{G^T(x)\Theta}{\lambda_1[1 + \|G(x)\|^2]}. \quad (12)$$

Where $\Theta \in R^{n\times L}$ and $\Theta = [\theta, 0, \cdots, 0]^T$. Then, the state x is ultimately consistently bounded by the set:

$$D = \left\{ x \in R^n : v_0(x) \leq \frac{\mu}{k_0\alpha}, \frac{\bar{k}_2}{\bar{k}_1} \leq k_0 \leq 1 \right\}, \quad (13)$$

with the following adaptive weight update law

$$\dot{W} = \begin{cases} 2k_0\dfrac{\partial v_0}{\partial x}S^T(x) & if\|W\| < M_W \\ -\beta W + 2k_0\dfrac{\partial v_0}{\partial x}S^T(x) & if\|W\| \geq M_W \end{cases}, \quad (14)$$

$$\dot{\theta} = -\gamma_1\theta + k_0\left|\frac{\partial v_0}{\partial x}\right|. \quad (15)$$

The parameters of $\lambda, \lambda_1, \bar{k}_1, \bar{k}_2, \alpha, and\ \mu$ can be determined as in the proof. The proof of the above theorem is divided into the following two steps: step 1, we prove that there exist a nominal controller $u_N = u^a + u^b + u^c$ and a Lyapunov function $v_0(x)$ for the normal system described by $\dot{x} = \zeta(x) + \Delta\zeta(x) + G(x)[u + \Delta g(x)]$, such that the closed-loop of the normal system is stable; step 2, we prove that the state x is ultimately consistently bounded, using the control law stated in the theorem.

Proof: step 1

Substituting the controller equations of (9-12) into system （1）, we have:

$$\dot{x} = \zeta(x) + \Delta\zeta(x) + G(x)[u^a + u^b + u^c + \Delta g(x)]$$

Define a positive function $v_0(x) = \overline{V}(x)$, then we have:

$$\dot{v}_0(x) = (\frac{\partial \overline{V}(x)}{\partial x})^T(\zeta(x) + G(x)u^a) + (\frac{\partial \overline{V}(x)}{\partial x})^T G(x)(u^b + \Delta g(x)) + (\frac{\partial \overline{V}(x)}{\partial x})^T(\Delta\zeta(x) + G(x)u^c)$$

From Assumption 1, we have

$$(\frac{\partial \overline{V}(x)}{\partial x})^T(\zeta(x) + G(x)u^a) \le -k_3\left|\frac{\partial \overline{V}(x)}{\partial x}\right|^2 \quad (16)$$

From Assumption 2 and the structure of $u^b(x)$, we have

$$(\frac{\partial \overline{V}(x)}{\partial x})^T G(x)(u^b + \Delta g(x)) = (G^T(x)\frac{\partial \overline{V}(x\ )}{\partial x_i})^T(u^b + \Delta g(x)) = 0$$

when $x \in E$, and

$$(\frac{\partial \overline{V}(x)}{\partial x})^T G(x)(u^b + \Delta g(x))$$
$$= (\frac{\partial \overline{V}(x)}{\partial x})^T G(x)(-\frac{G^T(x)\frac{\partial \overline{V}(x)}{\partial x}}{\left\|G^T(x)\frac{\partial \overline{V}(x)}{\partial x}\right\|}\xi(x) + \Delta g(x))$$
$$= -\left\|G^T(x)(\frac{\partial \overline{V}(x)}{\partial x})^T\right\|\xi(x) + (\frac{\partial \overline{V}(x)}{\partial x})^T G(x)\Delta g(x)$$
$$\le -\left\|G^T(x)(\frac{\partial \overline{V}(x)}{\partial x})^T\right\|\xi(x) + \left\|(\frac{\partial \overline{V}(x)}{\partial x})^T G(x)\right\|\left\|\Delta g(x)\right\|$$
$$\le 0,$$

when $x \notin E$. Hence

$$(\frac{\partial \overline{V}(x)}{\partial x})^T G(x)(u^b + \Delta g(x)) \le 0 \quad (17)$$

From Assumption 2 and structure of $u^c(x)$, we have

$$(\frac{\partial \overline{V}(x)}{\partial x})^T(\Delta\zeta(x) + G(x)u^c)$$
$$= (\frac{\partial \overline{V}(x)}{\partial x})^T\Delta\zeta(x) + (G^T(x)\frac{\partial \overline{V}(x\ )}{\partial x_i})^T u^c$$
$$\le \left\|(\frac{\partial \overline{V}(x)}{\partial x})^T\Delta\zeta(x)\right\| \le \rho(x) = 0,$$

when $x \in E$, and

$$(\frac{\partial \overline{V}(x)}{\partial x})^T(\Delta\zeta(x) + G(x)u^c)$$
$$= (\frac{\partial \overline{V}(x)}{\partial x})^T\Delta\zeta(x) + (\frac{\partial \overline{V}(x)}{\partial x})^T G(x)(-\frac{G^T(x)\frac{\partial \overline{V}(x)}{\partial x}}{\left\|G^T(x)\frac{\partial \overline{V}(x)}{\partial x}\right\|^2}\rho(x))$$
$$\le \left\|(\frac{\partial \overline{V}(x)}{\partial x})^T\Delta\zeta(x)\right\| - \rho(x)$$
$$\le 0,$$

When $x \notin E$. Hence

$$(\frac{\partial \overline{V}(x)}{\partial x})^T(\Delta\zeta(x) + G(x)u^c) \le 0 \quad (18)$$

Thus, we obtain the results

$$\dot{v}_0(x) \le -k_3\left|\frac{\partial \overline{V}(x)}{\partial x}\right|^2 \quad (19)$$

From (19), the stability of the normal system is proven.

Proof: step 2:

Define a Lyapunov function for system 1 of the following form:

$$V(x, \widetilde{W}, \widetilde{\theta}) = k_0 v_0(x) + \frac{1}{2}tr\{\widetilde{W}^T\widetilde{W}\} + \frac{1}{2}\widetilde{\theta}^2 \quad (20)$$

with $\widetilde{\theta} = \theta - \varepsilon$, then the derivatives of V is

$$\dot{V} = k_0\frac{\partial v_0}{\partial x}\{f(x) + \Delta f(x) + G(x)[u^a + u^b + u^c + \Delta g(x)]\} + k_0\frac{\partial v_0}{\partial x}g(x)u^F - k_0\frac{\partial v_0}{\partial x}\widetilde{W}S(x) + k\frac{\partial v_0}{\partial x}WS(x) + k_0\frac{\partial v_0}{\partial x}\varepsilon(x) + tr\{\dot{\widetilde{W}}^T\widetilde{W}\} + \widetilde{\theta}\dot{\theta} \quad (21)$$

Using (14), we obtain

$$\dot{V} = k_0\dot{v}_0 + k_0\frac{\partial v_0}{\partial x}G(x)u^F + k_0\frac{\partial v_0}{\partial x}WS(x) + k_0\frac{\partial v_0}{\partial x}\varepsilon(x) - \beta I_W tr\{W^T\widetilde{W}\} + \widetilde{\theta}\dot{\theta}$$

where I_W is the indicator function of W, and it satisfies

$$I_W = \begin{cases} 1 & if\ \|W\| \ge M_W \\ 0 & if\ \|W\| < M_W \end{cases} \quad (22)$$

As $tr\{W^T\widetilde{W}\} = \frac{1}{2}\|W\|^2 + \frac{1}{2}\|\widetilde{W}\|^2 - \frac{1}{2}\|W^*\|^2$, then

$$\dot{V} = k_0\dot{v}_0 + k_0\frac{\partial v_0}{\partial x}G(x)u^F + k_0\frac{\partial v_0}{\partial x}WS(x) + k_0\frac{\partial v_0}{\partial x}\varepsilon(x) - \frac{\beta}{2}tr\{\widetilde{W}^T\widetilde{W}\} + \frac{\beta}{2}(1-I_W) tr\{\widetilde{W}^T\widetilde{W}\} - \frac{\beta}{2}I_W\|W\|^2 + \frac{\beta}{2}I_W\|W^*\|^2 + \widetilde{\theta}\dot{\theta} \tag{23}$$

By substituting u^F (λ, λ_1) into (23), from Assumption 1, the derivatives of V satisfies that

$$\dot{V} \le -k_0k_3\left|\frac{\partial v_0}{\partial x}\right|^2 + k_0\left|\frac{\partial v_0}{\partial x}\right|\frac{\|G(x)\|^2\|W\|\|S(x)\|}{\lambda[1+\|G(x)\|^2]} + k_0\left|\frac{\partial v_0}{\partial x}\right|\|W\|\|S(x)\| + k_0\left|\frac{\partial v_0}{\partial x}\right|\frac{\|G(x)\|^2|\theta|}{\lambda_1[1+\|G(x)\|^2]} + k_0\left|\frac{\partial v_0}{\partial x}\right|\|\varepsilon(x)\| + \widetilde{\theta}\dot{\theta} - \frac{\beta}{2}tr\{\widetilde{W}^T\widetilde{W}\} + \frac{\beta}{2}(1-I_W)tr\{\widetilde{W}^T\widetilde{W}\} - \frac{\beta}{2}I_W\|W\|^2 + \frac{\beta}{2}I_W\|W^*\|^2 \tag{24}$$

As $\frac{\|G(x)\|^2}{1+\|G(x)\|^2} \le 1$, (24) can be rewritten as

$$\dot{V} \le -k_0k_3\left|\frac{\partial v_0}{\partial x}\right|^2 + k_0\left|\frac{\partial v_0}{\partial x}\right|\|W\|\|S(x)\|\left(1+\frac{1}{\lambda}\right) + k_0\left|\frac{\partial v_0}{\partial x}\right|\frac{|\theta|}{\lambda_1} + k_0\left|\frac{\partial v_0}{\partial x}\right|\theta - k_0\left|\frac{\partial v_0}{\partial x}\right|\theta + k_0\left|\frac{\partial v_0}{\partial x}\right|\varepsilon + \widetilde{\theta}\dot{\theta} - \frac{\beta}{2}tr\{\widetilde{W}^T\widetilde{W}\} + \frac{\beta}{2}(1-I_W)tr\{\widetilde{W}^T\widetilde{W}\} - \frac{\beta_i}{2}I_W\|W\|^2 + \frac{\beta}{2}I_W\|W^*\|^2 \tag{25}$$

Let $k_3 = \bar{k}_1 + \bar{k}_2 + \bar{k}_3$, (25) is transformed into:

$$\dot{V} \le -k_0\bar{k}_1\left|\frac{\partial v_0}{\partial x}\right|^2 - k_0\bar{k}_2\left|\frac{\partial v_0}{\partial x}\right|^2 - k_0\bar{k}_3\left|\frac{\partial v_0}{\partial x}\right|^2 + k_0s\left|\frac{\partial v_0}{\partial x}\right|\|W\|\left(1+\frac{1}{\lambda}\right) + k_0\left|\frac{\partial v_0}{\partial x}\right|\theta[1+\frac{1}{\lambda_1}] - \frac{\gamma_1}{2}\widetilde{\theta}^2 - \frac{\gamma_1}{2}\theta^2 + \frac{\gamma_1}{2}\varepsilon^2 - \frac{\beta}{2}tr\{\widetilde{W}^T\widetilde{W}\} + \frac{\beta}{2}(1-I_W)tr\{\widetilde{W}^T\widetilde{W}\} - \frac{\beta}{2}I_W\|W\|^2 + \frac{\beta}{2}I_W\|W^*\|^2 \tag{26}$$

Choosing

$$\lambda \ge \frac{k_0 s}{\sqrt{2\bar{k}_2\beta - sk_0}}, \quad \lambda_1 \ge \frac{k_0}{\sqrt{2k_0\bar{k}_2\gamma_1 - k_0}} \tag{27}$$

and

$$\beta > \frac{s^2k_0^2}{2\bar{k}_2}, \quad \gamma_1 > \frac{k_0}{2\bar{k}_2} \tag{28}$$

yields

$$\dot{V} \le -k_0\bar{k}_1\left|\frac{\partial v_0}{\partial x}\right|^2 - k_0\bar{k}_3\left|\frac{\partial v_0}{\partial x}\right|^2 - [\bar{k}_2\left|\frac{\partial v_0}{\partial x}\right|^2 - 2\sqrt{\frac{\bar{k}_2\beta}{2}}\left|\frac{\partial v_0}{\partial x}\right|\|W\| + \frac{\beta}{2}\|W\|^2] + [(\sqrt{k_0\bar{k}_2}\left|\frac{\partial v_0}{\partial x}\right|)^2 - 2\sqrt{\frac{k_0\bar{k}_2\gamma_1}{2}}\theta\left|\frac{\partial v_0}{\partial x}\right| + \frac{\gamma_1}{2}\theta^2] + \frac{\beta}{2}\|W\|^2 + \bar{k}_2\left|\frac{\partial v_0}{\partial x}\right|^2 - \frac{\gamma_1}{2}\widetilde{\theta}^2 - \frac{\gamma_1}{2}\theta^2 + \frac{\gamma_1}{2}\varepsilon^2 - \frac{\beta}{2}tr\{\widetilde{W}^T\widetilde{W}\} + \frac{\beta}{2}(1-I_W)tr\{\widetilde{W}^T\widetilde{W}\} - \frac{\beta}{2}I_W\|W\|^2 + \frac{\beta}{2}I_W\|W^*\|^2 \tag{29}$$

If $\frac{\bar{k}_2}{\bar{k}_1} \le k_0 \le 1$ is satisfied, then (29) can be changed into

$$\dot{V} \le -k_0\bar{k}_3\left|\frac{\partial v_0}{\partial x}\right|^2 - \frac{\beta}{2}\|\widetilde{W}\|^2 + \frac{\beta}{2}\|W^*\|^2 + \beta(1-I_W)tr\{\widetilde{W}^T\widetilde{W}\} - \frac{\gamma_1}{2}\widetilde{\theta}^2 + \frac{\gamma_1}{2}\varepsilon^2 + (1-I_W)\frac{\beta}{2}M_W^2 \tag{30}$$

Since

$$\beta(1-I_W)tr\{\widetilde{W}^T\widetilde{W}\} = \begin{cases}\beta tr\{\widetilde{W}^T\widetilde{W}\} & if |W| < M_W \\ 0 & if |W| \ge M_W\end{cases} \tag{31}$$

we obtain

$$\beta(1-I_W)tr\{\widetilde{W}^T\widetilde{W}\} \le \beta M_W^2 \tag{32}$$

Moreover, because

$$(1-I_W)\frac{\beta}{2}M_W^2 \le \frac{\beta}{2}M_W^2 \tag{33}$$

(30) can be transformed into the following form

$$\dot{V} \le -k_0\bar{k}_3\left|\frac{\partial v_0}{\partial x}\right|^2 - \frac{\beta}{2}\|\widetilde{W}\|^2 + \frac{\beta}{2}M_W^2 + \beta M_W^2 + \frac{\beta}{2}M_W^2 - \frac{\gamma_1}{2}\widetilde{\theta}^2 + \frac{\gamma_1}{2}\varepsilon^2 \tag{34}$$

By using (5), we have

$$\dot{V} \le -\frac{k_0\bar{k}_3k_4}{k_3}v_0(x) - \frac{\beta}{2}\|\widetilde{W}\|^2 - \frac{\gamma_1}{2}\widetilde{\theta}^2 + 2\beta M_W^2 + \frac{\gamma_1}{2}\varepsilon^2 \tag{35}$$

therefore $\dot{V} \le -\alpha V + \mu$,

with $\alpha = \min\left\{\frac{\bar{k}_3k_4}{k_3}, \beta, \gamma_1\right\}$,

$$\mu = 2\beta M_W^2 + \frac{\gamma_1}{2}\varepsilon^2 \tag{36}$$

Integrating both sides of (36) yields

$$V(t)\leq \frac{\mu}{\alpha}+\left[V(0)-\frac{\mu}{\alpha}\right]e^{-\alpha t},\quad \forall t\geq 0 \tag{37}$$

Due to (37), it can be deduced that $x, W(x), \theta(x)$ are bounded consistently. From (20), we have

$$k_0 v_0(x)\leq V \tag{38}$$

Therefore,

$$v_0(x)\leq \frac{\mu}{k_0\alpha}+\frac{1}{k_0}\left[V(0)-\frac{\mu}{\alpha}\right]e^{-\alpha t},\ \forall t\geq 0. \tag{39}$$

The above completes the proof that x is ultimately consistently bounded by the set D.

4. ILLUSTRATION EXAMPLE

This section takes a fermentation process as a nonlinear process example to show that the control design of section 3 can result in a stable closed-loop to ensure the system states to converge to zero in the presence of a fault.

The fermentation process is assumed to operate at a constant volume V, with the dynamics of biomass X, substrate S, and toxin concentration C_t, described by the follows:

$$\frac{dX}{dt}=\mu X-DX \tag{40}$$

$$\frac{dS}{dt}=-DS-\mu\frac{X}{y_s} \tag{41}$$

$$\frac{dC_t}{dt}=qX^{1/3}-DC_t \tag{42}$$

Where, the dilution rate, D, and the yield coefficient, y_s, are given by

$$D=\frac{F}{V},\ y_s=\frac{y\mu}{My+\mu},$$

and the nonlinear inhibited specific growth rate is $\mu=\mu_m[\frac{S}{K_s+S+S^3/K_i}][\frac{K_t}{K_t+C_t^2}]$.

The parameters of $y, q, \mu_m, K_s, K_i, K_t, M$ are given in Table 1 for the process.

Table 1: Fermentation model parameters

Volume	V	200[l]
Constant	y	0.417
Constant	M	0.0196
Toxin production constant	q	0.0296[l/h(g/l)$^{2/3}$
Maximum specific growth rate	μ_m	0.0135[l/h]
Monod constant	K_s	0.05[g/l]
Substrate inhibition constant	K_i	2150[l^2/g^2]
Toxin inhibition constant	K_t	5.5[g^2/l^2]

Defining the state as $x=[X\quad S\quad C_t]^T$, and the input $u=F/V$, the equations (40-42) become:

$$\begin{bmatrix}\frac{dX}{dt}\\ \frac{dS}{dt}\\ \frac{dC_t}{dt}\end{bmatrix}=\begin{bmatrix}\mu X\\ -(M+\mu/y)X\\ qX^{1/3}\end{bmatrix}+\begin{bmatrix}-X\\ -S\\ -C_t\end{bmatrix}u \tag{43}$$

Using the data in Table 1, we can find:

$$\zeta(x)=\begin{bmatrix}0.5x_1\\ -1.4x_1\\ 0.6x_1^{1/3}\end{bmatrix},\quad G(x)=\begin{bmatrix}-X\\ -S\\ -C_t\end{bmatrix}$$

Let $\Delta g(x)=\begin{bmatrix}\theta_1x_1x_2e^{x_2}\\ 2x_2^2e^{x_2}\sin\theta_2\\ \theta_3x_1e^{x_1}\end{bmatrix},\quad \Delta\zeta(x)=\begin{bmatrix}\theta_2x_1^2\cos\theta_1\\ x_1^2\sin\theta_2\\ \theta_3x_1^2\end{bmatrix}$

where $x=col(x_1,x_2,x_3)=\begin{bmatrix}\frac{dX}{dt} & \frac{dS}{dt} & \frac{dC_t}{dt}\end{bmatrix}^T$,

$\theta_1\in(-2,2)$ and $\theta_2,\theta_3\in(-1,1)$ are the uncertainty parameters. In this example, a radial basis function (RBF) network is chosen to represent the dynamic changes after the fault occurrence, with 10 hidden nodes and 10 centers that are distributed uniformly in region [-1,1].

Choose $\xi(x)=2|x|^2e^{|x|}$, $\rho(x)=2x_1^2$, $v_0=x^Tx=\|x\|^2$.

Then the control input is:

$$u^a=-0.4x_1^{2/3}+0.9x_2$$

$$u^b=\begin{cases}-2|x|^2e^{|x|} & x_1\neq 0\ and\ x_2\neq 0\ and\ x_3\neq 0\\ 0 & otherwise\end{cases}$$

$$u^c=\begin{cases}-\frac{2x_1^2}{(x_1^2+x_2^2+x_3^2)^{1/2}} & x_1\neq 0\ and\ x_2\neq 0\ and\ x_3\neq 0\\ 0 & otherwise\end{cases}$$

the unknown fault function is assumed to be $f(x)=\begin{pmatrix}2\cos x_1\\ 3\cos x_2\\ \cos x_3\end{pmatrix}$, this results in:

$$u^F=\frac{G^T(x)WS(x)}{0.005}+\frac{G^T(x)\begin{bmatrix}\theta\\ 0\end{bmatrix}}{0.005},$$

the weight adaptive law:

$$\dot{W}=2k_0\frac{\partial v_0}{\partial x}S^T(x),$$

$$\dot{\theta}=-0.0025\theta+k_0\left|\frac{\partial v_0}{\partial x}\right|,\ \text{and}$$

the set $D=\left\{x\in R^n: v_0(x)\leq\frac{1.6}{k_0}, 0.5\leq k_0\leq 1\right\}$

We choose $k_0=0.6$, the fault is introduced at $T=1s$, the control results are shown in Figures 1-6.

Figures 1, 3, and 5 depict the control responses of the three states without using of the proposed accommodation strategy. Obviously, the states diverge from the set-point after the occurrence of the fault at T=1. Converse to the above, the results of using the proposed accommodation control law show that all states converge despite of the fault, as shown in Figures 2,4, and 6. This suggest that the proposed control is effective.

5. CONCLUSION

An active fault-accommodation control law has been developed to ensure the closed-loop stability for a class of nonlinear systems, using a neural network approach. The application of the proposed design has been shown to be effective for a fermentation process.

References

1. Michael A. Demetriou and Marios M. Polycarpou. “Incipient fault diagnosis of dynamical systems using online approximators”, IEEE Trans. Automat. Contr., 1998, 43(11): 1612-1617.

2 Xiaodong Zhang, Marios M. Polycarpou and Thomas Parisini. “A robust and isolation scheme for abrupt and incipient faults in nonlinear systems”, *IEEE Trans. Automat. Contr.*, 2002, 47(4) : 576-592.

3 Guanghong Yang, Siying Zhang, James Lam and Jianliang Wang. “Reliable control using redundant controllers”, *IEEE Trans. Automat. Contr.*, 1998, 47(11) : 1558-1593.

4 Delin Chu and Michel Malabre. “Numerically reliable design for proportional and derivative state-feedback decoupling controller,” *15 Triennial world congress of the international federation of automatic control*, Barcelona, 2002: 599-604.

5 Veillette J. R. “Reliable linear-quadratic state feedback control”. *Automatica*, 1995, 31 : 137-143.

6 Patton J. R. fault-tolerant control : the 1997 situation. in Proc. *IFAC Symp. Fault detection, supervision safety for process*, 1997: 1033-1055.

7 Tortora, G., B. Kouvaritakis and D. W. Clarke. Simultaneous optimization of tracking performance and accommodation of sensor faults. *International Journal of Control*, 2002, 75(3) : 163-176.

8. Michael A. Demetriou and Marios M. Polycarpou, Incipient fault diagnosis of dynamical systems using online approach. *IEEE Trans. Automat. Contr.*, 1998, 43(11) : 1612-1617.

9. Boskovic D. J., Yu H. S. and Mehra K. R., Astable scheme for automatic control reconfiguration in the presence of actuator failures, in Proc. *Amer. Contol conf.*, 1998: 2455-2459.

10. Idan M., Johnson M., Calise J. A. and Kaneshge J., Intelligent aerodynamic /propulsion flight control for flight safety: A nonlinear adaptive approach, in Proc. *Amer. Contol conf.*, 2001: 2918-2923.

11. Proll T., Karim N. M. Nonlinear control of a bioreactor model using exact and I/O linearization. Int. J. Control, 1994, 60(4): 499-502.

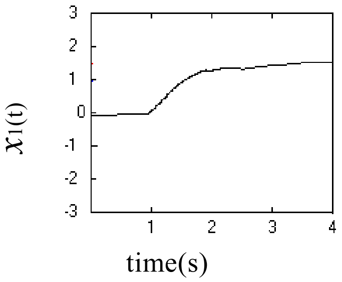

Fig.1: Control response of state $x_1(t)$ without fault accommodation.

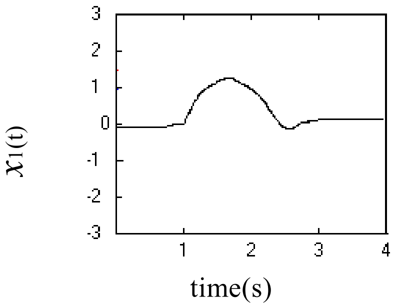

Fig.2: Control response of state $x_1(t)$ with the proposed fault accommodation.

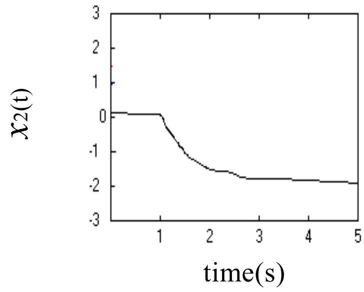

Fig.3: Control response of state $x_2(t)$ Without the fault accommodation

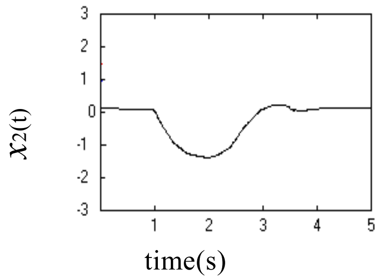

Fig.4: Control response of state $x_2(t)$ with the Proposed fault accommodation

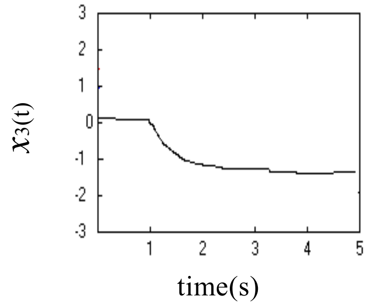

Fig.5: Control response of state $x_3(t)$ without the fault accommodation

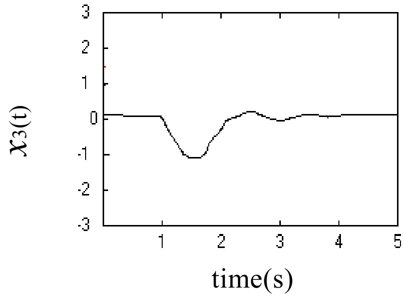

Fig.6: Control response of state $x_3(t)$ with the proposed fault accommodation.