A SUBSPACE APPROACH TO MIMO CONTROL PERFORMANCE MONITORING AND DIAGNOSIS

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Abstract: In this paper we begin with a state space model of a generally non-square process and derive the minimum achievable variance in a state feedback form. We propose a simple control performance calculation which uses orthogonal projection of filtered output data onto past closed-loop data. Finally, we propose a control performance monitoring technique based on the output covariance and diagnose the cause of suboptimal control performance using generalized eigenvector analysis. The proposed methods are demonstrated on an industrial wood waste burning power boiler.

Keywords: MIMO control performance monitoring; generalized eigenanalysis; covariance-based monitoring

1. INTRODUCTION

With the initial success of minimum variance single-loop performance assessment (Harris, 1989; Desborough and Harris, 1992; Qin, 1998; Harris *et al.*, 1999; Kozub, 1996; Harris and Seppala, 2002) and industrial case studies (Thornhill *et al.*, 1999; Miller *et al.*, 1998; Harris *et al.*, 1996*b*; Perrier and Roche, 1992; Weinstein, 1992; Desborough and Miller, 2002), research interest has shifted to the assessment of MIMO control systems using the minimum variance benchmark (Harris *et al.*, 1996*a*; Huang *et al.*, 1997; Huang, 1997; Shah *et al.*, 2002) and a covariance-based benchmark (McNabb and Qin, 2003)

Currently the MIMO performance monitoring benchmark has been a straightforward extension of the SISO variance ratio, which looks at only the trace of the covariance matrix. However, trace based monitoring index is insufficient for assessing the multivariate covariance of the control performance. Another drawback of the existing literature is that research emphasis has been placed on control performance assessment and little has been done regarding diagnosis.

The control performance monitoring tehcniques typically calculate a benchmark performance from closed-loop operation data based on some minimal knowledge, such as the time delay information. Due to interaction, the performance suboptimality in each variable is not independent from each other. Therefore, the suboptimality of a MIMO control system necessarily occupies a subspace instead of the entire output space. In this paper we propose a subspace approach to extract the major directions of suboptimality (MDS) and measure the variance inflation in each of the directions. To deal with the MIMO control performance diagnosis, we propose to use generalized eigenvectors to diagnose the directions of suboptimality.

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This paper is organized as follows. The extended state space model is given in Section 2. The minimum variance control solution in state space is shown in Section 3. Section 4 introduces the view of MVC as an optimal subspace and Section 5 proposes covariance-based monitoring and demonstrates the use of generalized eigenvector diagnosis techniques in this new framework with an industrial example. The paper ends with a few concluding remarks.

2. EXTENDED STATE SPACE PROCESS MODEL

We assume the open loop process is described by the following state space model in innovation form:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Ke(k) \\ y(k) &= Cx(k) + Du(k) + e(k) \end{aligned} \quad (1)$$

where $x \in \Re^n$, $u \in \Re^m$, $y \in \Re^p$, $e \in \Re^p$ are the state, input, output and innovation vectors. A, B, C, D, and K are matrices with appropriate dimensions. Denoting

$$y_{d+1}(k) = \begin{bmatrix} y(k) \\ y(k+1) \\ \vdots \\ y(k+d) \end{bmatrix}, u_{d+1}(k) = \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+d) \end{bmatrix}$$

$$e_{d+1}(k) = \begin{bmatrix} e(k) \\ e(k+1) \\ \vdots \\ e(k+d) \end{bmatrix}$$

$$\Gamma_{d+1} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{d-1} \\ CA^d \end{bmatrix}$$

$$H_{d+1} = \begin{bmatrix} D & 0 & 0 & \cdots & 0 \\ CB & D & 0 & \cdots & 0 \\ CAB & CB & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & D & 0 \\ CA^{d-1}B & CA^{d-2}B & \cdots & CB & D \end{bmatrix}$$

$$G_{d+1} = \begin{bmatrix} I & 0 & 0\cdots & \cdots & 0 \\ CK & I & 0 & \cdots & 0 \\ CAK & CK & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & I & 0 \\ CA^{d-1}K & CA^{d-2}K & \cdots & CK & I \end{bmatrix}$$

where d is the time delay order of this multivariate system. The notation $y_j(k)$ and $e_j(k)$ will be used throughout this paper to represent j-element vectors of $y(k)$ and $e(k)$ extending from time k to $k+j-1$.

We have the following extended equation:

$$\begin{aligned} y_{d+1}(k) = &\Gamma_{d+1}x(k) + H_{d+1}u_{d+1}(k) \\ &+G_{d+1}e_{d+1}(k) \end{aligned} \quad (2)$$

3. STATE SPACE MINIMUM VARIANCE CONTROL

Because of the time delay, not all elements of $y_{d+1}(k)$ are affected by $u(k)$. McNabb (McNabb, 2002) and McNabb and Qin (McNabb and Qin, 2003) present a new algorithm for deriving a multivariate time delay (MTD) matrix $L \in \Re^{(d+1)p\times d}$ for use with the extended state space model (2).

Denote

$$g_{d+1} \triangleq \begin{bmatrix} I \\ CK \\ CAK \\ \vdots \\ CA^{d-1}K \end{bmatrix}, \quad h_{d+1} \triangleq \begin{bmatrix} D \\ CB \\ CAB \\ \vdots \\ CA^{d-1}B \end{bmatrix}$$

$$G_{d+1} = \left[g_{d+1} \left| \begin{matrix} 0 \\ \hline G_d \end{matrix} \right. \right], \quad H_{d+1} = \left[h_{d+1} \left| \begin{matrix} 0 \\ \hline H_d \end{matrix} \right. \right],$$

$$e_d(k+1) = \begin{bmatrix} e(k+1) \\ \vdots \\ e(k+d) \end{bmatrix}$$

To extract the multivariate time delay, the algorithm proposed by McNabb (McNabb, 2002) uses the first few Markov parameters or the {A, B, C, D} matrices, which shifts the output $y(k)$ forward successively and forms a new output by a linear combinations of the shifted outputs,

$$\begin{aligned} y^{(j)}(k) &= L^{(j)}y_{j+1}(k) \\ &= C^{(j)}x(k) + D^{(j)}u(k) \end{aligned}$$

The matrix $L^{(j)} \in R^{P\times(j+1)P}$ is chosen such that $D^{(j)}$ keeps the maximum possible rank. The algorithm will terminate when $D^{(j)}$ reaches full column rank. At the end of the iteration, set

$$\begin{aligned} d &= j \\ L &= L^{(j)} = L^{(d)} \\ \Lambda &= D^{(j)} = D^{(d)} \\ \tilde{y}(k+d) &= y^{(d)}(k) \end{aligned}$$

The output $\tilde{y}(k+d)$ is known as the MTD-filtered output which has the following expression,

$$\tilde{y}(k+d) = Ly_{d+1}(k)$$
$$= L\Gamma_{d+1}x(k) + \Lambda u(k) + Lg_{d+1}e(k)$$
$$+L \begin{bmatrix} 0 \\ G_d \end{bmatrix} e_d(k+1) \quad (3)$$

The minimum variance control of system (1) is achieved by

$$u(k) = -\Lambda^+ L(\Gamma_{d+1}x(k) + g_{d+1}e(k)) \quad (4)$$

where Λ^+ is the Moore-Penrose pseudo-inverse. The feedback invariant term or the output under minimum variance control is

$$\tilde{y}_{mv}(k+d) = L \begin{bmatrix} 0 \\ G_d \end{bmatrix} e_d(k+1)$$

The filtered output shown in Equation (3) can be interpreted as the combination of two independent terms; an optimal d step ahead prediction of $\tilde{y}(k+d)$ and the associated prediction error

$$\tilde{y}(k+d) = \tilde{y}(k+d|k) + \tilde{y}_{mv}(k+d) \quad (5)$$

where

$$\tilde{y}(k+d|k) = L\Gamma_{d+1}x(k) + \Lambda u(k) + Lg_{d+1}e(k) \quad (6)$$

represents the optimal d step ahead prediction of $\tilde{y}(k+d)$.

McNabb and Qin (McNabb and Qin, 2003) show further that L corresponds to a unitary interactor and

$$E\left(\tilde{y}^T(k)\tilde{y}(k)\right) = E\left(y^T(k)y(k)\right)$$

In other words, the sum of the variance of each original output variable is minimized by the MVC law in Eq. 4.

4. CALCULATION OF MVC VARIANCE BY SUBSPACE PROJECTION

The optimal prediction $\tilde{y}(k+d|k)$ under a time-invariant controller is related to (McNabb, 2002),

$$\tilde{y}(k+d|k) = \Theta_r y_r(k-r+1) \quad (7)$$

where $y_r(k-r+1) = \left[y^T(k-r+1), \cdots, y^T(k)\right]^T$ and r is sufficiently large. As a consequence, Equation (5) can be rewritten as

$$\tilde{y}(k+d) = \Theta_r y_r(k-r+1) + \tilde{y}_{mv}(k+d) \quad (8)$$

Again $y_r(k-r+1)$ depends on data before time k and $\tilde{y}_{mv}(k+d)$ depends only on innovations from time $(k+1)$ through $(k+d)$, which are independent of $y_r(k-r+1)$. Therefore, $E\{\tilde{y}_{mv}(k+d)\cdot y_r^T(k-r+1)\} = 0$. Formulating three data matrices with column dimension N

$$\tilde{Y}_N = \begin{matrix}[\tilde{y}(k+d) & \tilde{y}(k+d+1) \\ \dots & \tilde{y}(k+d+N-1)]\end{matrix}$$
$$\tilde{Y}_{mv,N} = \begin{matrix}[\tilde{y}_{mv}(k+d) & \tilde{y}_{mv}(k+d+1) \\ \dots & \tilde{y}_{mv}(k+d+N-1)]\end{matrix}$$
$$Z_{r,N} = \begin{matrix}[y_r(k-r+1) & y_r(k-r+2) \\ \dots & y_r(k-r+N)]\end{matrix}$$

we have $\frac{1}{N}\tilde{Y}_{mv,N}Z_{r,N}^T \rightarrow E\{\tilde{y}_{mv}(k+d)y_r^T(k-r+1)\} = 0$ as $N \rightarrow \infty$. Therefore, defining $\Pi_Z^\perp = I - Z_{r,N}^T\left(Z_{r,N}Z_{r,N}^T\right)^{-1}Z_{r,N}$, we have

$$\tilde{Y}_{mv,N} = \tilde{Y}_N\Pi_Z^\perp$$

The MVC covariance is

$$cov(\tilde{y}_{mv}(k)) = \frac{1}{N-1}\tilde{Y}_{mv,N}\tilde{Y}_{mv,N}^T \quad \text{as } N \rightarrow \infty \quad (9)$$

and the associated multivariate control performance index is

$$\eta = \frac{tr\left\{cov(\tilde{y}_{mv}(k))\right\}}{tr\left\{cov(\tilde{y}(k))\right\}} = \frac{tr\left\{\tilde{Y}_{mv,N}\tilde{Y}_{mv,N}^T\right\}}{tr\left\{\tilde{Y}_N\tilde{Y}_N^T\right\}}$$

From the above derivation we demonstrate that:

1. (1) The output of the process under minimum variance control can be calculated by a single row projection of the MTD filtered output data onto the row space of the normal closed-loop output data, and
2. (2) The minimum variance output occupies an *optimal* subspace of the general closed-loop output.

The variance index has a value between 0 and 1, with 1 corresponding to the minimum variance. The limitation of η, however, is that it only considers the trace of the covariance matrix, ignoring the off-diagonal terms of the covariance.

5. PERFORMANCE MONITORING BASED ON OUTPUT COVARIANCE

Most of the MIMO performance indices are based on the sum of variances of each output, i.e., the trace of the covariance matrix. This type of index is adequate when all variables are fairly independent. In practice, however, the output variables are rarely independent, especially in the case of ill-conditioned plants and highly interacting plants. In these cases, it is more appropriate to use the covariance of the output to monitor controller performance. The benchmark covariance can be the minimum variance benchmark, but it can be any other benchmarks.

5.1 Covariance-based Indices and Suboptimality Directions

To find a direction in $\tilde{y}(k)$ along which the worst suboptimality occurs, we find the direction p with $\|p\| = 1$ and project $\tilde{y}(k)$ and $\tilde{y}_{mv}(k)$ to this direction:

$$\begin{aligned} \Pi_p \tilde{y}(k) &= p^T \tilde{y}(k)/p^T p &= p^T \tilde{y}(k) \\ \Pi_p \tilde{y}_{mv}(k) &= p^T \tilde{y}_{mv}(k)/p^T p &= p^T \tilde{y}_{mv}(k) \end{aligned}$$

The variance of the projections are, respectively,

$$\begin{aligned} var(\Pi_p \tilde{y}(k) &= p^T cov(\tilde{y}(k))p \\ var(\Pi_p \tilde{y}_{mv}(k) &= p^T cov(\tilde{y}_{mv}(k))p \end{aligned}$$

The direction p along which the largest variance ratio occurs is

$$p = arg\ max \frac{p^T cov(\tilde{y}(k))p}{p^T cov(\tilde{y}_{mv}(k))p} \tag{10}$$

The direction of p after maximization shows the direction with the most potential to improve the performance. The solution to this problem is a generalized eigenvector problem,

$$cov(\tilde{y}(k))p_i = \mu_i cov(\tilde{y}_{mv}(k))p_i$$

where p_i is the generalized eigenvector corresponding to the i^{th} largest generalized eigenvalue μ_i. The "volume" of the suboptimality or variance inflation due to poor control performance is:

$$\prod_{i=1}^{l} \mu_i$$

where l is the number of selected directions. The volume-based performance can be defined as

$$I_v(l) = \left(\prod_{i=1}^{l} \mu_i \right)^{-1}$$

McNabb and Qin (McNabb and Qin, 2003) show that for all possible projections Π,

$$cov\left(\Pi \tilde{y}_{mv}(k)\right) \leq cov\left(\Pi \tilde{y}(k)\right)$$

Therefore, $\mu_i \geq 1$ and I_v is between zero and one. When $\tilde{y}(k)$ achieves the minimum variance performance, I_v approaches one. On the other hand, I_v close to zero indicates a very poor performance.

The volume based performance index can be very small due to the multiplication effect of several small numbers. To normalize this effect, we define a radius-based performance index as follows:

$$I_r(l) = (I_v(l))^{1/l} = \left(\prod_{i=1}^{l} \mu_i \right)^{-1/l}$$

This index also ranges between zero and one. It provides a geometric average of the poor performance in all l directions. Note that $I_v(l)$ and $I_r(l)$ consider the covariance matrices of $\tilde{y}(k)$ and $\tilde{y}_{mv}(k)$, whereas the η index focuses on variance only.

After a significant suboptimality is detected by using $I_v(l)$ or $I_r(l)$, the major directions of suboptimality are already calculated as p_i. These directions can then be used to locate the main sources of suboptimality.

5.2 Industrial Example

This example uses industrial data from a wood waste (*hog fuel*) burning power boiler. Five second samples of process variables (PV) with their corresponding setpoints (SP) and controller outputs (OP) were collected from the DCS over an eleven day period. The process and instrumentation diagram of the boiler process is shown in Figure 1. We selected five PV's with associated SP's as shown in Table 1. Full open loop testing was not possible on the power boiler. We therefore assume that each PV had a single time delay associated with the full set of manipulated variables, corresponding to a diagonal interactor. The individual time delays for each of these loops (in units of sample periods) were 1, 34, 1, 2 and 1, respectively. All analysis was performed on data scaled to zero mean and unit variance.

Table 1. Power boiler tags used in analysis

Variable #	Tag	Description
1	FC1	Total air flow
2	PC1	Boiler master (900# header pressure)
3	PC2	Forced draft fan pressure
4	PC3	Furnace pressure
5	PC4	Overfire air pressure

Time series plots of $(PV - SP)$ for each of the five loops are shown in Figure 2 and are divided into 50 sequential 250 minute periods.

To apply the covariance based monitoring to this problem we first select the significant number of principal components of the five process variables. We choose the number of PC's to be four and calculate the volumetric performance index (I_v) and the radius-based performance index (I_r) as shown in Figure 3. Both indices show a clear drop from Period I to Period II. For Period I the I_r index is around 0.6, which means on average each

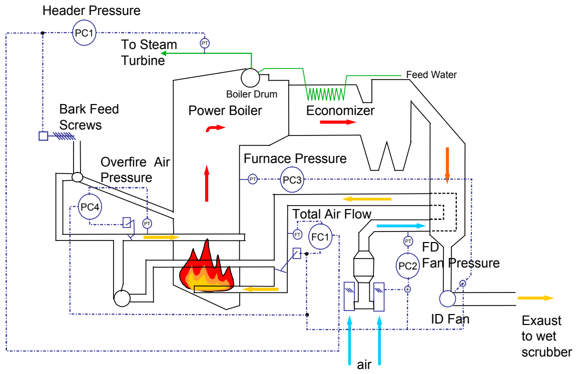

Fig. 1. Power boiler schematic

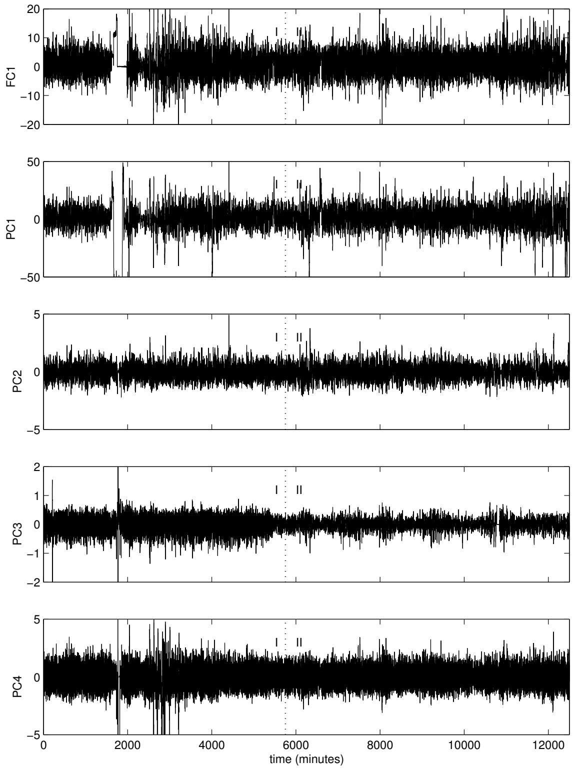

Fig. 2. Time series of setpoint error $(PV - SP)$ for five power boiler loops

variable is about 60% of its optimal performance. For Period II the average performance drops to about 40% of its optimal performance.

In reality, however, it is usually the case that some loops are worse performers than others. To identify the directions of suboptimality and the contributing variables, two 250 minute blocks were chosen for more detailed analysis. The period extending from 3500 minutes to 3750 minutes (the 15th analysis block) is used to represent the behavior before the shift in control performance and the period extending from 10000 minutes to 10250 minutes (the 41st analysis block) is used to represent the system behavior following the shift in control performance. The generalized eigenvector analysis was performed separately on these two periods as shown in Equation (10). The upper and lower plots on the left of Figure 4 show the eigenvalues for the two time periods (labeled as dataset #1 and dataset #2). The middle plots show the first eigenvectors and the right plots show the second eigenvectors for both time periods. It is clear that for both time periods, the major suboptimality lies in two directions, although there are five controlled variables. The first direction of suboptimality is dominant in both time periods, with dataset #1 about 70-fold and dataset #2 about 110-fold for potential improvement. Since the suboptimality is adequately captured in two directions, the covariance-based MIMO performance monitoring indicates that the suboptimality in the five controlled variables is highly correlated. It is possible that by improving the performance of one loop, the other loops are improved due to correlation or interaction. For both time periods the directions of suboptimality point to variable 1 (FC1) for the most potential to improve and variable 2 (PC1) for the least potential to improve. It is likely that by improving the performance of FC1

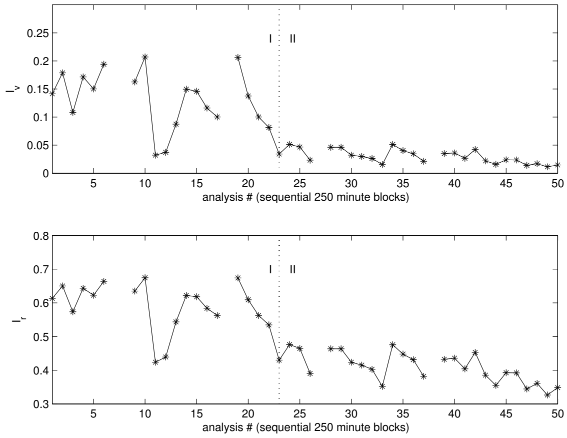

Fig. 3. Trends of the I_v index (top plot) and the I_r (bottom plot) index for 4 retained principal components $(l = 4)$

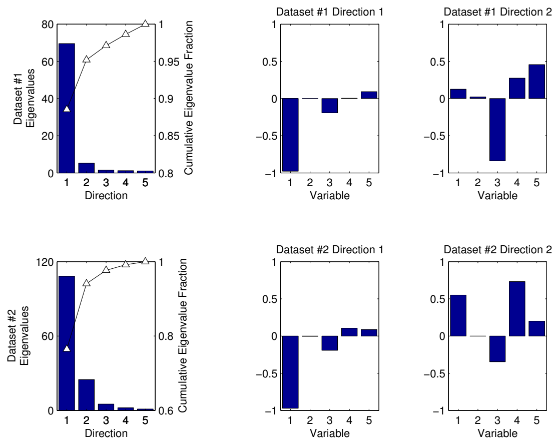

Fig. 4. Eigenvector decomposition of $cov(\tilde{y}_{mv}(k))^{-1}cov(\tilde{y}(k))$ for dataset # 1 (top 3 plots) and dataset # 2 (bottom 3 plots).

other loops will be improved due to interaction. By examining the process diagram in Figure 1, it is confirmed that this is likely because FC1 is the total air flow which directly affects all other loops except PC1, which is the steam header pressure. A significant shift in the second direction occurs between datasets #1 and #2. In dataset #1 PC2 dominates the second eigenvector but in dataset #2 FC1 and PC3 dominate.

6. CONCLUSIONS

The main contributions of this paper are the use of covariance based monitoring and the application of a generalized eigenvector based technique for identifying the major directions of suboptimality of MIMO feedback control systems. This framework provides a systematic performance diagnosis method as well as covariance-based performance assessment indices. Future work will focus on the impact of sensor and actuator faults on control performance.

7. ACKNOWLEDGMENTS

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