Fault Diagnosis and Fault Identification for Fault-Tolerant Control of Chemical Processes

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Abstract: Fault-tolerant control (FTC) of nonlinear systems is presented within an adaptive control framework. FTC can be accomplished by three subtasks; fault-diagnosis, fault identification and adaptive nonlinear control. In order to diagnose a fault at a time, a set of residual generators for fault diagnosis is designed by means of un-known input observers. When disturbances exist, disturbance-decoupled model could be derived for reliable diagnosis. Fault identification following fault diagnosis is an analogue to control task; the diagnosed fault is regarded as a control input and found out such that the residual from residual generator incorporating identification task is driven to reference zero. And, feedback linearizing control is liked with fault diagnosis and fault identification to compensate for a fault to the process. A three-tank system is taken as an example for demonstration of the presented FTC.

Keywords: fault diagnosis, fault identification, fault-tolerant control, nonlinear systems .

1. INTRODUCTION

Automated chemical process has yield to high quality and high efficiency of normal operation, but has become more complicated and more vulnerable to faults because processes have been integrated into wider operation platform and operation algorithms may be another fault sources. Furthermore, a simple fault could be amplified by the control system and developed into malfunction of the loop, even into a failure at the plant level. This requires advanced fault diagnosis and supervision to improve reliability and safety. A cost-effective way to achieve the goal is by means of a fault-tolerant control (FTC) (Blank et al., 2001).

The FTC could be achieved by merging the fault information obtained from fault diagnosis and fault identification into the control system for fault accommodation. Fault diagnosis scheme has to efficiently detect and identify a fault even when the process is under closed-loop control and varies over wide range, and following fault diagnosis, fault identification estimates time-varying behaviors of the diagnosed fault which then is reflected into the control law to accommodate the fault.

Nonlinear observer-based fault diagnosis where the research that has been made around a linear system has been lately extended to nonlinear system (Frank and Ding, 1997) is presented. The model used for observer design could be decoupled from disturbances and/or a dedicated fault by means of state transformation, and so resulting observer is unaffected by disturbances and possess structured sensitivities to the faults. The state transformation is based on the concept of fault (disturbance) detectability index and so it contains outputs and their derivatives. A bank of residual generators providing generalized residuals set and decisions function such as fixed threshold are needed to diagnose a fault. The design method and conditions of such nonlinear observers will be presented.

When a fault is detected and localized, its magnitude and time-evolving behavior should be identified to take a countermeasure keeping process performances. The same model and the concept of fault relative order as those of fault diagnosis are utilized to obtain design model which allows to reconstruct the fault and on which the residual generator is designed. Fault identification is formulated into control task problem such that the residual from the residual generator driven by identified fault is forced to zero. The method is based on the conditions of input observability (Hou and Patton, 1998; Kabore and Wang, 1999); the residual responds to only a specific fault and reconstructs the actual fault.

Fault-tolerant control is achieved by combining input-output feedback linearizing control law based on the fault-parameterized model with fault informations obtained from fault diagnosis fault identification, which is reduced to the adaptive linearizing control (Sastry and Isidori, 1989).

2. ON-LINE FAULT DIAGNOSIS

Consider nonlinear systems affine in control, disturbance and fault mode;

$$\dot{x} = g_0(x) + \sum_{s=1}^m g_s(x) u_s(t) + \sum_{s=1}^{n_d} D_s(x) d_s(t) + \sum_{s=1}^{n_f} e_s(x) f_s(t)$$
(1a)
$$y = h(x)$$
(1b)

where $x \in \Gamma \subset \mathbb{R}^n$, $u \in \Omega_u \subset \mathbb{R}^m$, $y \in \mathbb{R}^p$ are the state vector, the control input vector, and the output vector. $d \in \Omega_d \subset \mathbb{R}^{n_d}$ is the disturbance vector including model errors, and $f \in \Omega_j \subset \mathbb{R}^{n_f}$ is the fault mode vector for component faults and actuator faults, and both vectors consist of unknown time-varying functions. $g_s(x), s = 0, 1, ..., m; D_s(x), s = 1, ..., n_d$ and $e_s(x), s = 1, ..., n_f$ are smooth vector fields, and $h_s(x), s = 1, ..., q$ are smooth scalar fields, respectively, on Γ and they are known. Here, Γ is a physically feasible and bounded set, and Ω with subscripts are bounded sets for corresponding inputs.

FDI Strategy: When the process is subject to disturbances, model-based FDI strategy may give misleading analytical redundancies. Thus, to improve the FDI performance, a means of creating analytical redundancy not affected by disturbances should be devised as in the following steps;

- **S1:** Obtain a reduced model decoupled from disturbances, but still affected by faults.
- **S2:** For the detection of faults remained at step S1, produce a residual generator based on disturbance-decoupled model obtained at step S1.
- S3: For fault isolation among the faults at step S1,

partition faults into isolable fault subsets and generate a bank of residual generators in which each one is dedicated to each fault subset. It is based on a fault-added disturbance-decoupled model and provides generalized residuals set giving different sensitivities to different fault subsets. This feature enables to uniquely isolate a fault subset by checking the values of all residuals.

Fault Detectability: A fault is said to be detectable if a fault affects at least one of observable outputs. From this point of view, the detectability of a fault can be characterized by using the concept of relative order known as a fault detectability index in fault diagnosis field (Liu and Si, 1997), which is defined as the smallest integer, r_{f_c} , such that

$$L_{e_j} L_{g_0}^{r_{j}-1} h_i(x) \neq 0 \qquad \exists i \in [1, ..., p], \forall x \in \Gamma$$

$$\tag{2}$$

where $e_j(x)$ is a fault vector field of a fault, f_j and $h_i(x)$ is a scalar function of the output, y_i . If fault relative order, r_{f_j} , is less than or equal to n, the fault is detectable. Otherwise, the fault is not detectable. Disturbance relative order, r_{d_j} , for the disturbance,

 d_{i} , can be defined in the same way.

Disturbance-Decoupled Model: Residual generators will be obtained through the design of observers based on disturbance-decoupled or a fault-added disturbance-decoupled models. The conditions that guarantee a state transformation inducing disturbance-decoupled nonlinear model are based on the concept of well-defined disturbance relative order and are analogies to those of feedback linearization in nonlinear control theory (Isidori, 1989).

Consider the system with only disturbances;

$$\dot{x} = g_0(x) + \sum_{s=1}^{n_d} D_s(x) d_s(t)$$
(3a)
$$y = h(x)$$
(3b)

$$y = h(x) \tag{3b}$$

For above system, if conditions below are met,

C1. The relative order, r_{d_i} , of the output, y_i , i = 1,..,l (l < p), with respect to disturbance vector, d, is well defined.

C2. The characteristic matrix, $C_D(x)$, formed at the r_{d_i} th times derivative of each output, y_i , before the disturbance vector, d, has full row rank.

$$C_{D}(x) = \begin{bmatrix} L_{D_{1}}L_{g_{0}}^{r_{d_{1}}-1}h_{1} & \cdots & L_{D_{n_{d}}}L_{g_{0}}^{r_{d_{1}}-1}h_{1} \\ \vdots & \vdots & \vdots \\ L_{D_{1}}L_{g_{0}}^{r_{d_{n_{d}}}-1}h_{n_{d}} & \cdots & L_{D_{n_{d}}}L_{g_{0}}^{r_{d_{n_{d}}}-1}h_{n_{d}} \end{bmatrix} (x)^{-(4)}$$

C3. The distribution, $\Delta(x) = span\{D_1(x),...,D_{n_d}(x)\}$, identified and spanned by disturbance vector fields has constant rank, $\overline{q} (\leq q)$ and it is involutive, i.e., Lie brackets of any pair of vector fields belonging to $\Delta(x)$ belong to $\Delta(x)$ again.

then, there exists a state transformation

$$z = \Phi(x) = \begin{bmatrix} \mathbf{x}^{i} \\ \mathbf{\hbar} \end{bmatrix} = \begin{bmatrix} y_{i} \\ \vdots \\ y_{i}^{(r_{i}-1)} \\ \mathbf{\hbar} \end{bmatrix} (x) = \begin{bmatrix} h_{i} \\ \vdots \\ L_{s_{0}}^{(q_{i}-1)} h_{i} \\ \mathbf{\hbar} \end{bmatrix} (x) \quad \forall i \in [1,..,p] \quad (5)$$

where $\mathbf{x} = \begin{bmatrix} \mathbf{x}^{i}, \dots, \mathbf{x}^{i} \end{bmatrix}^{T}$, $\mathbf{x}^{i} = \begin{bmatrix} \mathbf{x}_{1}^{i}, \dots, \mathbf{x}_{r_{d}}^{i} \end{bmatrix}^{T} = \begin{bmatrix} y_{1}, \dots, y_{1}^{(r_{d}-1)} \end{bmatrix}^{T}$, and $\mathbf{\overline{h}} \in \mathbb{R}^{n-r_{d}}$ with $r_{d} = r_{d_{1}} + \dots + r_{d_{n_{d}}}$ consists of scalar fields, $\mathbf{\overline{h}}_{i}$, such that $L_{D}\mathbf{\overline{h}}_{i}(\mathbf{x}) = 0$ and makes the state transformation locally invertible (Isidori, 1989);

The system in the transformed state is

$$\dot{\mathbf{x}}_{i_{a_{i}}}^{i} = L_{g_{0}}^{t_{a_{i}}}h_{i}(x) + \begin{bmatrix} L_{g_{1}} f_{g}^{i}^{-1}h_{i} & \cdots & L_{g_{m}} f_{g}^{i}h_{j}^{-1}h_{j}^{T}(x)u + \begin{bmatrix} L_{g_{1}} f_{g_{0}}^{i}h_{i} & \cdots & L_{g_{m}} f_{g}^{i}^{-1}h_{j} \end{bmatrix}^{T} d \\ + \begin{bmatrix} L_{e_{i}} L_{g_{0}}^{i}^{-1}h_{i} & \cdots & L_{e_{s_{j}}} f_{g_{0}}^{i}h_{i} \end{bmatrix}^{T}(x)f \mid_{x=0^{-1}(x,h)}$$

$$\begin{bmatrix} \dot{\mathbf{x}}_{i}^{i} \\ \vdots \\ \dot{\mathbf{x}}_{i_{q}-1}^{i} \end{bmatrix} = A_{i}\begin{bmatrix} \mathbf{x}_{i}^{i} \\ \vdots \\ \mathbf{x}_{i_{q}-1}^{i} \end{bmatrix} + B_{i}\mathbf{x}_{i_{q}}^{i} + \begin{bmatrix} L_{g_{1}}h_{i} & \cdots & L_{g_{m}}h_{i} \\ \vdots & \cdots & \vdots \\ L_{g_{1}}L_{g_{0}}^{g_{q}-2}h_{i} & \cdots & L_{g_{m}}L_{g_{0}}^{g_{q}-2}h_{i} \end{bmatrix} (x)u \\ + \begin{bmatrix} L_{e_{1}}h_{i} & \cdots & L_{e_{s_{j}}}h_{i} \\ \vdots & \cdots & \vdots \\ L_{e_{1}}L_{g_{0}}^{i} & -h_{i} & \cdots & L_{e_{s_{j}}}h_{i} \\ \vdots & \cdots & \vdots \\ L_{e_{1}}L_{g_{0}}^{i} & -h_{i} & \cdots & L_{e_{s_{j}}}h_{i} \end{bmatrix} (x)f \mid_{x=0^{-1}(x,h)}$$

$$\dot{\mathbf{h}} = \mathbf{h}(\mathbf{x}, \mathbf{h}, u, f)$$

$$(7c)$$

where A_i is the $(r_i-1)\times(r_i-1)$ matrix and B_i is the $(r_i-1)\times1$ vector and (A_i, B_i) is in a canonical form. The transformed system is divided into two subsystems according to explicit dependence on the disturbances. Disturbance-decoupled model consists of lower subsystems (7b)(7c) and it will be utilized for the design of nonlinear observers that are robust to disturbances but sensitive to faults.

Remark: The model is driven by the faults and as well $\mathbf{x}_{t_{a_i}}^i$ as new inputs, not directly available. Its estimation is to use a differentiator filter But, since each filter is driven by a measured output, the effects of the faults are reflected into estimated output and its successive derivatives, and the disturbance decoupled model actually useful for FDI is limited to the (7c). But, their estimates from filters will be used for provision of unavailable states with (7c). This means that the faults whose relative orders are more than two cannot be separable through the resulting decoupled model unless all states are available.

Fault Detection: The design of a nonlinear observer for FD (Fault Detection) is performed on the model (7c) and various design methods of an observer in the literatures can be considered. If state estimates from differentiator filters are sufficiently accurate, the basis model becomes;

$$\dot{\boldsymbol{\hbar}} = \boldsymbol{\hbar} \left(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{u}, \boldsymbol{f} \right) \tag{8a}$$

$$Y_{ex} = C\bar{h} \tag{8b}$$

Under some assumptions below,

A1 Extra outputs, Y_{ex} , not involved in disturbance decoupling are available and linear in the state.

A2 The basis model is observable from extra outputs.A3 The basis model can be put into a time-varying linear system with a Lipschitz nonlinear perturbation;

$$\dot{\boldsymbol{h}} = Q(\boldsymbol{x}, u)\boldsymbol{h} + N(\boldsymbol{h}, \boldsymbol{x}, u) + E(\boldsymbol{h}, \boldsymbol{x}, f)$$
⁽⁹⁾

$$\left| N\left(\boldsymbol{\hbar}, \boldsymbol{x}, \boldsymbol{u}\right) - N\left(\boldsymbol{\hat{\hbar}}, \boldsymbol{x}, \boldsymbol{u}\right) \right| \leq \boldsymbol{\bar{\hbar}}_{o} \left| \boldsymbol{\hbar} - \boldsymbol{\hat{\hbar}} \right|$$
(10)

where \hbar_{a} is a constant.

A4 Constant matrix, K, can be chosen such that

$$I\left[\left(\underline{Q}(\mathbf{x},u)-KC\right)^{T}+\left(\underline{Q}(\mathbf{x},u)-KC\right)\right]<-\mathbf{a}\quad\forall\mathbf{x},u$$
(11)

where $I[t; \cdot]$ denotes the eigenvalues of time-varying matrix at time t and a > 0 is a constant. A candidate observer can be taken as

$$\dot{\mathbf{\hbar}} = Q(\mathbf{x}, u)\mathbf{\hbar} + N(\mathbf{\hbar}, \mathbf{x}, u) + KC(\mathbf{\hbar} - \mathbf{\hbar})$$
(12)

and then the error dynamics with a residual is

$$\dot{e} = (Q(\mathbf{x}, u) - KC) e + N(\mathbf{\hbar}, \mathbf{x}, u) - N(\mathbf{\hbar}, \mathbf{x}, u) + E(\mathbf{\hbar}, \mathbf{x}, f)$$
(13a)

$$r = \left(e^T C^T C e\right)^{1/2} \tag{13b}$$

where $e = \hbar - \hat{\hbar}$.

Assumption A3 can be easily met since the basis model is reduced and a nonlinear perturbation term is allowed, and when the model is defined over bounded domain. Assumption A4 makes the error dynamics stable if the design matrix, K, can be chosen such that for $a > 2\hbar_0$ it is satisfied (Slotine and Li, 1984).

In the absence of any fault (f=0), the error dynamics are made stable around the equilibrium, e=0 and so the residual is decaying to zero, indicating no fault. But, in the presence of any fault $(f \neq 0)$, the error no longer stays at zero due to nonlinear nonvanishing effects by a fault and so resulting nonzero residual indicates the occurrence of a fault.

Fault Is olation: Due to the allowance for disturbance decoupling, the generalized observer scheme (GOS) (Chen and Patton, 2000) providing a generalized residuals set for fault isolation is adopted. To implement the GOS, the model decoupled from disturbances and as well a fault subset is needed, and it can be obtained in the same way as before for augmented disturbances. Robust fault isolation between two faults in sense of the GOS can be checked by;

$$\operatorname{rank}\left(\left(\frac{\partial \mathbf{h}}{\partial x}\right)(e_i(x), e_j(x))\right) = \operatorname{rank}\left(e_i(x), e_j(x)\right) = 2 \quad (14)$$

where $e_i(x)$ and $e_j(x)$ are vector fields of considered faults. This condition makes sure that two faults are not only reflected into disturbance-decoupled model but also not decoupled at a time. The observability of a fault-added disturbance-decoupled model is assumed.

The design of residual generators for fault isolation is based on a designated fault and disturbance decoupled model and their designs proceed as before. When one fault occurs at a time, the right fault isolation can be done by checking values of all residuals.

3. FAULT IDENTIFICATION

When the focus is made on the design of fault tolerant control, in addition to fault detection and is olation, fault identification identifying the size of the fault and its time varying behavior has to be solved. Faults to be identified are limited to the faults isolated by fault diagnosis.

Fault identification problem can be reformulated into control task problem. In this approach, fault signal is regarded as a control input to the system and a feedback control is found such that the residual tracks a zero reference. Since the residual is usually given as the difference between measured output and estimated output, the fault forces the estimated outputs from residual generator to track measured outputs. When there is no fault, the feedback control will decay to zero, indicating no fault, while in the presence of a fault, the feedback control will provide a control input which is actually an estimate of the fault. Consider the system with single fault vector;

$$\dot{x} = g_0(x) + \sum_{s=1}^m g_s(x)u_s(t) + \sum_{s=1}^{n_{f_i}} e_s(x) f_s(t)$$
(15a)

$$y = h(x) \tag{15b}$$

where f_s is one element of fault vector, $\overline{f_i}$, and descriptions of variables, x, y, u, and functions, g_s, e_s, h_j , are the same as those of the system (1). A1 the fault detectability index, k_j , for the fault vector, $\overline{f_i}$, such that

$$L_{e_s} L_{g_0}^{k_j - 1} h_j(x) \neq 0 \quad s \in \{1, ..., n_{f_i}\}, j \in \{1, ..., \overline{p}\}, \forall x \in \Gamma(16)$$

is well defined.

A2. the matrix,

$$E_{i}(x) = \begin{pmatrix} E_{1,k_{1}}^{i}(x) \\ E_{2,k_{2}}^{i}(x) \\ \vdots \\ E_{\overline{p},k_{\overline{p}}}^{i}(x) \end{pmatrix} = \begin{pmatrix} \sum_{s=1}^{n_{f_{1}}} L_{e_{s}} L_{g_{0}}^{k_{1}-1} h_{1}(x) \\ \sum_{s=1}^{n_{f_{1}}} L_{e_{s}} L_{g_{0}}^{k_{2}-1} h_{2}(x) \\ \vdots \\ \sum_{s=1}^{n_{f_{1}}} L_{e_{s}} L_{g_{0}}^{k_{\overline{p}}-1} h_{\overline{p}}(x) \end{pmatrix}$$
(17)

is a full column rank for all $x \in \Gamma$.

Then, the following state coordinate can be taken

$$\mathbf{x}^{j} = \begin{pmatrix} \mathbf{x}_{1}^{j} \\ \mathbf{x}_{2}^{j} \\ \vdots \\ \mathbf{x}_{k_{j}}^{j} \end{pmatrix} = \begin{pmatrix} h_{j}(x) \\ L_{s0} h_{j}(x) \\ \vdots \\ L_{s0}^{k_{j}-1} h_{j}(x) \end{pmatrix}, \quad j = 1, ..., \overline{p}$$

$$(18)$$

and the system is transformed into following form, which is the same as (7);

$$\dot{\mathbf{x}}^{j} = A_{j}^{i} \mathbf{x}^{j} + B_{j}^{i} G_{0,k_{j}}^{1}(\mathbf{x}, \mathbf{V}) + \sum_{s=1}^{m} G_{s,j}^{1}(\mathbf{x}, \mathbf{V}) u_{s} + E_{j}^{i}(\mathbf{x}, \mathbf{V}) \overline{f_{i}}^{(19a)}$$

$$\mathbf{V} = G_{0}^{2}(\mathbf{x}, \mathbf{V}) + \sum_{s=1}^{m} G_{s}^{2}(\mathbf{x}, \mathbf{V}) u_{s}$$

$$y_{j} = C_{j}^{i} \mathbf{x}^{j} = \mathbf{x}_{1}^{j}$$
(19b)

where $\mathbf{x}^{j} = (\mathbf{x}_{1}^{j}, ..., \mathbf{x}_{k_{j}}^{j})$, $\mathbf{x} = (\mathbf{x}^{1}, ..., \mathbf{x}^{\overline{p}}) = \Phi^{1}(x) \in \mathbb{R}^{k}$ with $k = \sum_{s=1}^{\overline{p}} k_{s}^{s}$, $\mathbf{V} = \Phi^{2}(x) \in \mathbb{R}^{n-k}$ such that $\mathbf{h} = (\mathbf{x}, \mathbf{V})^{T}$ is invert-

ible.

$$G_{s,j}^{l} = \left[0, ..., L_{g, k}^{j} \tilde{L}_{d}^{-l} h_{j}(x)\right]^{T}, s = 0, 1, ..., \overline{p}$$

$$E_{j}^{i} = \left[0, ..., \sum_{s=1}^{n_{\ell}} L_{e_{s}} L_{g_{0}}^{k_{j}-l} h_{j}(x)\right]^{T}, \text{ and } (A_{j}^{i}, C_{j}^{i}) \text{ is in a observ}$$

able canonical form and (A_j^i, B_j^i) is in a controllable canonical form.

Based on the structure of the above systems, a candidate residual generator incorporating fault identification is taken as (Kabore and Wang, 1999):

$$\hat{\mathbf{x}}^{j} = A^{i}_{j} \hat{\mathbf{x}}^{j} + B^{i}_{j} G^{i}_{0,k_{j}}(\hat{\mathbf{x}}, \mathbf{V}) + \sum_{s=1}^{m} G^{i}_{s,j}(\hat{\mathbf{x}}, \mathbf{V}) u_{s} + E^{i}_{j}(\hat{\mathbf{x}}, \mathbf{V}) \hat{f}^{-}_{i} - K^{i}_{j}(C_{j} \hat{\mathbf{x}}^{j} - y_{j})$$
(20a)
(20b)

$$\mathbf{V} = G_0^2 (\mathbf{x}, \mathbf{V}) + \sum_{s=1}^{s} G_s^2 (\mathbf{x}, \mathbf{V}) u_s$$

$$r_j^i = C_j^i \mathbf{x}^j - y_j$$
(20c)

$$\widehat{\overline{f}}_{i} = E_{i}^{-1}(\widehat{\mathbf{x}}, \mathbf{V}) \begin{bmatrix} v - \begin{pmatrix} G_{0,k_{1}}^{1}(\widehat{\mathbf{x}}, \mathbf{V}) + \sum_{s=1}^{m} G_{s,k_{1}}^{1}(\widehat{\mathbf{x}}, \mathbf{V}) u_{s} \\ G_{0,k_{2}}^{1}(\widehat{\mathbf{x}}, \mathbf{V}) + \sum_{s=1}^{m} G_{s,k_{2}}^{1}(\widehat{\mathbf{x}}, \mathbf{V}) u_{s} \\ \vdots \\ G_{0,k_{\overline{\nu}}}^{1}(\widehat{\mathbf{x}}, \mathbf{V}) + \sum_{s=1}^{m} G_{s,k_{\overline{\nu}}}^{1}(\widehat{\mathbf{x}}, \mathbf{V}) u_{s} \end{bmatrix} \end{bmatrix}$$
(21)

where v is a proper new input and taken as

$$v = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_{\overline{p}} \end{pmatrix} = \begin{pmatrix} y_1^{(k_1)} \\ y_2^{(k_2)} \\ \vdots \\ y_{\overline{p}}^{(k_{\overline{p}})} \end{pmatrix}$$
(22)

and observer gain, K_j^i , is chosen such that each observable form may be stable and makes the error exponentially converge to zero.

4. FAULT-TOLERANT CONTROL

To accommodate identified faults, the linearizing control law can be linked with fault identification, which is reduced to adaptive linearizing control (Sastry and Isidori, 1989; Teel et. al., 1991; Hu, 1999). When the resulting control law is applied to the faulty process, quasi-linear system results in.

$$\dot{e} = A_{e} e + W(e, \mathbf{h}, u_{a})\Theta$$
⁽²³⁾

where A_c is a $(r+1) \times (r+1)$ Hurwitz matrix, u_a is approximately linearizing control law and e is error coordinate. Θ is the fault error and the matrix, $W(\cdot)$ is nonlinear functions before the fault error. As fault error is decaying to zero, the quasi-linear system becomes asymptotically linearized. The stability of the perturbed system is ensured if the perturbing term is bounded over the domain and the poles of the Hurwitz linear system are placed sufficiently deep into the left half of s-plane (Zak, 1990).

5. APPLICATION

Figure 1 shows a schematic of a three-tank system. Using the mass balance and mass flows by Torricell's law, the system can be described as;

$$\dot{x}_{1} = (u_{10} - a_{1} x_{3} \operatorname{sgn}(x_{1} - x_{3}) \sqrt{2g |x_{1} - x_{1}|} / A$$

$$\dot{x}_{2} = (u_{20} - a_{3} s_{32} \operatorname{sgn}(x_{2} - x_{3}) \sqrt{2g |x_{2} - x_{3}|} - a_{2} s_{20} \sqrt{2g x_{2}}) / A$$

$$\dot{x}_{3} = (a_{1} x_{3} \operatorname{sgn}(x_{1} - x_{3}) \sqrt{2g |x_{1} - x_{3}|} - a_{3} s_{32} \operatorname{sgn}(x_{3} - x_{2}) \sqrt{2g |x_{3} - x_{2}|}) / A$$

where the state, x, is the level of each tank and u_{10} ,

 u_{20} are mass inflows. A is the cross-section of tank and s_{13} , s_{32} , s_{20} are the cross sections of interconnected and outlet pipe, respectively. a_i is scaling constants and g is the gravity constant. f_i denotes faults caused by various reasons such as leaks, clogging and pump failures. Levels are available. Fault distribution matrix, E(x), to the fault, f, is given as;

$$H_{xy} = \begin{bmatrix} \sqrt{2g_{x}} & 0 & 0 & \frac{u_{b}}{A} & 0 & \frac{g_{y}(x_{1} - x_{y})}{2g_{x} - x_{1}} & 0 & 0 \\ 0 & \sqrt{2g_{x}} & 0 & 0 & \frac{u_{b}}{A} & 0 & \frac{g_{y}(x_{2} - x_{y})}{A} & \frac{g_{y}(x_{2} - x_{y})}{2g_{x} - x_{1}} & \frac{\sqrt{2g_{x}}}{A} \\ 0 & 0 & \sqrt{2g_{x}} & 0 & 0 & \frac{g_{y}(x_{1} - x_{x})}{A} & \frac{g_{y}(x_{2} - x_{y})}{A} & \frac{g_{y}(x_{2} - x_{y})}{A} & 0 \end{bmatrix}$$

Unknown disturbance is not considered. And, all modeled faults have the fault relative order of one. Outputs carry with measurement noises.

Residual generator for fault detection will be designed based on the whole model (Figure2) and isolable fault subsets for fault isolation are as follows;

$$S_1 = \{f_1, f_4\}, S_2 = \{f_2, f_5, f_8\}, S_3 = \{f_3\}, S_4 = \{f_6\}, S_5 = \{f_7\}$$

Generalized residuals set, $\{r_1, r_2, r_3, r_4, r_5\}$, is generated from residual generators via observers based on the models in new states, \mathbf{h}_j such that $L_{e_i}\mathbf{h}_j = 0$ where e_i is all fault vector fields belonging to a fault subset, S_i (Figure 3).

As for fault identification, only one fault at a time is estimated and its fault relative order is one. The state of \mathbf{V} is available from extra outputs and the first derivative of the output, y_i , is obtained by a differentiator filter. The results are shown in Figure 4.

Performance of linearizing control linked with fault identification is compared with simple linearizing control as shown in Figure 5.

ACKNOWLEDGEMENT

We acknowledge the financial aid for this research provided by the Brain Korea 21 Program supported by the Ministry of Education. In addition, we would like to thank the Automation & Systems Research Institute and Institute of Chemical Engineering of Seoul National University.

REFERENCES

Blanke, M., Staroswieki, M. and Wu, N.E. (2001) Concepts and Methods in Fault-Tolerant Control. Proc. American Control Conference, Arlington, June, 2606-2620.

Frank, P. M. and Ding, X. (1997) Survey of Robust Residual Generation and Evaluation Methods in Observer-Based Fault Detection Systems, *J. Proc. Contr.* **7**(6), 403-423.

Hou, M. and Patton, R.J. (1998) Input Observability and Input Reconstruction. *Automatica*. **34**(6), 789-794.

Kabore, P. and Wang, H. (1999) On the Design of Fault Diagnosis Filters and Fault Tolerant Control. American Control Conference, San Diego, California, June.

Liu, B. and Si, J. (1997) "Fault Isolation Filter Design for Linear Time-Invariant Systems," *IEEE Trans. Autom. Control*, **42**, 704-707.

Isidori, A. (1989) Nonlinear Control Systems, 2nd, Springer-Verlag.

Sastry, S. and Isidori, A. (1989) "Adaptive Control of Linearizable Systems," *IEEE Trans. Autom. Control*, **34**, 1123-1131. Chen, J. and Patton, R. J. (2000) *Robust Model-based Fault Diagnosis for Dynamic Systems*. Kluwer Academic Publishers, Boston/Dordrecht/London.

Slotine, J-J. E. and Li, W. (1991) *Applied Nonlinear Control*, Prentice Hall, New Jersey.

Hu, Q. and Rangaiah, G.P. (1999), "Adaptive Internal Model Control of Nonlinear Processes," *Chem. Engng. Sci.*, 54, 1205-1220. (6.7)

Zak, S.H. (1990) On the Stabilization and Observation of Nonlinear/Uncertain Dynamic Systems. *IEEE Trans. Automatic Control.* **35**, 604.



Figure1. Schematic of a three-tank system



Figure3. Generalized residuals set $(r_i, i=1,...,5)$ for fault isolation



Figure5. Performance by FTC and simple nonlinear control