DISTURBANCE ATTENUATION WITH ACTUATOR CONSTRAINTS BY MOVING HORIZON $\mathcal{H}_\infty$ CONTROL

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Abstract: Exploiting the moving horizon strategy, we provide in this paper a solution of the constrained L_2-gain attenuation control problem that is less conservative than a recently suggested switching approach based on off-line controller computations. The main advantage of the presented scheme is its capability of automatically relaxing or tightening the performance specification in order to obey hard control constraints while achieving the best possible performance in a suitable class of LMI-generated control gains.

Keywords: L_2-gain attenuation, control constraints, dissipation theory, on-line optimization

1. INTRODUCTION

In the past two decades H_∞-control has received considerable interest, in particular for the possibility to beneficially manage the trade-off between high performance requirements and high control action. It is somewhat unfortunate, however, that the designer has only influence onto closed-loop transfer function shapes in the frequency domain and that there is no direct way to enforce hard time-domain constraints on the control inputs. To overcome this drawback, a large variety of approaches have been proposed in the literature such as anti-windup techniques (Kothare *et al.*, 1994), saturation avoidance methods based on maximal output invariant sets (Gilbert and Tan, 1991), model predictive control (Mayne *et al.*, 2000) and switching techniques (Hirata and Fujita, 2000). For a survey we refer to (Scherer *et al.*, 2002) and the references therein.

In this paper, we provide a moving horizon scheme for the L_2-gain attenuation problem with hard control constraints, where a constrained $\mathcal{H}_\infty$ problem is solved on-line and updated by the new measurement. The scheme has the capability to automatically trade-off constraint satisfaction and performance by relaxing or tightening the performance specification, which leads to performance improvements. The feedback gain is determined on-line such that the ellipsoids, where constraints are respected, are shaped according to the actual state and hence performance can be further improved, whereas the off-line controller construction in (Scherer *et al.*, 2002) is based on extremal solutions of the Riccati equation corresponding to the $\mathcal{H}_\infty$ problem. Therefore, this paper can be viewed as a direct extension of (Scherer *et al.*, 2002) towards a non-conservative solution of the constrained L_2-gain attenuation control problem. In a similar fashion, it is suggested in (Kothare *et al.*, 1996) to use the moving horizon strategy in order to ensure robust stability while minimizing an upper bound of a quadratic cost, whereas our scheme explicitly strives for L_2-gain performance guarantees for the overall closed-loop system.

The paper is organized as follows. In Section 2 we describe an off-line solution to the constrained $\mathcal{H}_\infty$ control, using the concept of state-space ellipsoids and reachable sets (Boyd *et al.*, 1994). In Section 3 we derive the crucial condition to guarantee dissipation after briefly showing why the naive implementation of the moving horizon strategy might fail. Then, an extended LMI optimization problem is formulated that will be solved on-line at each sampling time to determine the feedback gain, updated with the actual state. An algorithm for a concrete implementation of the proposed scheme is given in Section 4. Simulation results for the same open-loop unstable continuous stirred tank reactor as in (Scherer *et al.*, 2002) are presented in Section 5.

2. PRELIMINARIES

Consider a discrete system described by

$$x(k+1) = Ax(k) + Bw(k) + B_u(k) \tag{1a}$$
$$z(k) = Cx(k) + Dw(k) + D_u u(k) \tag{1b}$$

subject to control constraints

$$|u_i(k)| \le u_{i,max}, \forall k \ge 0, i = 1, 2, \cdots, m_2. \tag{2}$$

Here $x \in \mathbb{R}^n$ denotes the states, $w \in \mathbb{R}^{m_1}$ the external disturbances, $u \in \mathbb{R}^{m_2}$ the control inputs and $z \in \mathbb{R}^p$ the controlled outputs.

With state-feedback control $u = Kx$, the closed-loop system is

$$x(k+1) = A_{cl}x(k) + Bw(k) \tag{3a}$$
$$z(k) = C_{cl}x(k) + Dw(k) \tag{3b}$$

where $A_{cl} = A + B_u K$ and $C_{cl} = C + D_u K$. Let us briefly recap the case without control constraints. The discrete time closed-loop L_2-gain from w to z is smaller than γ if and only if there exists a symmetric $P > 0$ such that

$$\begin{pmatrix} P & 0 & A_{cl}^T P & C_{cl}^T \\ 0 & \gamma^2 I & B^T P & D^T \\ PA_{cl} & PB & P & 0 \\ C_{cl} & D & 0 & I \end{pmatrix} > 0 \tag{4}$$

It is easily seen that (4) implies Schur stability of A_{cl}, and with $V(x) = x^T P x$ one easily obtains the dissipation inequality

$$V(x(k)) + \sum_{i=0}^{k-1} \left(\|z(i)\|^2 - \gamma^2 \|w(i)\|^2\right) \le V(x(0)) \tag{5}$$

for any trajectory $x(\cdot), w(\cdot)$ of the closed-loop system (3). Due to $V(x) \ge 0$, for $x(0) = 0$ we can conclude that the discrete L_2-gain of the closed-loop system is not larger than γ. With the substitution $Q = P^{-1}$ and $Y = KQ$ and by performing a congruence transformation with $diag(Q, I, Q, I)$, (4) is equivalent to

$$\begin{pmatrix} Q & * & * & * \\ 0 & \gamma^2 I & * & * \\ AQ + B_u Y & B & Q & * \\ CQ + D_u Y & D & 0 & I \end{pmatrix} > 0 \tag{6}$$

which is an LMI in γ^2, Q, Y. Let γ_{opt} denote the infimal value for which (6) with $P > 0$ is feasible.

Let us now come back to the case with control constraints. For this purpose we assume that the disturbance energy is bounded as

$$\sum_{i=0}^{\infty} \|w(i)\|^2 \le \alpha^2. \tag{7}$$

Due to (5), the output energy is bounded as

$$\sum_{i=0}^{\infty} \|z(i)\|^2 \le r \tag{8}$$

and the state trajectory remains in the ellipsoid

$$\mathcal{E}_1(P, r) := \{x \in \mathbb{R}^n : V(x) \le r\} \tag{9}$$

if the initial state $x(0)$ is contained in the ellipsoid

$$\mathcal{E}_2(P, r, \alpha) := \{x \in \mathbb{R}^n : \gamma^2 \alpha^2 + V(x) \le r\}. \tag{10}$$

Exploiting $u = YQ^{-1}x$, we infer (Boyd *et al.*, 1994)

$$\begin{aligned} \max_{k \ge 0} |u_i(k)|^2 &= \max_{k \ge 0} \left| \left(YQ^{-1}\right)_i x(k) \right|^2 \\ &\le \max_{x \in \mathcal{E}_1} \left| \left(YQ^{-1}\right)_i x \right|^2 \le r \left\| \left(YQ^{-\frac{1}{2}}\right)_i \right\|_2^2. \end{aligned} \tag{11}$$

Therefore the control constraints (2) can be enforced by guaranteeing that Q and Y also satisfy

$$\begin{pmatrix} \frac{1}{r} X & Y \\ Y^T & Q \end{pmatrix} \ge 0, \quad X_{ii} \le u_{i,max}^2 \tag{12}$$

for some X. We note that (12) is an LMI in X, Y, Q for fixed r, and that the constraint $\xi \in \mathcal{E}_2(P, r, \alpha)$ can as well be re-formulated as

$$\begin{pmatrix} r - \gamma^2 \alpha^2 & \xi^T \\ \xi & Q \end{pmatrix} \ge 0 \tag{13}$$

which is an LMI in γ^2 and Q for fixed α.

This leads to an algorithm for solving the constrained L_2-gain attenuation problem. For fixed α, r and $\xi = x(0)$ solve the LMI optimization problem

$$\min_{\gamma^2, Q, Y, X} \gamma^2 \text{ subject to (6), (12) and (13).} \quad (14)$$

Suppose that (γ_0, Q_0, Y_0) is an (almost) optimal solution of (14). If the system is controlled with the state-feedback gain $K_0 = Y_0 Q_0^{-1}$, we conclude with $P_0 = Q_0^{-1}$ from the above discussion that

- the control constraints (2) are respected for all disturbances satisfying (7);
- the disturbances are attenuated in the sense of

$$\sum_{i=0}^{\infty} \left(\|z(i)\|^2 - \gamma_0^2 \|w(i)\|^2 \right) \le x(0)^T P_0 x(0).$$

Remarks.

- In this construction the bound α reflects the a priori knowledge on the disturbance, whereas both the output energy bound r and the corresponding optimal value $\gamma_0 = \gamma_0(\alpha, r)$ are measures for disturbance attenuation. It is simple to extract various limits of these two parameters for feasibility of (14), such as $r \ge \gamma_{opt}^2 \alpha^2$ (where the choice with equality is too ambitious due to control constraints).
- If $\xi = x(0) = 0$, the optimal value γ_c of problem (14) satisfies $\gamma_{opt} \le \gamma_c$, reflecting a performance degradation due to control constraints. Moreover it follows from (13) that $\gamma_c \le \gamma_0$ which relates to a further performance degradation due to non-zero initial conditions.

The above construction is a pretty standard approach to guaranteeing disturbance attenuation by constrained control. It clearly reflects an inherent trade-off between satisfying the constraints and achieving high controller performance. If having to be prepared for unforeseen large disturbances one has to choose a large value of α (and hence large r) which leads to large γ or low performance, even if the actual disturbance affecting the system is rather mild and admits a smaller bound on its energy. On the other hand, enforcing high performance levels (small γ) requires to either reduce α or r, which might result in control constraint violation in case that the system is affected by unexpectedly large disturbances. This motivates an on-line scheme to trade-off the satisfaction of constraints and the level of performance. To this end, the moving horizon strategy, which is well-known in the literature of model predictive control, serves as a candidate.

3. MOVING HORIZON STRATEGY

The basis of the moving horizon strategy in model predictive control is solving an optimal control problem on-line at each sampling time, updated by the new measurement (Mayne *et al.*, 2000).

Exploiting the moving horizon strategy, one would solve on-line the LMI optimization problem (14) with the actual state $x(k)$ at each time k, which contains the past information on internal dynamics, external disturbances and controls. In this scheme, the current state $x(k)$ serves as "feedback" not only for the computation of control values but also for the choice of the feedback gain. The latter provides an opportunity to trade-off constraint satisfaction and performance. By minimizing the performance level γ on-line, one obtains the best possible performance, while keeping constraints satisfied. Unfortunately, this simple implementation of the moving horizon strategy might fail to guarantee dissipation for the controlled system, as shown in detail in (Scherer *et al.*, 2002). In the same reference it is also shown how to recover dissipation, and we repeat in this paper the key points for the discrete-time problem and in view of its implementation in a moving horizon scheme.

Assume that the LMI optimization problem (14) admits a solution for the closed-loop state $x(k)$ at each sampling time k, denoted as (γ_k, Q_k, Y_k). The feedback control is defined by

$$u(k) = K_k x(k), \; k = 0, 1, 2, \cdots$$

with $K_k = Y_k P_k$ and $P_k = Q_k^{-1}$.

At time $k = 0$, according to the principle of moving horizon strategy, $u(0) = K_0 x(0)$ will be applied to the system until the next sampling instant $k = 1$. With the actual state $x(1)$ as initial condition, the LMI optimization problem (14) will be solved again. Let us investigate whether the solution at time $k = 1$ keeps the closed-loop system dissipative. We first observe that

$$\|z(0)\|^2 - \gamma_0^2 \|w(1)\|^2 \le x(0)^T P_0 x(0) - x(1)^T P_0 x(1)$$
$$\|z(1)\|^2 - \gamma_1^2 \|w(1)\|^2 \le x(1)^T P_1 x(1) - x(2)^T P_1 x(2)$$

and hence

$$\sum_{i=0}^{1} \|z(i)\|^2 - \max\{\gamma_0, \gamma_1\}^2 \|w(i)\|^2 \le x(0)^T P_0 x(0) -$$
$$- \left[x(1)^T P_0 x(1) - x(1)^T P_1 x(1)) \right] - x(2)^T P_1 x(2).$$

If $\left[x(1)^T P_0 x(1) - x(1)^T P_1 x(1) \right] \ge 0$, dissipation holds with level $\max\{\gamma_0, \gamma_1\}$. The solution of the LMI optimization problem (14) at the time $k = 2$ with $x(2)$ leads in a similar fashion to

$$\sum_{i=0}^{2} \left[\|z(i)\|^2 - \max\{\gamma_0, \gamma_1, \gamma_2\}^2 \|w(i)\|^2 \right] \le$$
$$\le x(0)^T P_0 x(0) - \left[x(1)^T P_0 x(1) - x(1)^T P_1 x(1)) \right] -$$
$$- \left[x(2)^T P_1 x(2) - x(2)^T P_2 x(2)) \right] - x(3)^T P_2 x(3).$$

To guarantee dissipation one requires

$$x(0)^T P_0 x(0) - \left[x(1)^T P_0 x(1) - x(1)^T P_1 x(1))\right] - \\ - \left[x(2)^T P_1 x(2) - x(2)^T P_2 x(2))\right] \leq x(0)^T P_0 x(0).$$

In general, just solving the LMI optimization problem (14) at times $k = 1$ and $k = 2$ with respective initial conditions does not guarantee this inequality to hold. Therefore, the naive implementation of the moving horizon strategy will generally fail. However, the discussion reveals the crucial strategy to guarantee dissipation as follows: Define

$$p_{k-1} := x(0)^T P_0 x(0) - \\ - \sum_{j=1}^{k-1} [x(j)^T P_{j-1} x(j) - x(j)^T P_j x(j)]. \quad (15)$$

For dissipation one has to enforce at iteration k that

$$p_{k-1} - \left[x(k)^T P_{k-1} x(k) - x(k)^T P_k x(k)\right] \leq p_0. \quad (16)$$

Moreover p_k can be recursively updated as

$$p_k := p_{k-1} - \left[x(k)^T P_{k-1} x(k) - x(k)^T P_k x(k)\right].$$

It is easy to include the dissipation constraint (16) in the optimization problem (14) to end up with the following extended LMI problem at time k with the actual state $x(k)$:

$$\min_{\gamma^2, Q, Y, X} \gamma^2 \quad (17)$$

subject to (6), (12), (13) for $\xi = x(k)$, and

$$\begin{pmatrix} p_0 - p_{k-1} + x(k)^T P_{k-1} x(k) & x(k)^T \\ x(k) & Q \end{pmatrix} \geq 0. \quad (18)$$

The implementation of this on-line scheme is possible since P_{k-1} and p_{k-1} have been determined at the previous time instant $k-1$ and are held fixed. Let us suppose that (17) admits an (almost) optimal solution (γ_k, Q_k, Y_k) and define the feedback gain $K_k = Y_k Q_k^{-1}$ as well as $P_k = Q_k^{-1}$. Controlling the system with $u(k) = K_k x(k)$ then implies that

- the control constraints (2) are respected;
- the controller automatically relaxes the performance requirement if necessary not to violate constraints and it enhances the performance level if possible and in such a manner that the closed-loop system is guaranteed to obey the dissipation inequality

$$\sum_{i=k}^{l} \|z(i)\|^2 - \gamma^2 \|w(i)\|^2 \leq x(k)^T P_k x(k)$$

for $0 \leq k \leq l$ and with $\gamma = \max\{\gamma_k, \dots, \gamma_l\}$.

Let us stress that the feature of automatic performance adaptation is viewed to be the most relevant progress over (Scherer *et al.*, 2002). Moreover we recall that the off-line controller construction in (Scherer *et al.*, 2002) was based on extremal solutions of the Riccati equation corresponding to the $\mathcal{H}_\infty$ problem, whereas the present scheme picks the solution (shapes of ellipsoids) depending on the individual system state which leads to performance improvements.

For the actual on-line implementation of this scheme it is essential that the LMI optimization problem (17) is feasible at each time-instant k, which gives rise to the need for an on-line adaptation of the parameters α and r as suggested in the algorithm in the next section. If the LMI's are not feasible for all combinations of α and r one could either relax the control constraint to enforce feasibility (which is always successful for stabilizable systems but which might not be practically possible) or one could switch to a standard MPC scheme with quadratic cost which incurs a loss of guaranteed disturbance suppression properties.

4. ALGORITHM FOR MOVING HORIZON IMPLEMENTATION

Let us now discuss a concrete implementation of the suggested scheme, together with one out of a multitude of possibilities how to adapt the parameters α and r. In fact we keep α fixed while we try to enforce feasibility of (17) by increasing r (from a given r_0) whenever necessary. Moreover, for the given α, r_0 and with $x(0) = 0$, we consider the controller $K_c = K_c(\alpha, r_0)$ - defined with $P_c = P_c(\alpha, r_0)$ - as the one with best performance, and at each time k we first check whether this best gain guarantees dissipation and constraint satisfaction in order to avoid unnecessary on-line computations.

Algorithm

Step 1 Initialization. Let α and r_0 be given. Solve the LMI optimization problem (14) with $\xi = x(0) = 0$ and compute $K_c = YQ^{-1}$ and $P_c = Q^{-1}$.

Step 2 At time $k = 0$, set $r = r_0$. If $x(0) = 0$, set $K_0 = K_c$, $P_0 = P_c$, $p_0 = 0$ and go to Step 6. If $x(0) \neq 0$, solve the LMI optimization problem (14) with $\xi = x(0)$. If it admits a solution, compute $K_0 = YQ^{-1}$, $P_0 = Q^{-1}$, $p_0 = x(0)^T P_0 x(0)$ and go to Step 6. If not feasible, increase r until feasibility is retained.

Step 3 At time k, set $r = r_0$. If $x(k) \in \mathcal{E}_2(P_c, r_0, \alpha)$ and $p_{k-1} - x(k)^T P_{k-1} x(k) + x(k)^T P_c x(k) \leq p_0$, then set $K_k = K_c$, $P_k = P_c$, and go to Step 5.

Step 4 Solve the LMI optimization problem (17) with $\xi = x(k)$. If it admits a solution, compute $K_k = YQ^{-1}$, $P_k = Q^{-1}$, and go to Step 5. If not feasible, increase r and repeat Step 4.

Step 5 Prepare for the next computation:
$p_k = p_{k-1} - \left[x(k)^T P_{k-1} x(k) - x(k)^T P_k x(k)\right].$

Step 6 Apply $u(k) = K_k x(k)$ to control the system. Replace k by $k+1$ and continue with Step 3.

5. EXAMPLE: CONTROL OF AN UNSTABLE CSTR

We take the same example as in (Scherer *et al.*, 2002) for demonstrating the proposed moving horizon scheme. This is a continuous stirred tank reactor, in which the substance B is produced from the initial reactant A in the main reaction, and unwanted parallel and consecutive reactions form by-products D and C, as A $\xrightarrow{r_1}$ B $\xrightarrow{r_3}$ C and A $\xrightarrow{r_2}$ D. The reaction velocities r_i are assumed to depend on the concentration and/or the temperature nonlinearly. The inflow of the CSTR contains only the substance A and is assumed to come from an upstream unit. Therefore, the concentration and temperature in the inflow can be viewed as external disturbances. The control objective is to maintain the concentration of the main product B despite these inflow variations. As control inputs we may choose the inflow rate normalized by the reactor volume and the heat removal, which suffer saturation. A more detailed description of the CSTR can be found in (Allgöwer, 1996).

We discretize the linearized model given in (Scherer *et al.*, 2002) with a sampling time of $\delta = 0.1\,\text{min}$. We obtain a system in the form of (3) with

$$A = \begin{pmatrix} 0.9739 & -0.0942 & -0.4378 \\ -0.0012 & 1.0321 & 0.1567 \\ -0.0162 & 0.0640 & 1.0648 \end{pmatrix}$$

$$(B_1|B_u) = \left(\begin{array}{rr|rr} 0.0592 & -0.0017 & 0.0022 & 0.0502 \\ 0 & 0.0006 & -0.0008 & -0.0103 \\ -0.0005 & 0.0082 & -0.0103 & -0.0028 \end{array}\right),$$

where $x \in \mathbb{R}^3$ represent the normalized concentrations of substances A and B, and the normalized reaction temperature, respectively; $w \in \mathbb{R}^2$ and $u \in \mathbb{R}^2$ denote the normalized disturbances and controls, respectively. It is assumed that controls are bounded as $|u_i(k)| \le 1,\ \forall k \ge 0,\ i = 1, 2$. We further choose the same controlled output $z = \mathrm{col}(Hx, Eu)$ as in (Scherer *et al.*, 2002) with $H = \mathrm{diag}(0.5, 1, 1)$ and $E = \mathrm{diag}(0.1, 0.1)$. An (almost) optimal attenuation level for the unconstrained $\mathcal{H}_\infty$ problem is $\gamma_{opt} = 0.1819$.

Let us assume that the disturbance could be occasionally very large and the energy is bounded as $\sum_{i=0}^{i=\infty} \|w(i)\|^2 \le 6$. Following the algorithm given in Section 4, we implement a moving horizon controller with $\alpha = 0.1$ and $r_0 = 200\gamma_{opt}^2\alpha^2$. A much smaller α is chosen, since it is allowed for the moving horizon controller and leads to better performance. According to the discussion in Section 3, the moving horizon controller respects the control constraints while keeping the closed-loop system dissipative. For reasons of comparison, we design a fixed controller by solving LMI optimization problem (14) with $\alpha_f^2 = 6$, $r_f = 4.6\gamma_{opt}^2\alpha_f^2$ and $\xi = x(0) = 0$. The subscript f is affixed for the fixed controller. This design ensures that for any disturbance with energy bounded by 6, the fixed controller satisfies the control constraints and admits a performance level of $\gamma_f = 0.3893$.

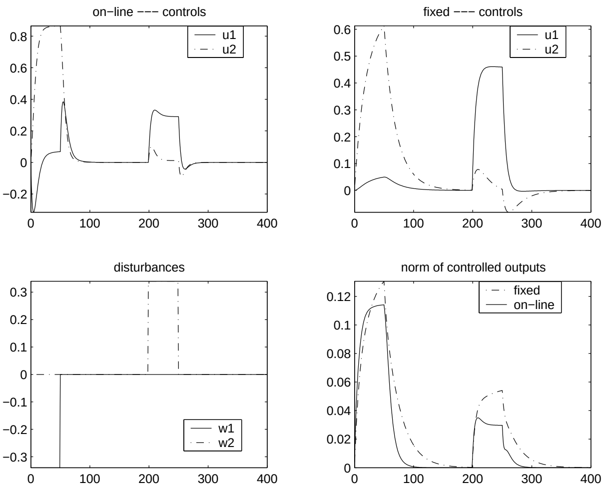

Fig. 1. Comparison of disturbance attenuation. Both moving horizon and fixed controllers guarantee dissipation and constraint satisfaction.

Fig. 1 shows the results of attenuating an impulse variation in inflow concentration and inflow temperature, respectively. The impulse is with a width of 50 sampling periods and an energy of about 6. For these disturbances, no on-line adaptation of γ happens in the moving horizon controller, nevertheless performance improvement over the fixed controller can be clearly seen in the bottom-right picture of Fig. 1, which is achieved by allowing to choose smaller α.

When unexpected stronger disturbances affect the systems, the fixed controller may violate the hard constraints. In this case, we just clip the control signals to keep them within bounds, which implies the loss of dissipation guarantee for the fixed controller. Fig. 2 and Fig. 3 present the results for such disturbances, from both the moving horizon controller and the fixed controller. The disturbances consist of a sinusoidal variation and an impulse with high intensity as plotted in the bottom-left picture of Fig. 2 and as defined by

$$w_1(k) = \begin{cases} s_1 + \bar{w}_1(k) & \text{for } 0 \le k \le 50 \\ \bar{w}_1(k) & \text{for } k > 50 \end{cases}$$

$$\bar{w}_1(k) = a_1 \sin(-0.024(k+100)) \sin(0.2(k+100))$$

$$w_2(k) = \begin{cases} \bar{w}_2(k) & \text{for } 0 \le k < 200 \\ s_2 + \bar{w}_2(k) & \text{for } 200 \le k \ge 250 \\ \bar{w}_2(k) & \text{for } k > 250 \end{cases}$$

$$\bar{w}_2(k) = a_2 \sin(-0.024(k+110)) \cos(0.2(k+110))$$

with $a_1 = a_2 = 0.02$, $s_1 = -0.41$ and $s_2 = 0.75$. Automatic performance adaptations are indicated clearly in the bottom-right picture of Fig 2, which leads to better performance of the moving horizon controller as illustrated in the bottom-right picture of Fig 3. More precisely, the moving horizon controller makes the best of the control constraints to achieve the improvement during the first impulse; relaxing on-line performance so as to avoid actuator saturation leads to the improvement during the second impulse. Moreover, the performance improvement around $k = 100$ and $k = 300$ is obtained by tightening the performance specification when the impulse variations in the inflow are removed.

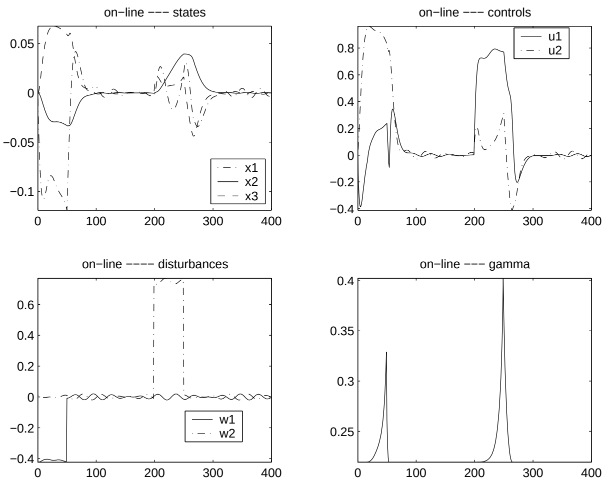

Fig. 2. Responses for moving horizon controller.

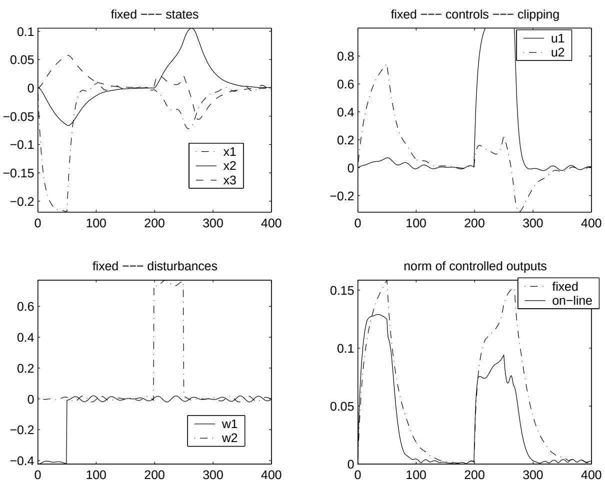

Fig. 3. Responses for fixed controller and comparison of disturbance attenuation with moving horizon controller.

6. CONCLUSIONS

In combining the moving horizon paradigm with dissipation theory, we proposed in this paper an on-line optimization scheme to solve the L_2-gain attenuation problem with hard control constraints. Technically, the feedback gain is determined on-line by solving a constrained $\mathcal{H}_\infty$ control problem updated by the actual state, while a dissipation constraint is introduced to guarantee disturbance attenuation for the closed-loop system. This scenario automatically manages the trade-off between satisfying constraints and achieving high performance, which is viewed as the most relevant progress over (Scherer *et al.*, 2002) with corresponding improvements of performance.

It should be pointed out that the tuning mechanism for the parameters α and r in the proposed scheme requires further investigation. If the disturbances are not directly feed through to the outputs, it is straightforward to include output constraints as well, and other extensions pointed out in (Scherer *et al.*, 2002) are under consideration.

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REFERENCES

Allgöwer, F. (1996). *Näherungsweise Ein-/Ausgangs-Linearisierung nichtlinearer Systeme.* Fortschr.-Ber. VDI Reihe 8 Nr. 582, VDI Verlag. Düsseldorf.

Boyd, S., L. El Ghaoui, E. Feron and V. Balakishnan (1994). *Linear Matrix Inequalities in System and Control Theory.* SIAM. Philadelphia.

Gilbert, E.C. and K.C. Tan (1991). Linear systems with state and control constraints: The theory and application of maximal output admissible sets. *IEEE Trans. Automat. Contr.* **AC-36**, 1008–1020.

Hirata, K. and M. Fujita (2000). Measurement feedback switching control for systems with state and control constraints. In: *Proc. Amer. Contr. Conf.*. Chicago, Illinois. pp. 3321–3325.

Kothare, M. V., P. J. Campo, M. Morari and C. N. Nett (1994). A unified framework for the study of anti-windup designs. *Automatica* **30**(12), 1869–1883.

Kothare, M. V., V. Balakrishnan and M. Morari (1996). Robust constrained model predictive control using linear matrix inequalities. *Automatica* **32**(10), 1361–1379.

Mayne, D.Q., J.B. Rawlings, C.V. Rao and P.O.M. Scokaert (2000). Constrained model predictive control: Stability and optimality. *Automatica* **36**, 789–814.

Scherer, C. W., H. Chen and F. Allgöwer (2002). Disturbance attenuation with actuator constraints by hybrid state-feedback control. In: *Proc. 41th IEEE Conf. Decision Contr.*