YOULA–KUČERA PARAMETRISATION IN SELF–TUNING LQ CONTROL OF A CHEMICAL REACTOR

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Abstract: The contribution deals with the application of self–tuning LQ control of a laboratory CSTR (Continuous Stirred Tank Reactor). The strategy of the linear control system is based on a recursive identification of the dual YK (Youla–Kučera) parameter of the plant and subsequent calculation of a new YK parameter of the controller. This YK parameter is determined via a non-conventional LQ control design where squared derivative of the manipulated variable and control error are considered.

Keywords: CSTR, LQ control, Youla–Kučera parameterisation, spectral factorisation, Diophantine equation

1. INTRODUCTION

To improve the quality of the products it is necessary to improve the automation (control) of production of these products. One of the most important control problems in the chemical industry is a control of chemical reactors. Chemical reactors represent a typical class of plants with nonlinear behavior.

A traditional approach to design a control system for such plants includes nonlinear strategies (Kanter *et al.*, 2002), robust strategies (Aguilar *et al.*, 2002; Sampath *et al.*, 2002) or adaptive strategies (Dostál *et al.*, 1999).

For the most part of theoretical works reference signal is assumed to be from a class of stochastic functions. However, in technologic practice, references belong always to a class of deterministic functions. Moreover, practical needs of control show, that it is not always sufficient to restrict the output and control signals only. Very often, the manipulated variable derivatives should be restricted as well. The solution of such a control problem represents then a non-conventional LQ problem (Dostál *et al.*, 1994).

This paper describes adaptive non-conventional LQ control of the CSTR. The nonlinear model of the CSTR is for adaptive application approximated by an external SISO (Single-input Single-output) linear model and then it is possible to apply any of control techniques introduced for linear systems (Čirka *et al.*, 2002*b*; Čirka *et al.*, 2002*a*).

The main aim of this paper is to present an adaptive LQ control design involving both the controller and plant model YK parameterisations and demonstrate its feasibility on the CSTR. Dual YK parameter has been identified using IDTOOL – identification toolbox for Simulink (Čirka and Fikar, 2000). The identification algorithm has been presented in papers (Mikleš, 1990; Mikleš *et al.*, 1992).

The paper is organised as follows. Section 2 recalls the results of the Youla-Kučera parameterisation.

The simulation and experimental results obtained from control of a laboratory chemical reactor are in Section 3. Finally, Section 4 offers the conclusions.

1.1 Notation

For simplicity, the arguments of polynomials are omitted whenever possible - a polynomial $X(s)$ is denoted by X. We denote $X^*(s) = X(-s)$ for any rational function $X(s)$.

2. CONTROL ALGORITHM

Mathematical model of the reactor is described by the system of nonlinear differential equations with variable parameters. The modern control theory is the best developed for linear systems. One of the possible control solutions for nonlinear systems is to find an adequate linear mathematical approximation of the nonlinear object and to apply a self–tuning algorithm. The procedure presented here is based on the Youla–Kučera parameterisation: in each step the dual YK parameter of the plant model is estimated and subsequently a new YK parameter of the controller is designed. It is assumed that an initial plant model and stabilising controller are available.

2.1 System Description

Consider the closed-loop system illustrated in Fig. 1. A continuous-time linear time-invariant input-output nominal representation of the plant to be controlled is considered

$$Ay = Bu \tag{1}$$

where y, u are process output and controller output, respectively. A and B are polynomials in complex argument s that describe the input-output properties of the plant.

We assume that the condition $\deg B \leq \deg A$ holds (i.e. transfer function of the plant is proper) and A and B are coprime polynomials.

The reference w is considered to be from a class of functions expressed as

$$Fw = H \tag{2}$$

where H, F are coprime polynomials and $\deg H \leq \deg F$.

The feedback controller is described by the equations

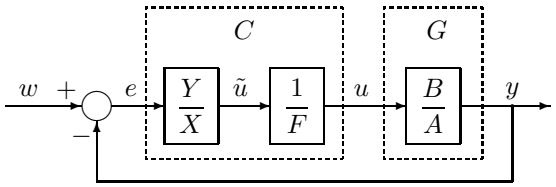

Fig. 1. Block diagram of the nominal closed-loop system

$$X\tilde{u} = Ye, \quad Fu = \tilde{u} \tag{3}$$

where X, Y are coprime polynomials and $X(0)$ is nonzero. The second equation assures that the controller tracks the class of references specified by (2).

Consider the nominal plant and the nominal controller transfer functions in the fractional representations

$$G = \frac{N_G}{D_G}, \quad C = \frac{N_C}{D_C}, \tag{4}$$

where

$$N_G = \frac{B}{M_1}, \quad D_G = \frac{A}{M_1} \tag{5}$$

$$N_C = \frac{Y}{M_2}, \quad D_C = \frac{FX}{M_2} \tag{6}$$

and M_1, $M_2 \in \mathcal{S}$ with degrees $\deg(M_1) \geq \deg(A)$ and $\deg(M_2) \geq \deg(FX)$, D_G, N_G, D_C and $N_C \in \mathcal{RH}_\infty$. $\mathcal{S}$ denotes the set of stable polynomials and $\mathcal{RH}_\infty$ the set of stable proper rational transfer functions.

A stabilising controller is then given by solution of a Diophantine equation

$$D_G D_C + N_G N_C = 1 \tag{7}$$

Substituting equations (5) and (6) into (7), the condition of stability in $\mathcal{S}$ takes the form

$$AFX + BY = M_1 M_2. \tag{8}$$

2.2 Identification Part

The identification is based on the idea which was first introduced by Hansen and Franklin (1988) in view of closed-loop experiment design. It uses the dual YK parameterisation of all linear time invariant (LTI) plants that are stabilised by a given known controller. In order to describe this method, we need the following theorem.

Theorem 1. Let a nominal model plant $G = N_G/D_G$, with N_G and D_G coprime over $\mathcal{RH}_\infty$, be stabilised by a controller $C = N_C/D_C$, with N_C and D_C coprime over $\mathcal{RH}_\infty$. Then the set of all plants stabilised by the controller C is given by

$$G(Q) = \frac{N_q}{D_q} = \frac{N_G + D_C Q}{D_G - N_C Q}, \tag{9}$$

where

$$Q \in \mathcal{RH}_\infty \tag{10}$$

Proof Dual to that of (Vidyasagar, 1985) □

Since our method involves polynomials rather than polynomial fractions, we present a short overview of the corresponding transformation between both descriptions.

Corollary 2. Let a nominal model plant $G = N_G/D_G = B/A$, with N_G, D_G, B and A defined by (5), be stabilised by a controller $C = N_C/D_C = Y/FX$, with N_C, D_C, Y and FX defined by (6). Then the set of all plants stabilised by the controller C is given by

$$G(Q) = \frac{B_q}{A_q} = \frac{B_m Q_d + F X_m Q_n}{A_m Q_d - Y_m Q_n}, \tag{11}$$

where

$$Q = \frac{Q_n}{Q_d} \in \mathcal{RH}_\infty,\ A_m = AM_2,$$
$$B_m = BM_2,\ X_m = XM_1,\ Y_m = YM_1 \tag{12}$$

Corollary 2 represents the standard parameterisation of the class of all plants that are stabilised by the actual controller C. Based on the closed-loop system in Fig. 2, Hansen and Franklin (1988) showed that the parameter Q satisfies the relation

$$z = Qx \tag{13}$$

where the signals x and z can be reconstructed by filtering the measured data u, y with filters that depend on known factors of the nominal plant P and the nominal controller C, respectively

$$x = N_G y - D_G u \tag{14}$$
$$z = D_C y + N_C u \tag{15}$$

Then, the YK parameter Q can be identified from reconstructed auxiliary signals x and z according the following prediction error equation

$$\varepsilon(t, \theta) = z(t) - Q(\theta) x(t) \tag{16}$$

where θ represents the identified parameters.

2.3 Controller Design Part

The goal of optimal deterministic LQ tracking is to design a controller that enables the control system to satisfy the basic requirements

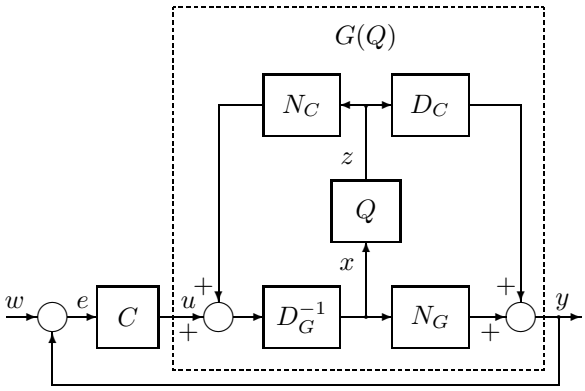

Fig. 2. The dual YK parameterisation

- stability of the closed-loop system
- asymptotic tracking of the reference

and in addition the control law that minimises the cost function

$$J = \int_0^\infty \left(\varphi \tilde{u}^2(t) + \psi e^2(t)\right) dt \tag{17}$$

where $e = w - y$ denotes the control error and $\varphi > 0$, $\psi \geq 0$ are weighting coefficients. The cost function (17) can be rewritten using Parseval's theorem, to obtain an expression in the complex domain

$$J = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \left(\tilde{u}^* \varphi \tilde{u} + e^* \psi e\right) ds \tag{18}$$

For controller design we propose to use the method described in Čirka *et al.* (2002*b*). Let us at first summarise the known results dealing with parameterised systems:

Theorem 3. Consider the closed-loop system with the configuration in Fig. 3 defined by $G(Q)$ and $C(S)$ where $Q = Q_n/Q_d$ and $S = FS_n/S_d$ are stable proper rational functions. The closed-loop system is stable if and only if Q and S together define a stable loop.

Proof (Tay *et al.*, 1989). □

Theorem 4. Consider the minimisation of the cost function (17) with respect to the YK parameter S that is specified as a transfer function. Assume that the nominal system $G = N_G/D_G = B/A$ is stabilised by a nominal controller $C = N_C/D_C = Y/FX$ and that a stable transfer function Q is known. Solve spectral factorisation equations for stable D_c and D_f

$$D_c^* D_c = \varphi A_q^* A_q F^* F + \psi B_q^* B_q \tag{19}$$

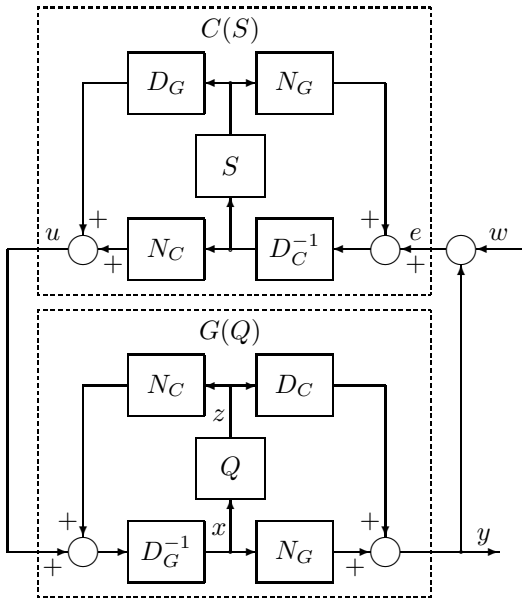

Fig. 3. Block diagram of the closed-loop system

$$D_f^* D_f = A_q^* A_q H^* H \tag{20}$$

and the coupled bilateral Diophantine equations for S_n and S_d

$$\begin{aligned} D_c^* S_n = &-\varphi D_f A_q^* F^* Y_m + \psi D_f B_q^* X_m \\ &-Q_d D V^* \end{aligned} \tag{21}$$

$$\begin{aligned} D_c^* S_d = &\varphi D_f A_q^* F^* A_m F + \psi D_f B_q^* B_m \\ &+Q_n F D V^*. \end{aligned} \tag{22}$$

The optimal YK parameter is then given as

$$S = \frac{F s_n}{s_d} = \frac{F S_n}{S_d}, \tag{23}$$

where

$$s_n = \frac{S_n}{D_c D_f} \text{ and } s_d = \frac{S_d}{D_c D_f}$$

Proof (Čirka *et al.*, 2002*b*). □

2.4 Combined Algorithm

The adaptive control algorithm is realised in the following steps:

1. Suppose the initial model $P = B/A$ is known and stabilised by a nominal controller $C = Y/X$.
2. The filtered variables x and z are obtained from equations (14) and (15).
3. The dual YK parameter Q is recursively estimated from equation (13) in the discrete time intervals $t_k = kT_s$ with the sampling period T_s. Here, the modified LDDIF (Čirka and Fikar, 2000) identification procedure was used.
4. The polynomials D_f and D_c in spectral factorisations (19) and (20) are calculated.
5. The YK parameter S is updated on the base of solution of the Diophantine equations (21) and (22).
6. Jump to step 2.

3. RESULTS AND DISCUSSION

3.1 Mathematical Model

The control algorithm has been tested on control of exothermic reactor. The mathematical model of CSTR was developed in the form (Mikleš *et al.*, 1999)

$$\begin{aligned} \frac{dc_A}{dt} &= \frac{1}{V_r}\big(q_A c_{Ai} - (q_A + q_B)c_A\big) - \upsilon \\ \frac{d\vartheta_r}{dt} &= \frac{q_A + q_B}{V_r}(\vartheta_{ri} - \vartheta_r) - \frac{A\alpha}{c_{pr} V_r \rho_r}(\vartheta_r - \vartheta_c) \\ &\quad + \frac{1}{c_{pr}\rho_r}(-\Delta H)\upsilon - \frac{k_s A\alpha}{c_{pr} V_r \rho_r}(\vartheta_r - \vartheta_{out}) \\ \frac{d\vartheta_c}{dt} &= \frac{q_c}{V_c}(\vartheta_{ci} - \vartheta_c) + \frac{A\alpha}{c_{pc} V_c \rho_c}(\vartheta_r - \vartheta_c) \end{aligned}$$

with initial conditions:
$c_A(0) = c_A^s$, $\vartheta_r(0) = \vartheta_r^s$ and $\vartheta_c(0) = \vartheta_c^s$.

The reaction rate is expressed as

$$\upsilon = 2k c_A^y c_B^z e^{\frac{E(\vartheta_r - \vartheta_0)}{R\vartheta_r\vartheta_0}}$$

$$c_B = \frac{c_{Bi} q_B}{q_A + q_B}$$

where

c	concentrations [mol m^{-3}]
V	volumes [m^3]
ϑ	temperatures [K]
ρ	densities [kg m^{-3}]
c_p	specific heat capacities [J kg^{-1} K^{-1}]
q	flow rates [m^3 min^{-1}]
A	heat exchange surface area [m^2]
α	heat transfer coeficient [J s^{-1} m^{-2} K^{-1}]
$-\Delta H$	heat of reaction [J mol^{-1}]
k	reaction rate constant [mol cm^{-3} s^{-1}]
E	activation energy [J mol^{-1}]
R	gas constant [J mol^{-1} K^{-1}]
y, z	the orders of reaction [-]
k_s	loss of heat coefficient [-]

The subscripts are $(\cdot)_r$ for the reactant mixture, $(\cdot)_c$ for the coolant, $(\cdot)_i$ for feed (inlet) values and the superscript $(\cdot)^s$ for steady-states values.

This mathematical model was tested by different identification methods and parameters in Table 1 were found. For control purposes, the controlled output and control input are defined as $y = \vartheta_r - \vartheta_r^s$ and $u = q_c - q_c^s$.

Table 1. Parameter values, inlet values and initial conditions

Values of all parameters	
V_r	$= 940\,\mathrm{cm}^3$
V_c	$= 90\,\mathrm{cm}^3$
c_{pr}	$= 4180\,\mathrm{J\,kg}^{-1}\,\mathrm{K}^{-1}$
c_{pc}	$= 4180\,\mathrm{J\,kg}^{-1}\,\mathrm{K}^{-1}$
ρ_r	$= 0.001\,\mathrm{kg\,cm}^{-3}$
ρ_c	$= 0.001\,\mathrm{kg\ cm}^{-3}$
q_A	$= 15\,\mathrm{cm}^3\,\mathrm{min}^{-1}$
q_B	$= 6\,\mathrm{cm}^3\,\mathrm{min}^{-1}$
$-\Delta H$	$= 98300\,\mathrm{J\,mol}^{-1}$
E	$= 3.0917\,10^4\,\mathrm{J\,mol}^{-1}$
k_s	$= 0.007$
k	$= 0.091\,\mathrm{mol\,cm}^{-3}\,\mathrm{s}^{-1}$
z	$= 0.875$
y	$= 1.641$
$A\alpha$	$= 116.09\,\mathrm{J\,min}^{-1}\,\mathrm{K}^{-1}$
ϑ_0	$= 297.65\,\mathrm{K}$
R	$= 8.314\,\mathrm{J\,mol}^{-1}\,\mathrm{K}^{-1}$
ϑ_{out}	$= 293.15\,\mathrm{K}$
Feed values	
ϑ_{ri}	$= 293.15\,\mathrm{K}$
ϑ_{ci}	$= 298.15\,\mathrm{K}$
c_{Ai}	$= 2.64\,10^{-3}\,\mathrm{mol\,cm}^{-3}$
c_{Bi}	$= 1.5297\,10^{-4}\,\mathrm{mol\,cm}^{-3}$
Steady-state values	
ϑ_r^s	$= 303.46\,\mathrm{K}$
ϑ_c^s	$= 303.06\,\mathrm{K}$
c_A^s	$= 1.4784\,10^{-4}\,\mathrm{mol\,cm}^{-3}$

Designed LQ adaptive control was verified in simulations as well as on the real plant. For the verification of control algorithm in laboratory conditions it was assumed with the high probability that data in Table 1 were valid in experiments, too. The results are very similar.

The goal of the adaptive control has been to track specified temperature ϑ_r in the reactor with exothermic reaction. The temperature ϑ_r is controlled by the flow rate q_c of the coolant.

The experiment was realised in two steps:

1. Control simulation of the CSTR model. The advantage of the control simulation of the chemical reaction with thermal effects is to prevent run-away problems, which can occur experimentally, particularly for such a reaction.
2. Control of real laboratory CSTR.

In both cases, the nominal transfer function of the reactor is of the form

$$G = \frac{B}{A} = \frac{-3.7}{840s + 1} \tag{24}$$

and the nominal controller is determined as

$$C = \frac{Y}{FX} = \frac{-0.117}{s} \tag{25}$$

The task was

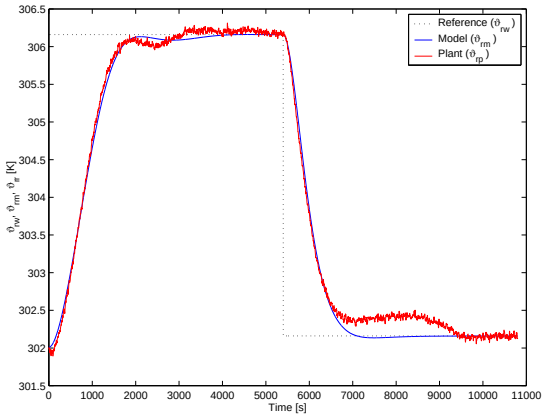

Fig. 4. Controlled output time responses

- to identify in each step the dual YK parameter of the plant (model) based on the input (the coolant feed) and output (the temperature in the reactor) data, respectively. The structure of the identified dual YK parameter was chosen of the form $Q = q_0$,
- and subsequently to design a new YK parameter of the controller.

The following references were tracked:

Step No.	1	2
Time [s]	0	5400
Temperature [K]	306.16	302.16

The boundary of the control input were used within $0 \leq u(t) \leq 140\ \mathrm{cm}^3\mathrm{min}^{-1}$. The weighting coefficients in cost (18) were $\varphi = 1$ and $\psi = 0.003$.

The obtained time responses of the temperature in the reactor (for both cases) are compared in Fig. 4. The small differences between the responses of the model and real plant were caused by the behaviour of the coolant temperature (Fig. 5) in real experiment. Fig. 6 and 7 shows simulation and experimental coolant feed and identified dual YK parameter.

Note 1. We have realised several experiments with various control structures (fixed controller and classic self-tunning controller). Both control structures performed well. However, the fixed controller cannot handle parameter changes of the reactor and its performance can deteriorate.

4. CONCLUSIONS

In this paper an adaptive LQ controller design procedure was presented. The design method is based on the idea of YK parameterisation of the controller and the plant model. The algorithm was applied to a CSTR. The advantage of this method resides in the fact that in the case of CSTR only one parameter needs to be identified in order to update the controller.

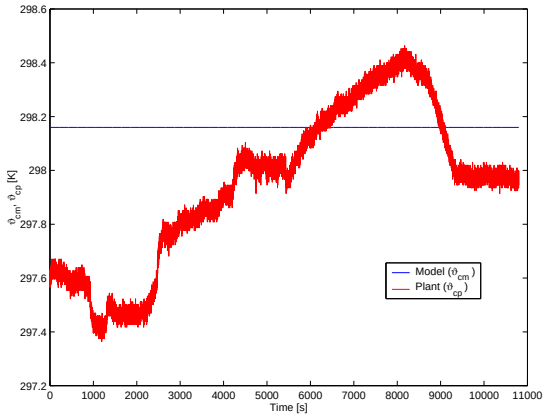

Fig. 5. Coolant temperature

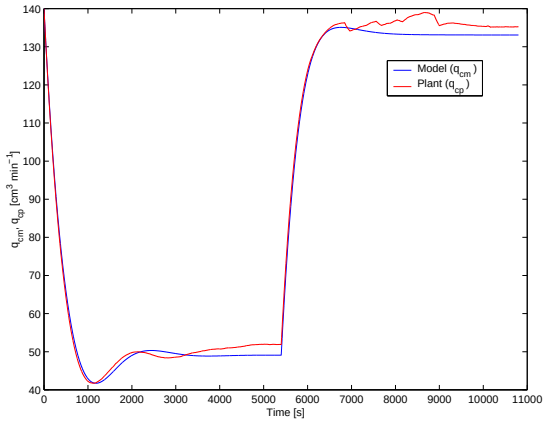

Fig. 6. Coolant feed

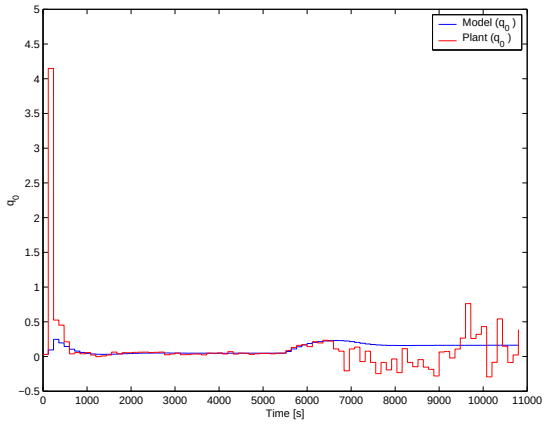

Fig. 7. Identified dual YK parameter

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