Nonlinear Control of a Fluid Catalytic Cracking Unit

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Abstract- The dynamics of a fluid catalytic cracking unit (FCCU) is a typical nonlinear system. The purpose of this study is to develop a nonlinear control of a FCC unit via feedback linearization. Based on the mechanistic model of a FCC unit, a nonlinear controller is designed. The servo and tracking properties of the closed loop system are verified by simulation for a sample FCC unit. The simulation shows that the controller is valid and can be applied to practical FCC units.

Key words- nonlinear system, FCC unit, feedback linearization.

1. INTRODUCTION

The catalytic cracking has been one of the most important processes in petroleum refining. FCC units are known to be very difficult to modeling and control because of complex kinetics and dynamics of both cracking and coke burning reaction, multivariable character and the strong interaction between the riser and regenerator. Early papers on FCC unit modeling are mainly concerned with the reaction kinetics or steady-state process behavior. Weekman[9] proposed the three-lump and 10-lump cracking reaction models. Since Kurihara[6] firstly presented a dynamic model for bed-cracking type FCC unit in 1967, dynamic models for the riser-type FCC unit have been developed in recent years. Zheng [11, 12, 13] described a comprehensive dynamic model for side-by-side type FCC unit, including the reactor and the tow stage regenerator.

Most of controllers successfully used in FCC units are PID. Recently, model predictive control (MPC) based advanced control has been proposed and implemented in some plants [2, 8]. Since the dynamic model of the FCC unit is strongly nonlinear, the goal of the paper is to design a nonlinear controller based on the mechanistic model. By applying the theory of feedback linearization which developed mature in differential geometric control [3, 4, 5, 7], a nonlinear controller is synthesized. For a sample FCC unit, the simulation shows that the proposed controller assures not only the setpoint tracking but also the inner stability of the closed loop system.

2. FEEDBACK LINEARIZATION OF MIMO NONLINEAR SYSTEMS

Consider a following affine nonlinear system

$$\begin{cases} \dot{X} = f(X) + g_1(X)u_1 + g_2(X)u_2 \\ \qquad + \cdots + g_m(X)u_m \\ y_1(t) = h_1(X) \\ \cdots\cdots \\ y_m(t) = h_m(X) \end{cases} \tag{2.1}$$

where $X \in R^n$ is the state vector; $f(X)$ and $g_i(X)$ are n dimension vector fields; u_i are the control variables; $y_i(t)$ are the controlled variable; $h_i(X)$ is a scalar function of X, $i=1,2,\cdots,m$. Assume X_0 is an equilibrium point, i.e. $f(X_0)=0$, $h_1(X_0)=h_2(X_0)=\cdots=h_m(X_0)=0$.

If system (2.1) has relative degree $r = \{r_1, r_2, \cdots, r_m\}$ [7], where r satisfies $r_1+r_2+\cdots+r_m = r \le n$, following nonlinear coordinate transformation can be acquired:

$$z_1^1 = h_1(X), z_2^1 = L_f h_1(X), \cdots, z_{r_1}^1 = L_f^{r_1-1} h_1(X)$$

$$\cdots$$

$$z_1^m = h_m(X), z_2^m = L_f h_m(X), \cdots, z_{r_m}^m L_f^{r_m-1} h_m(X)$$

If $r \neq n$, the other $n-r$ nonlinear coordinate transformation should be chosen:

$$z_{r+1} = \eta_1(x), \cdots, z_n = \eta_{n-r}(x)$$

If the Jacobian matrix $\dfrac{\partial Z(X)}{\partial X}$ is nonsingular at $X = X_0$, following system can be obtained

$$\begin{cases} \dot{z}_j^i = z_{j+1}^i, \\ \dot{z}_{r_i}^i = b^i(z,\eta) + \sum_{k=1}^{m} a_k^i(z,\eta) u_k = v_i \\ \dot{\eta} = q(z,\eta) + \sum_{k=1}^{m} p_k^i(z,\eta) u_k \\ y_i = z_1^i \end{cases}$$

where, $i=1,2,\cdots,m$ and $1 \le j \le r_i - 1$

Since *A(z)* is nonsingular, solved the following equation

$$\begin{bmatrix} \dot{z}_{r_1}^1 \\ \vdots \\ \dot{z}_{r_m}^m \end{bmatrix} = \begin{bmatrix} b_1(z) \\ \vdots \\ b_m(z) \end{bmatrix} + \begin{bmatrix} a_1^1(z) & a_2^1(z) & \cdots & a_m^1(z) \\ \vdots & \vdots & \cdots & \vdots \\ a_1^m(z) & a_2^m(z) & \cdots & a_m^m(z) \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix}$$

can obtain *u*

$$u = A^{-1}(z)(-b(z) + v)$$

If the zero dynamics of the system $\dot{\eta} = q(0,\eta) + p(0,\eta)[-A^{-1}(0,\eta)b(0,\eta)]$ is asymptotically stable, let $v_i = (s^{ri} + c_{ri-1}s^{ri-1} + \cdots + c_1 s + c_0) z_1^i, i = 1, \cdots, m$. Here the c_i are chosen so that $s^{ri} + c_{ri-1}s^{ri-1} + \cdots + c_1 s + c_0$ is a Hurwitz polynomial. Then, an asymptotically stable closed loop system is obtained.

3. FEEDBACK LINEARIZATION OF A FCC UNIT

3.1 Process model of a FCC unit

Fig.1. shows the reactor and regenerator part of a FCC unit. The feed injected into the reactor riser, where it mixes with hot regenerated catalyst and vaporizes. The hot catalyst provides the heat of vaporization and the heat of reaction. As a result of the cracking reactions, a carbonaceous material (coke) is deposited on the surface of the catalyst. The catalyst and gas are separated in the settler. Then spent catalyst is transported from the reactor to the regenerator. In the regenerator, catalyst is fluidized with air flow injected from the bottom of the regenerator. Carbon and hydrogen on the catalyst react with oxygen to produce carbon monoxide, carbon dioxide and water. Regenerated catalyst flows into the reactor riser.

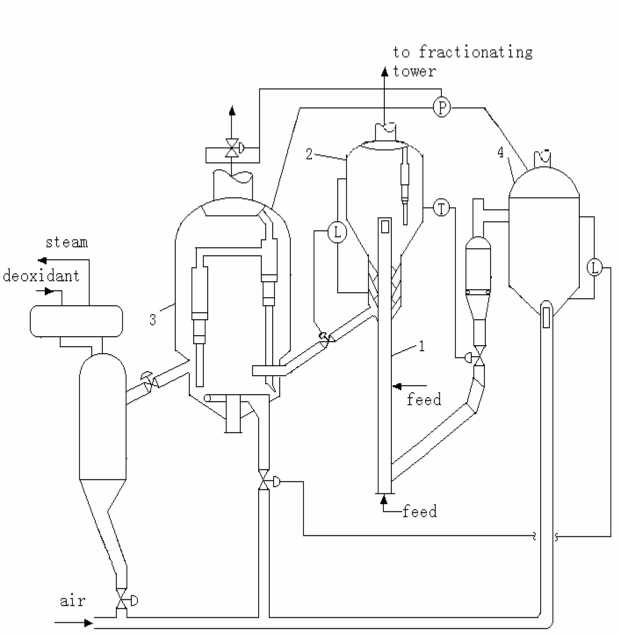

Fig. 1. the reactor &regenerator of FCC unit
1. reator 2. settler 3. the first stage of regenerator 4. the second stage of regenerator

Manipulated variables are mass flow rate of the catalyst entering the riser (R_C), mass flow rate of the spent catalyst entering the spent catalyst transport line (R_{CS}) and mass flow rate of the regenerated catalyst in the first stage of regenerator (R_{CG1}).Controlled variables are temperature of reactor riser outlet (T_{RA2e}), inventory of catalyst in the settler (H_S) and inventory of catalyst in the second stage of regenerator (H_{GC2}).

Based on the references [11] [12] and [13], a dynamic mathematical model of a FCC unit can be obtained

$$\frac{dT_{RA1e}}{dt} = - \frac{1}{S_{T1}} * \frac{1}{1+\prod_1} (\mathrm{T_{RA1e}} - \mathrm{T_{RA1i}}) - \frac{1}{S_{T1}} * \frac{1}{1+\prod_1} \wedge_1 \frac{R_C}{R_{O1}} \mathrm{C_{CA1e}} \quad (3.1.1)$$

$$\frac{dT_{RA2e}}{dt} = - \frac{1}{S_{T2}} * \frac{1}{1+\prod_2} (\mathrm{T_{RA2e}} - \mathrm{T_{RA2i}}) - \frac{1}{S_{T1}} * \frac{1}{1+\prod_1} \wedge_2 \frac{R_C}{R_{O2}} (\mathrm{C_{CA2e}} - \mathrm{C_{CA1e}}) \quad (3.1.2)$$

$$\frac{dH_S}{dt} = R_{CS} - R_C \quad (3.1.3)$$

$$\frac{dH_{GC1}}{dt} = R_{CS} - R_{CG1} \quad (3.1.4)$$

$$\frac{dC_{G1}}{dt} = \frac{1}{H_{GC1}} [R_{CS}(C_S - C_{G1}) - \frac{R_{A1}}{v_{OC}} (0.21 - O_{FG1})] \quad (3.1.5)$$

$$\frac{dT_{RG1}}{dt} = \frac{1}{H_{GC1} C_{PC}} [\frac{R_{A1}}{v_{OC}} (0.21 - O_{FG1})(\Delta H_{CB}) - R_{CS} C_{PC} (T_{RG1} - T_S) - R_{A1} C_{PA} (T_{RG1} - T_A)] \quad (3.1.6)$$

$$\frac{dH_{GC2}}{dt} = R_{CG1} - R_{CG2} \quad (3.1.7)$$

$$\frac{dC_{G2}}{dt} = \frac{1}{H_{GC2}} [R_{CG1}(C_{G1} - C_{G2}) - \frac{R_{A2}}{v_{OC}} (0.21 - O_{FG2})] \quad (3.1.8)$$

$$\frac{dT_{RG2}}{dt} = \frac{1}{H_{GC2} C_{PC}} [\frac{R_{A2}}{v_{OC}} (0.21 - O_{FG2})(\Delta H_{CB}) - R_{CS} C_{PC} (T_{RG2} - T_{RG1}) - R_{A2} C_{PA} (T_{RG2} - T_A)] \quad (3.1.9)$$

The physical significance of each variable shows in appendix A. the detailed expressions of Π, Λ, C_{CAe}, T_{RAi} and O_{FG} are shown in references [11] [12] and [13].

3.2 Design of the controller

Before designing the controller, we assumpt that the total inventory of catalyst is a constant; that is to say, the losing of catalyst is neglected.

Then we can draw that the inventory of catalyst in the first stage of regenerator H_{GC1} can be computed by using the following equation:

$$H_{GC1} = H - H_S - H_{GC2}$$

where H is the total inventory of catalyst.

Moreover, because of the characteristics of catalyst transport line, the system has another two qualities:

1: Mass flow rate of the catalyst entering the riser R_C, mass flow rate of the spent catalyst entering the spent catalyst transport line R_{CS}, mass flow rate of the regenerated catalyst in the first stage of regenerator R_{CG1} and mass flow rate of the regenerated catalyst in the second stage of regenerator R_{CG2} are all constrained:

$$0 < R_C, R_{CS}, R_{CG1}, R_{CG2} < R$$

2: mass flow rate of the regenerated catalyst in the second stage of regenerator is equal to mass flow rate of the catalyst entering the riser:

$$R_{CG2} = R_C$$

It shows that the reactor and regenerator part of a FCC unit is a nine stage nonlinear system with three inputs and three outputs, but it is not an affine system. We introduce a new variable $u_1$defined as below

$$\dot{R}_C = u_1$$

Let, $R_{CS} = u_2$; $R_{CG1} = u_3$; $T_{RA1e} = x_1$; $T_{RA2e} = x_2$; $R_C = x_3$; $H_s = x_4$; $C_{G1} = x_5$; $T_{RG1} = x_6$; $H_{GC2} = x_7$; $C_{G2} = x_8$; $T_{RG2} = x_9$.

Assume that some disturbance exists such as mass flow rate of feed R_O , temperature of feed T_{Oi} and pressure of regenerator P_{RG} . Then, an affine system can be obtained as follows

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_3) + d_1 \\ \dot{x}_2 = f_2(x_1, x_2, x_3) + d_2 \\ \dot{x}_3 = u_1 \\ \dot{x}_4 = x_3 - u_2 \\ \dot{x}_5 = f_5(x_4, x_5, x_6, x_7) + g_5(x_4, x_5, x_7)u_2 + d_5 \\ \dot{x}_6 = f_6(x_4, x_5, x_6, x_7) + g_6(x_4, x_6, x_7)u_2 + d_6 \\ \dot{x}_7 = u_3 - x_3 \\ \dot{x}_8 = f_8(x_7, x_8, x_9) + g_8(x_5, x_7, x_8)u_3 + d_8 \\ \dot{x}_9 = f_9(x_7, x_8, x_9) + g_9(x_6, x_7, x_9)u_3 + d_9 \\ Y = [x_2, x_4, x_7]^T \end{cases} \tag{3.2.1}$$

The relative degree of system (3.2.1) is $\{r_1, r_2, r_3\} = \{2,1,1\}$.

Let, $z_1 = x_2, z_2 = L_f x_2, z_3 = x_4, z_4 = x_7$, $z_\eta = \{\eta \mid \eta \in x \quad \eta \neq x_2, x_3, x_4, x_7\} \subset R^{n-4}$. Then, system (3.2.1) can be changed to:

$$\begin{cases} \dot{z}_1 = z_2 + d_1 \\ \dot{z}_2 = v_1 \\ \dot{z}_3 = v_2 \\ \dot{z}_4 = v_3 \end{cases} \tag{3.2.2a}$$

$$\dot{\eta} = q(\xi, \eta) + w_\eta d_\eta \tag{3.2.2b}$$

$$[y_1, y_2, y_3]^T = [z_1, z_3, z_4]^T \tag{3.2.2c}$$

The feedback linearization controller of system (3.2.2) is:

$$u_1 = \frac{v_1 - (\partial f_2/\partial x_1)f_1 - (\partial f_2/\partial x_2)f_2}{\partial f_2/\partial x_3};$$

$$u_2 = x_3 - v_2 ; u_3 = x_3 + v_3 ,$$

where,

$$v = -[\alpha_{11}(z_1 - y_{1d}) + \alpha_{12}z_2 \quad \alpha_2(z_3 - y_{2d}) \quad \alpha_3(z_4 - y_{3d})]^T ,$$

$\alpha_{11}, \alpha_{12}, \alpha_2, \alpha_3 > 0$ and y_{1d}, y_{2d}, y_{3d} are desired outputs.

If (3.2.2b) is asymptotically stable at the steady point, the output of the closed loop system can asymptotically track the desired output [5].

3.3 The verification of stability of zero dynamics

Based on the above analysis, if the zero dynamics of system (3.2.2b) is asymptotically stable at the steady point, the closed loop system is stable.

Because zero dynamics of this system is very complex, proving its stability by using Lyapunov function is difficult. Therefore, we prove zero dynamics stability by computing the eigenvalues of the Jacobian matrix of zero dynamics (3.2.2b) at some steady point. If the eigenvalues are in the left-half plane, the zero dynamics (3.2.2b) is asymptotically stable in the neighborhood of the steady point. We choose several steady points, and calculate the Jacobian matrix of system (3.2.2b) in each steady state. The results show that the eigenvalues of each Jacobian matrix are in the left-half plane. Therefore, the stability of the closed loop system can be assured.

4. RESULTS OF SIMULATION

The values of the variables in steady states are shown in appendix 2. In the simulation, the desired outputs: temperature of reactor riser outlet T_{RA2e}, inventory of catalyst in the settler H_S and inventory of catalyst in the second stage of regenerator H_{GC2} are changed from [800.5754,990,32.662] to [790,1000,260]. Fig.2, Fig.3 and Fig.4 show the comparison results of tracking reference trajectories of outputs by using feedback linearization and PID controllers respectively. It is shown that the controller designed by using feedback linearization makes the closed loop have better properties than a PID controller does. (the outputs of feedback linearization: —, the outputs of PID: ----). Fig.5 shows the capability of rejecting disturbance of feedback linearization controller.

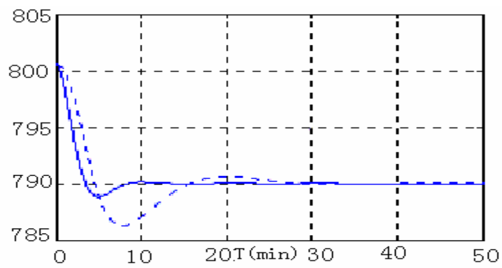

Fig.2. Temperature of riser outlet

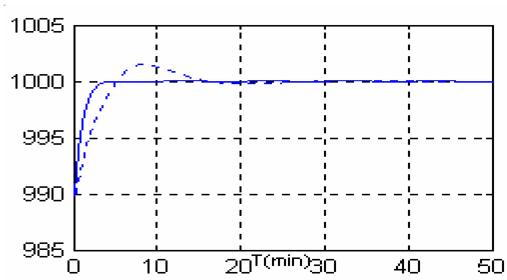

Fig.3. Inventory of catalyst in settler

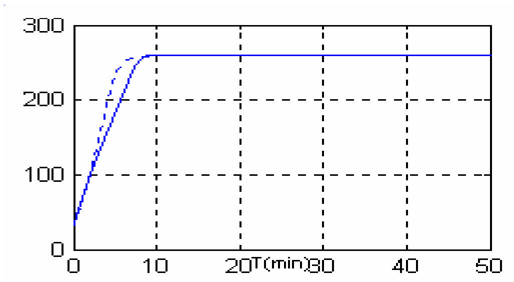

Fig.4. Inventory of catalyst in the second stage of regenerator

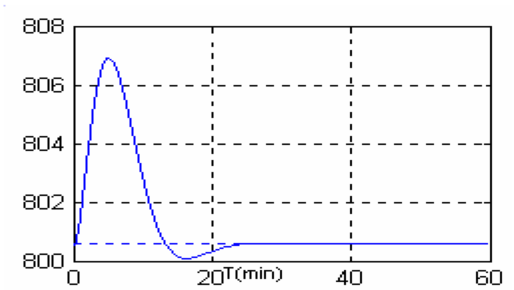

Fig.5. Temperature of riser out let when flow rate of feed reduce 5%

To show the stability of the closed loop system, the Jacobian matrix of zero dynamics of system in the steady point (appendix B) is computed as follows:

$$\begin{bmatrix} -17.3 & 0 & 0 & 0 & 0 \\ 0 & -0.3 & -8.6\times10^{-62} & 0 & 0 \\ 0 & -1.1\times10^{-52} & -0.3 & 0 & 0 \\ 0 & 5.7 & 0 & -2.8\times10^{3} & -1.8\times10^{-15} \\ 0 & 0 & 5.7 & 1.4\times10^{-19} & -5.7 \end{bmatrix}$$

The eigenvalues of the Jacobian matrix are-17.3, -5.7, -0.3, -2.8*10^3 and -0.3, which are in the left-half plane. Therefore, the closed loop system is asymptotically stable.

Fig.6, Fig.7 and Fig.8 show the simulation results of the temperature of the second feed inlet, the temperature of the second stage of regenerator and the mass fraction of coke in regenerated catalyst at the second stage of regenerator. The results validate the stability of zero dynamics.

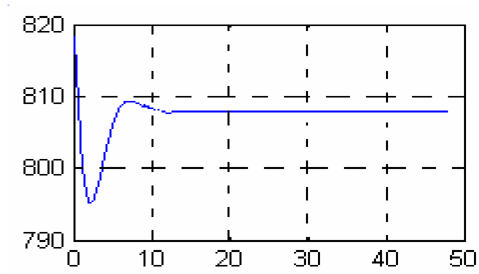

Fig.6. Temperature of the second feed inlet of riser

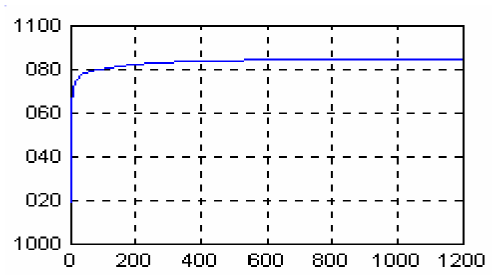

Fig.7. Temperature of the second stage of regenerator

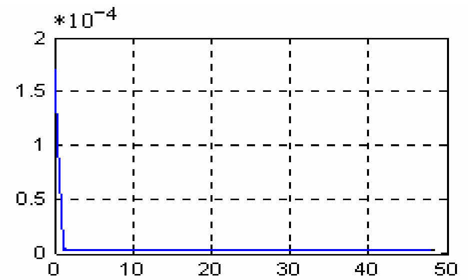

Fig.8. Mass fraction of coke in catalyst at the second stage of regenerator

5. CONCLUSIONS

In this paper, by designing a nonlinear controller of a FCC unit, we can draw the following conclusions.

1. The feedback linearization method can be used in the controller design of a FCC unit. The simulation results show that the proposed nonlinear controller can guarantee not only the closed loop stability, but also setpoint tracking.

2. The controller can effectively reject some disturbance which shows the robustness of the controller. When some disturbance, such as R_O, T_{Oi} and P_{RG}, exist in the system, the controller can still assure the closed loop stability.

The above analysis demonstrates that the controller proposed in this paper can be applied in practical plants.

APPENDIX A. NOMENCLATURE

Riser

C_{CA} Mass fraction of reactive carbon in catalyst, [0-1]
C_G Mass fraction of carbon in regenerated catalyst
C_S Mass fraction of carbon in spent catalyst
ΔH_{CR} Heat of reaction J/Kg catalyst
P_{RA} Pressure of reactor Pa
R Universal gas constant 8.315J/mol*K
R_C Mass flow rate of catalyst Kg/S
R_O Mass flow rate of feed Kg/S
R_W Mass flow rate of vapor Kg/S
S_T Feed space time S
T_{RA} Temperature of reactor K
t Time S
Y_A Mass fraction of unconverted hydrocarbons
H_S Inventory of catalyst in settler, Kg

Regenerator

C_G Mass fraction of carbon in regenerated catalyst
C_S Mass fraction of carbon in spent catalyst
C_{PA} Specific heat of air KJ/(Kg*K)
C_{PC} Specific heat of catalyst J/Kg*K
H_{GC} Inventory of catalyst in regenerator, Kg
ΔH_{CB} Heat of carbon burning , KJ/Kg carbon
O_{FG} Mole fraction of oxygen in flue gas [0-1]
R_A Volume flow rate of flue gas, Nm^3/h
R_{CG} Mass flow rate of regenerated catalyst
R_{CS} Mass flow rate of spent catalyst
T_A Temperature of air at regenerator inlet
T_A Temperature of air
T_{RG} Temperature of regenerator
T_S Temperature of spent catalyst
ν_{OC} Stoichiometric coefficient, Nm^3oxygen/Kg carbon

APPENDIX B

The values of the variable in steady states are shown as follow. The definition of symbols which were not shown in this paper can be found in reference [11], [12] and [13].

$K_{A0}=199.476$	$K_{C0}=0.04916$	$K_{\varphi 0}=0.05652$
$E_A=79100$	$E_C=41300$	$P_{RA}=101325$
$R=8.315$	$V_{r1}=14.2545$	$V_{r2}=25.7634$
$V_{l1}=5.50128$	$V_{l2}=6.93312$	$\rho_{Oi}=0.8$ $\rho_L=2050$
$R_O=250$	$R'_O=50$	$R_C=1578.085$
$R_{w1}=16$	$R_{w2}=3$	$T_{o1i}=473.15$
$T_{c1i}=1012.15$	$T_{w1i}=523.15$	$T_{o2i}=606.65$
$T_{w2i}=523.15$	$\Delta H_{CR}=257200$	$\Delta H_V=40000$
$C_{po}=800$	$C_{pc}=260$	$C_{pw}=200$
$C_{pl}=0.21$	$P_a=2.8$	$n=2$
$K_{R0}=83600000$	$E_{CB}=50$	$K_{DO}=1.43$

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