# Model-based Auto-tuning System Using Relay Feedback

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Abstract: An on-line model based autotune system that employes conventional ATV test is proposed. The ATV responses from normalized FOPDT and SOPDT processes are grouped into two zones in a space, which has normalized amplitude and normalized period as coordinates. In terms of the amplitude and the period of constant cycles of an ATV test, model-based tuning rules are also prepared for two types of process, one for FOPDT in one zone and one for under-damped SOPDT in the other zone. Thus, from an ATV test, the amplitude and period of constant cycles are used in an identification step to select tuning rules to be applied to a given process. Then, PID controller settings are computed according to the selected tuning rules. The system can be inplemented as simple as the conventional autotune system, and the resulting control performance is is compatible to that from a model-based controller design.

Keywords: Relay Feedback, Inverse-based PID Controller, On-line Autotuning

### **1.INTRODUCTION**

The PID auto-tuning has shorten the time to commission control system, and facilitated control optimization through regular retuning. The relay feedback auto-tuning method proposed by Åström and Hägglund was attractive owing to its simplicity and robustness. The important ingredients in the auto-tuning system are identification (parametric or nonparametric) and rules for controller tuning. Many researches on these two aspects have been reported in recent years. For examples, improvements on the accuracy and efficiency by saturation relay feedback test (Yu, et al., 1995) or by reducing high-order harmonic terms or using the Fourier analysis (Lee, et al., 1995; Sung, et al., 1995; Wang, et al., 1997) have been reported. On the other hand, extensive works on the PID tuning formulae and refinement have been published. Extensions of Auto-tuning system to other different cases, such as: time varying delay (Leva, 1993), cascade controllers (Hang, et al., 1994), the Smith Preidictor (Plamor, et al., 1994) and the FSA (finite spectrum assignment) controller (Wang, et al.,1995) have also been found in literature. It used to observe that use of model-based tuning rules results in better control performance. But, the trade-off is lots of efforts are required to identify an appropriate model to apply these model-based tuning rules. As has been mentioned, although methods of identification using relay or ATV test has been extensively addressed in literature, the absence of an model-based auto-tuning system as simple and robust as the one of Åström and Hägglund is a simple fact. In this paper, we will reformulate the inverse-based tuning rules in terms of ultimate gain and frequency so that can be easily



Figure 1: Relay Feedback System

applied using those data obtained from a simple ATV test. On the other hand, a simple method based on the same ultimate gain and frequency obtained is proposed to select the tuning rules which are formulated specifically for different simple dynamics.

## 2. RELAY FEEDBACK IDENTIFICATION

#### 2.1 Ideal Relay Test

In 1984, Åström and Hägglund (Åström et al., 1984) suggest the relay feedback test to generate sustained oscillation as an alternative to the conventional continuous cycling technique. This relay feedback test was soon (Luyben, 1987) referred as autotune variation (abbr. ATV) test. As shown in Fig.6 is the ATV test. Controller tuning using ATV test is attractive, because it is operated under closed loop and no a priori knowledge of system is need. The test provides ultimate gain and ultimate period for applying Z-N rules to tune a PID controller.



Figure 2: Autotune with ATV Test

The ultimate gain and ultimate frequency are extimated from the ATV test as:

$$k_{cu} = \frac{4h}{\pi A} \tag{1}$$

$$\omega_u = \frac{2\pi}{P_u} \tag{2}$$

By making use of the data obtained, PID controller parameters can be computed using Z-N rules. The control performance thus obtained is in general crude compared with those tuned with model-based rules, such as IMC. This fact is most obvious when the process has underdamped second order dynamics. But, with one simple ATV test, it is usually not sufficient for identifying parametric model of processes other than one that has exactly FOPDT dynamics. This makes spaces for research on developing the parametric models from ATV tests. There are quite a few researchers (e.g. Luyben, 1987; Li, et al.,1991, Ching et al., 1992; Lee and Sung, 1993, Shen et al., 1996, Sung and Lee, 1997, Wang, et al., 1997, Huang, et al., 2000) worked on this problem. Nevertheless, methods from those published works have different extents of sophisicatcy and complexities, and is not convenient for apllication to autotune systems, where a simple and robust method is desirable. As a result, there remains space for developing effective parametric autotune methods to enhance the controller tuning and achieve better performance.

In model-based methods, tuning formulas are derived for each specific type of transfer function model. But, as far as control performance is concerned, to differentiate FOPDT dynamics from the overdamped dynamics seems not so crucial. In general, the FOPDT-based formula works quite well for over-damped or over-damped-like processes, but not for under-damped ones. Tuning rules for underdampled SOPDT process are indeed required. In this sense, the identification in a autotune system needs only to select models from two types, that is the FOPDT one or an under-damped SOPDT one. In the following, we shall illustrate this identification using one ATV test.

#### 2.2 Relay Feedback Response Curves

Consider the FOPDT and SOPDT models for representing general dynamics in chemical plants. The original and normalized models are shown in the following.

#### ➤ FOPDT Model

$$G(s) = \frac{K_p e^{-\theta s}}{\tau s + 1}$$

$$G(s) = \frac{K_p e^{-\bar{s}}}{\bar{\tau}\bar{s} + 1}$$
where  $\bar{s} = \theta s, \bar{\tau} = \frac{\tau}{\theta}$ 

► SOPDT Model

$$G_p(s) = \frac{K_p e^{-\theta s}}{\tau^2 s^2 + 2\tau \zeta s + 1}$$
$$G_p(\bar{s}) = \frac{K_p e^{-\bar{s}}}{\bar{\tau}^2 \bar{s}^2 + 2\bar{\tau} \zeta \bar{s} + 1}$$

where  $\bar{s} = \theta s, \bar{\tau} = \frac{\tau}{\theta}$ .

For these two types of processes, extensive ATV tests have been conducted over wide ranges of normalized parameters. For example, for SOPDT process,  $\bar{\tau}$  ranges from 0.1 to 100 and  $\zeta$  from 0.1 to 100. The resulting normalized ultimate gains and ultimate period are plotted and is as shown in Figure(3).



Figure 3: Generalized ATV Responses

2.3Identification

In Figure(3), it is thus possible to divide the ATV results into different zones. As has been mentioned, for PID controler tuning, overdamped processes and some of underdamped processes that has damping factors close to one can be approximated by FOPDT model. To differentiate the FOPDT and overdamped SOPDT dynamics from those of underdamped SOPDT, the curve having  $\zeta$  of 0.7 is selected as a criterion. This is because that, in Bode's plot, an SOPDT process does not have resonance peak when its  $\zeta$  is greater than or equal 0.707. Thus, as shown in Figure (4), there are two zones (Zone I and Zone II) separated by two curves. Among the curves, one is on the edge of zone I that represents FOPDT dynamics. The normalized time constant (i.e.  $\bar{\tau}$ ) of this FOPDT process ranges from 0.1 to 100. The other is on the edges of the zoneI and Zone II. Zone II represents processes of underdamped dynamics. With Figure (4), identifying an unknown system using ATV test can thus be conducted using the resulting A and  $P_u$ . By locating the experimental results on the fingure, parameters can be estimated, provide that  $k_p$  and  $\theta$  is known, using the following equations. In case of both values of  $k_p$ and  $\theta$  are not available, estimation method will be introduced later in the section of tuning procedures.



Figure 4: Simplified ATV Responses

#### 1. For point located in FOPDT zone:

$$\tau = \frac{\sqrt{K_u^2 - 1}}{\omega_u} = \frac{P_u}{2\pi}\sqrt{K_u^2 - 1}$$
$$\theta = \frac{\pi - \tan^{-1}\sqrt{K_u^2 - 1}}{\omega_u}$$

where,  $K_u = k_p k_{cu}$ 

#### 2. For point located in underdamped SOPDT zone:

$$2\tau\zeta\omega_u = Y = K_u\sqrt{\frac{\tan^2(\gamma)}{1+\tan^2(\gamma)}}$$
$$\tau\omega_u = X = \sqrt{1+\frac{Y}{\tan(\gamma)}}$$

where, 
$$K_u = k_p k_{cu}$$
 and  $\gamma = \omega_u \theta$ 

### 3. TUNING RULES

An inverse-based design is used to synthesize PID controller for autotune. The controller is synthesized so as to have a loop transfer function (abbr. LTF) of two standard forms of the following. That is:

$$G_{loop}(s) = \frac{0.65}{\theta} \frac{(1 + 0.4\theta s \ e^{-\theta s})}{s(1 + \tau_f s)}$$
(3)

or,

$$G_{loop}(s) = \frac{0.4}{\theta} \frac{e^{-\theta s}}{s(1+\tau_f s)} \tag{4}$$

These loop transfer function can provide the system reasonable stability robustness and control performance. According to these standard forms, the PID controller is systemesized for FOPDT dynamic process and is given as follows:

$$G_c(s) = K'_c(1 + \frac{1}{\tau'_R s}) \frac{1 + \tau'_D s}{1 + \tau_f s}$$
(5)

where,

$$\begin{array}{lll} (K_{c}^{'}) & = & 0.65 * \frac{\theta}{\tau} * \frac{1}{K_{p}} \\ (\tau_{R}^{'}) & = & \tau \\ (\tau_{D}^{'}) & = & 0.4\theta \end{array}$$

On the other hand, the PID controller for SOPDT type dynamic process is:

$$G_c(s) = K_c (1 + \frac{1}{\tau_R s} + \tau_D s)(1 + \tau_f s) \tag{6}$$

$$K_c = \frac{2\tau\zeta}{K_p\theta} * 0.4$$
  

$$\tau_R = 2\tau\zeta$$
  

$$\tau_D = \frac{\tau^2}{2\tau\zeta}$$

Since, all controller parameters are in terms of the dynamic parameters as idtentified in the previous section, these controller parameters can be re-formulated to be in terms of ultimate gain and ultimate period. In other words, the tuning parameters can be directly computed from the results of a ATV test. As an example, for FOPDT process, the PID parameters can be re-written as:

$$\begin{array}{rcl}
K_{c}^{'} &=& (K_{c})_{ZN} \times 0.54 \\
\tau_{R}^{'} &=& (\tau_{R})_{ZN} \times F_{1} \\
\tau_{D}^{'} &=& (\tau_{D})_{ZN} \times F_{2} 
\end{array}$$
(7)

where,

$$F_1 = \frac{\sqrt{K_{cu}^2 K_p^2} - 1}{\pi}$$

$$F_2 = \frac{1.6}{\pi} \left[ 1 - \frac{tan^{-1}\sqrt{K_{cu}^2 K_p^2 - 1}}{\pi} \right]$$

A comparison of this inverse-based tuning with that of Z - N tuning is as shown in Figure(5).



Figure 5: Comparisons of Inverse-based And Z-N Controller Settings

## 4. AUTOTUNE PROCEDURES

As has been methioned, when the values of  $k_p$  and the apparent deadtime,  $\theta$ , are available, the autotune procedure proceeds just like the conventional one, except that the selection of tuning rules by locating  $(A/(k_ph), P_u/\theta)$  on Figure (4) is required as a proceeding step, and the tuning rules itself are different.

In case of where  $k_p$  and  $\theta$  are not available, the ATV test has to make a slightly modification. The excitation to the relay feedback system is introduced through an external input at the output of relay. As shown in Figure (6). The external input has to be small like up to 10% of h, the level of relay output. Nomenclatures for computing amplitudes and period are given in Fighue (??). Then,  $k_p$  is estimated by the following equation:

$$k_p = \frac{\int_t^{t+P/2} y(\tau) d\tau}{\int_t^{t+P/2} u(\tau) d\tau}$$
(8)

On the other hand, the apparent deadtime for applying the identification zones in Figure (4) are estimated as an average value of true deadtime detected at the very begining of the test and q quater of period later at constant cycling, as shown in Figure.(??). This is because the deadtime interms of SOPDT dynamics is always lying between te two values aforementioned.

To calcult the amplitudes for identification, the centerline of the cycling has to be updated to to the bias form the input to a height of  $y_{\delta} = 0.5(A_1 - |A_2|)$ .



Figure 6: ATV Test with External Excitation



Fig 7: External Excitation ATV Test

Besides the preceding steps just mentioned, all other remaining procedures are the same as the previously described ones. Thus, by estimating the normalized amplitude and the normmalized period, and by making use of Figure (4), tuning rules can be selected and controller parameters can be computed.



Fig 8: Estimation for Apparent Deadtime

# 5. SIMULATION RESULTS

The following are the results of applying this autotune procedure to a few example processes.

Ex.1

Process 
$$\frac{1}{(s+1)^5}e^{-s}$$

Table1: Simulation Results

$K_p$	а	h	$K_{cu}$	$K_u$	$\omega_u$
1.0000	0.6444	1.000	1.9758	1.9758	0.5689
-	$P_u$	L	D	d	-
-	11.0444	2.8337	1.500	2.0206	-

Table2: PID Parameters and IAE values

	Z-N	proposed
$k_c$	1.1623	0.3348
$ au_I$	5.5222	2.2904
$ au_D$	1.3805	1.4776
IAE	8.518	6.685



Figure 7: Ex.1 Tuning result

Ex.2

Process 
$$\frac{1.08}{(s+1)^2(2s+1)^3}e^{-10s}$$

Table3: Simulation Results

$K_p$	а	h	$K_{cu}$	$K_u$	$\omega_u$
1.0826	1.0487	1.000	1.2141	1.3144	0.1807
	$P_u$	L	D	d	
	34.78	11.2275	11	11.1138	

Table 4: PID parameters and IAE values

	Z-N	proposed
$k_c$	0.7141	0.2094
$ au_I$	17.3900	4.6926
$ au_D$	4.3475	5.3974
IAE	38.325	21.749





Ex.3

Process 
$$\frac{1}{(s^2 + 0.6s + 1)}e^{-s}$$

Table5: Simulation Results

$K_p$	а	h	$K_{cu}$	$K_u$	$\omega_u$
1.0000	1.727	1.000	0.7373	0.7373	1.1464
-	$P_u$	L	D	d	-
-	5 4806	1 3609	1 000	1 1805	_

Table6: PID parameters and IAE values

	Z-N	proposed
$k_c$	0.4337	0.2128
$ au_I$	2.7403	0.6280
$ au_D$	0.6851	1.4044
IAE	7.3250	4.0262

## 5. CONCLUSION

In this paper, we have presented a autotune system which make uses of model-based tuning rules to enhance control performance. The presented system works in a similiar way to the one of Astrom and Hugglund in the sense it uses a conventional AYV test and the resulting amplitude and period of constant cycles. Because of using parametric models, additional steps it needs are the estimation of  $k_p$  and  $\theta$ . The tuning rules are derived from an inverse-based approach,



Figure 9: Ex.3 Tuning result



Figure 10: Identification Results

which is similar to the IMC method but simpler. From the simulation results, we can observe and conclude that this autotune system is efficient and self-contained.

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