ROBUST ITERATIVE LEARNING CONTROL DESIGN BASED ON GRADIENT METHOD

Shan Liu, Tiejun Wu

National Laboratory of Industrial Control Technology, Institute of Intelligent Systems and Decision Making, Zhejiang University, Hangzhou 310027, P.R. China

Abstract: An iterative learning control (ILC) method using both feedback and feedforward actions is proposed for a class of uncertain linear systems to achieve precise tracking control. A sufficient condition for the plant uncertainty and feedback controller, which guarantees the robust convergence of the learning, is given. The procedure of designing the robust algorithm has two steps: At first, the feedback controller with robust performance is synthesized based on H_∞ optimal approach according to the request of the sufficient condition; secondly, the incremental feedforward input signal is derived by gradient method with fixed step size. It is shown that the feedforward action has relation to the adjoint system of the closed nominal system.

Keywords: Iterative learning control, H_∞ optimal control, gradient method, plant uncertainty.

1. INTRODUCTION

Iterative learning control (ILC) is a kind of feedforward control technique suitable to be applied systems or processes with repetitive operation in order to improve sequentially the control accuracy by performing the same task iteratively (Arimoto, *et al.*, 1984; Moore, 1998). While ILC has drawn continuous attention by its simplicity and tracking accuracy, most studies have been focused on the robustness of ILC schemes (Amann, *et al.*, 1996b; Furuta, *et al.*, 1987; Heinzinger, *et al.*, 1989; Moon, *et al.*, 1998; Padieu and Su, 1990). The initial robustness analysis concentrated on the open-loop ILC updating law (Heinzinger, *et al.*, 1989), but the purely open-loop ILC scheme may fail to work in practical applications because of the drawback that the tracking error may possibly grow quite large in the early stages of learning. Thus, the feedback controller is commonly employed along with the ILC to ensure the closed-loop stability. In this situation, the ILC algorithms designed for robustness against the original plant uncertainty are possibly unfit for the uncertainty of the closed-loop system.

Furuta, *et al.* (1987) derived a gradient-type ILC scheme by using a steepest-descent algorithm to minimize a performance criterion, but their approach is a pure feedforward type and consequently suffers from a lack of robustness in practice yet. Amman, *et al.* (1996a) proposed an ILC law using feedback and feedforward actions on the basis of an optimization principle. Although their method has some degree robustness, it is unable to be applied to the plant with model uncertainty. Some researchers synthesize the ILC schemes based on the robust control theory. For example, Padieu, *et al.* (1990) presented general convergence conditions of ILC on the basis of H_∞ approach. Amann, *et al.* (1996b) showed a sufficient convergence condition for ILC with feedback controller and proposed a two-step design procedure based on H_∞ optimization. Moon, *et al.* (1998) reformulated the ILC design problem as a general robust control problem setup and gave a robust ILC procedure systematically. Most of the aforementioned approaches are designed and analyzed based on contraction mapping in frequency domain purely, but for any practical ILC process, a trial must end after a finite time. Thus, the conditions for convergence of these approaches can be overly restrictive because of the infinite time analysis of the feedforward action, sometimes leading to a requirement that the plant is invertible. For example, it is impossible to use general convergence condition in frequency domain for non-minimum-phase plants or strictly proper plants. Although a band-pass filter can be employed along with the feedforward action in order to relax the conservative convergence condition, the performance degradation that the tracking error will not converge to zero perfectly is caused in this case.

This paper focuses on the gradient-type ILC architecture of using both feedback and feedforward actions, and their performances are analyzed in the time domain and frequency domain respectively. A sufficient condition to guarantee the robust convergence of the learning strategy is given for a class of uncertain linear systems. Based on the derived sufficient condition, a two-step procedure of designing the robust ILC algorithm is proposed. First of all, the feedback controller with robust performance is synthesized based on H_∞ optimal approach according to the request of the sufficient condition; secondly, the incremental feedforward input signal is derived by gradient method with fixed step size.

This paper is organized as follows. Section 2 describes the ILC architecture with feedback controller and gives the relationship between original plant uncertainty and closed-loop system uncertainty. In section 3 a gradient-type ILC approach is proposed and a sufficient condition for robust convergence is derived in operator form. The design procedure based on H_∞ optimal approach and gradient method is presented in section 4. Section 5 verifies the usefulness of the proposed method through a simulation. Some concluding remarks are given in section 6.

2. ILC USING FEEDBACK AND FEEDFORWARD ACTIONS

Consider the controlled linear plant represented in operator form as

$$y = \tilde{P}u \tag{1}$$

where $u \in U$ and $y \in Y$, U and Y are a real Hilbert space with inner products $\langle \cdot,\cdot \rangle_u$ and $\langle \cdot,\cdot \rangle_y$ respectively. $\tilde{P}$ is the system input-output operator, describing the dynamics of plants with unstructured uncertainties

$$\mathcal{P} = \left\{ \tilde{P} : \tilde{P} = P\left(I + \Delta_P W\right), \left\| \Delta_P \right\| \le 1 \right\} \tag{2}$$

where P is the nominal plant operator, W is a fixed operator, Δ_P is a variable operator. Here $\|\cdot\|$ is the induced norm of operator.

Let $r(t)$ be a desired reference trajectory defined on the finite time interval $[0, T]$. An ILC system provides robustness against the plant uncertainty if it provides convergence of the iterative process for every $\tilde{P}$ in $\mathcal{P}$. It is equivalent to finding an ILC algorithm to construct a sequence of control inputs $\{u_k(t)\}$ converging to u_∞ as $k \to \infty$, such that u_∞ is the solution for the optimization problem

$$\min_u \left\{ J(u) = \|e\|_y^2 : e = r - y,\ y = \tilde{P}u \right\} \tag{3}$$

A block diagram of ILC using both feedback and feedforward actions is shown as Fig.1. In this figure, C is a feedback control operator and L is a feedforward control operator. The ILC approach in this case can be expressed as

$$\left. \begin{aligned} u_k &= v_k - Cy_k \\ v_{k+1} &= v_k + Le_k \end{aligned} \right\} \tag{4}$$

where $e_k = r - y_k$ and v_k is feedforward input. In this scheme, the feedback controller ensures closed-loop stability and suppresses exogenous disturbances, and the feedforward controller provides improved tracking performance utilizing past control results.

The closed-loop operator from feedforward input v_k to system output y_k is easily given by

$$y_k = \left(I + \tilde{P}C\right)^{-1} \tilde{P}v_k = \tilde{P}\left(I + C\tilde{P}\right)^{-1} v_k \tag{5}$$

The closed-loop operators corresponding to the uncertain system $\tilde{P}$ and the nominal system P are represented by $\tilde{G}$ and G, respectively. Hence, it yields

$$\tilde{G} = G\left(I + \Delta_P WCG\right)^{-1}\left(I + \Delta_P W\right) \tag{6}$$

From (6) it follows that

$$\tilde{G} = G\left[I + \left(I + \Delta_P WCP\left(I + CP\right)^{-1} \right)^{-1} \times \Delta_P W \left(I + CP\right)^{-1} \right] \tag{7}$$

By defining the sensitivity and complementary sensitivity operators associated with the nominal system, i.e.,

$$S = \left(I + CP\right)^{-1},\ T = CP\left(I + CP\right)^{-1} \tag{8}$$

and denoting the uncertainty of the closed-loop system by

$$\Delta_G = \left(I + \Delta_P WT\right)^{-1} \Delta_P WS \tag{9}$$

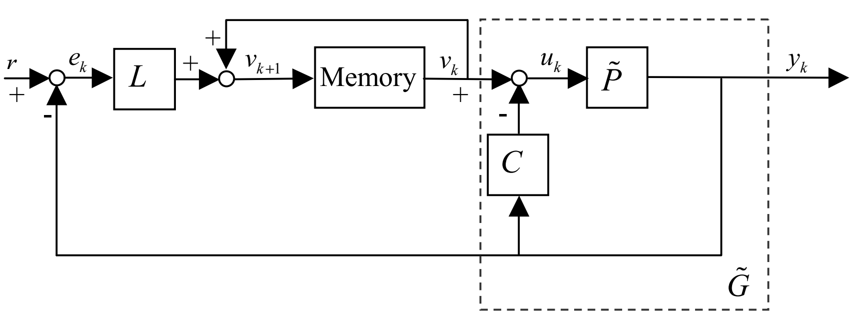

Fig.1. ILC using feedback and feedforward actions

the (7) can be rewritten as $\tilde{G} = G(I + \Delta_G)$. (9) implies the exact relationship between the uncertainty of the controlled system $\tilde{P}$ and that of the closed-loop system $\tilde{G}$ with feedback controller C.

3. ROBUST CONVERGENCE OF GRADIENT-TYPE ILC SCHEME

According to the aforementioned ILC configuration, if C as a fixed controller has already synthesized in advance to make the closed-loop system G of the nominal plant to achieve internal stable, the feedforward controller L can be designed based on the gradient method (Furuta, *et al.*, 1987). The gradient of J in (3) on v_k is

$$\Delta v_k = -G^* e_k \quad (10)$$

where G^* is the adjoint operator to G. Then the gradient-type ILC algorithm can be expressed as

$$v_{k+1} = v_k + \alpha G^* e_k \quad (11)$$

where α is a fixed step size of learning. For the actual closed-loop system $\tilde{G}$, by using the relation $e_k = r - \tilde{G} v_k$, from (11) it yields the error relation in the recursive form as

$$e_{k+1} = \left(I - \alpha \tilde{G} G^*\right) e_k \quad (12)$$

Hence

$$\begin{aligned} \|e_{k+1}\|_y^2 &= \left\|\left(I - \alpha \tilde{G} G^*\right) e_k\right\|_y^2 \\ &= \|e_k\|_y^2 + \alpha^2 \left\|\tilde{G} G^* e_k\right\|_y^2 - 2\alpha \left\langle e_k, \tilde{G} G^* e_k \right\rangle_y \end{aligned} \quad (13)$$

which leads to, by using the property of induced norm and the Schwarz inequality,

$$\begin{aligned} &\|e_{k+1}\|_y^2 - \|e_k\|_y^2 \\ &= \alpha^2 \left\|\tilde{G} G^* e_k\right\|_y^2 - 2\alpha \left\|G^* e_k\right\|_u^2 \\ &\quad -2\alpha \left\langle G^* e_k, \Delta_G G^* e_k \right\rangle_u \\ &\le \alpha \left(\alpha \left\|\tilde{G}\right\|^2 + 2\|\Delta_G\| - 2\right) \left\|G^* e_k\right\|_u^2 \end{aligned} \quad (14)$$

Note that if $\|\Delta_G\| \le \infty$, then $\|\tilde{G}\| \le \|G\|\left(1 + \|\Delta_G\|\right) \le \infty$. From (14), a sufficient condition can be obtained to ensure that the error sequence $\left\{\|e_k\|_y\right\}$ is monotonically non-increasing in k is

$$\alpha \left\|\tilde{G}\right\|^2 + 2\|\Delta_G\| - 2 < 0 \quad (15)$$

In view of the fact that (15) gives $\|\Delta_G\| < 1 - \frac{1}{2}\alpha \left\|\tilde{G}\right\|^2$, due to the step size $\alpha > 0$, the above sufficient condition implies that $\|\Delta_G\| \le \delta < 1$, where δ is a positive number. Thus, in order to ensure the robust convergence of the ILC law (11), the step size α need to be chosen in the range defined in the following

$$0 < \alpha < \frac{2 - 2\delta}{\|G\|^2 (1+\delta)^2} \quad (16)$$

As a result, from (14), (15) and (16) it follows that

$$\begin{aligned} &\|e_{k+1}\|_y^2 - \|e_k\|_y^2 \\ &\le \alpha \left[\alpha \|G\|^2 (1+\delta)^2 + 2\delta - 2\right] \left\|G^* e_k\right\|_u^2 \le 0 \end{aligned}$$

Thus the ILC law (11) is convergent for every plant satisfying the condition $\|\Delta_G\| \le \delta < 1$.

If δ is much close to 1, however, the condition (16) will become very strict and conservative, i.e., the step size α has to be chosen much small and, as a result, the convergence speed will be very slow.

The following theorem shows that the condition of choosing step size can be relaxed while the robustness for convergence of the ILC process will be maintained in the presence of plant uncertainty.

Theorem 1. Assume a set of uncertain systems as

$$\left\{\tilde{G} : \tilde{G} = G(I + \Delta_G), \|G\| < \infty, \|\Delta_G\| \le \delta < 1\right\} \quad (17)$$

and the ILC updating law is given by (11). If the learning step size α satisfies

$$0 < \alpha \le \frac{1}{\|G\|^2} \quad (18)$$

then for every plant $\tilde{G}$ satisfying (17),

1) the ILC tracking error sequence $\{e_k\}$ converges to a limited e_∞ when $k \to \infty$;
2) the ILC tracking error sequence $\{e_k\}$ converges to zero if $\ker\left(G^*\right) = 0$.

Proof:
Substituting condition (17) into (13) yields

$$\begin{aligned} &\|e_{k+1}\|_y^2 - \|e_k\|_y^2 = \alpha^2 \left\|G(I + \Delta_G) G^* e_k\right\|_y^2 \\ &\quad -2\alpha \left\langle e_k, G G^* e_k \right\rangle_y - 2\alpha \left\langle e_k, G \Delta_G G^* e_k \right\rangle_y \\ &= \alpha^2 \Big(\left\|G G^* e_k\right\|_y^2 + \left\|G \Delta_G G^* e_k\right\|_y^2 \\ &\quad + \left\langle \left(\Delta_G^* G^* G + G^* G \Delta_G\right) G^* e_k, G^* e_k \right\rangle_u \Big) \\ &\quad -\alpha \left(2\left\|G^* e_k\right\|_u^2 + \left\langle \left(\Delta_G^* + \Delta_G\right) G^* e_k, G^* e_k \right\rangle_u \right) \\ &= \alpha \Big\langle \Big(\alpha \left(G^* G + \Delta_G^* G^* G \Delta_G + \Delta_G^* G^* G \right. \\ &\quad \left. + G^* G \Delta_G \right) - 2I - \left(\Delta_G^* + \Delta_G\right) \Big) G^* e_k, G^* e_k \Big\rangle_u \end{aligned} \quad (19)$$

In view of the fact that

$$\begin{aligned} &\alpha \left(G^* G + \Delta_G^* G^* G \Delta_G + \Delta_G^* G^* G + G^* G \Delta_G \right) \\ &-2I - \left(\Delta_G^* + \Delta_G\right) \\ &= \left(I + \Delta_G^*\right) \left[\left(\alpha G^* G - I\right) + \left(\alpha G^* G \Delta_G - I\right) \right] \\ &\quad + \Delta_G^* - \Delta_G \\ &= \left(I + \Delta_G^*\right) \left[\left(\alpha G^* G - I\right)\left(I + \Delta_G\right) + \left(\Delta_G - I\right) \right] \\ &\quad + \Delta_G^* - \Delta_G \\ &= \left(I + \Delta_G^*\right) \left(\alpha G^* G - I\right) \left(I + \Delta_G\right) \\ &\quad + \left(I + \Delta_G^*\right)\left(\Delta_G - I\right) + \Delta_G^* - \Delta_G \\ &= \left(I + \Delta_G^*\right) \left(\alpha G^* G - I\right) \left(I + \Delta_G\right) + \Delta_G^* \Delta_G - I \end{aligned} \quad (20)$$

using the property that $\|G\|^2 = \left\|G^*\right\|^2 = \left\|G^* G\right\|$ and (18) we obtain

$$\left\| \alpha G^* G \right\| \le 1 \tag{21}$$

Thus $\left(I+\Delta_G^*\right)\left(\alpha G^* G - I\right)\left(I+\Delta_G\right)$ is a non-positive definite self-adjoint operator. From the system uncertainty condition (17), $\left\|\Delta_G\right\| \le \delta < 1$, it follows that $\Delta_G^* \Delta_G - I$ is a negative definite self-adjoint operator. Note that $\alpha > 0$, hence

$$\left\|e_{k+1}\right\|_y^2 - \left\|e_k\right\|_y^2 < \alpha\left(\delta^2 - 1\right)\left\|G^* e_k\right\|_u^2 \le 0 \tag{22}$$

Thus the sequence $\left\{\left\|e_k\right\|_y^2\right\}$ is monotonically non-increasing and converges to a limited limit, and $\lim_{k\to\infty}\left|\left\|e_{k+1}\right\|_y^2 - \left\|e_k\right\|_y^2\right| = 0$. Then let $k \to \infty$ in (22), it can be concluded that

$$0 \le \alpha\left(\delta^2 - 1\right)\lim_{k\to\infty}\left\|G^* e_k\right\|_u^2 \le 0 \tag{23}$$

i.e., $\lim_{k\to\infty}\left\|G^* e_k\right\|_u = 0$, or, equivalently, $\lim_{k\to\infty} G^* e_k = 0$. If the nominal plant satisfies $\ker\left(G^*\right) = 0$, there exists no non-zero e such that $G^* e = 0$, and it follows that $\lim_{k\to\infty} e_k = 0$. ■

Remark 1. From a result of the operator theory , i.e., $\overline{\Re(G)} = \left[\ker\left(G^*\right)\right]^{\perp}$, where $\Re$ denotes the range of an operator and $\overline{\Re(G)}$ the closure of $\Re(G)$, it follows that $\ker\left(G^*\right) = 0$ implies $Y = \overline{\Re(G)}$ or $\Re(G)$ is dense in Y (Amann, *et al.*, 1996a). Hence, the condition $\ker\left(G^*\right) = 0$ means that for any desired output trajectory r, there exists input u_d that drives the nominal plant to produce the output $y = r$. This condition can be weakened to that the desired output trajectory $r \in \Re(G)$ or $r \in \overline{\Re(G)}$, which implies the given desired output trajectory can be tracked exactly by the nominal plant output. These are the reasonable assumptions usually made in ILC studies.

Remark 2. The condition (17) in theorem 1 is not only a sufficient condition for the gradient-type ILC process to converge but also a condition for synthesizing the feedback controller C.

It is worthwhile to point out that theorem 1 can not directly applicable to the design of controller C, because the plant uncertainty and controller C are implicitly involved in the sufficient condition (17). The following theorem gives a realizable solution for designing the feedback controller C.

Theorem 2. Suppose the uncertain system $\tilde{P}$ described as (1) and (2), and the ILC updating law is given by (11) and (18). If the condition

$$\|WT\| + \|WS\| < 1 \tag{24}$$

is satisfied, then the ILC system is convergent.

Proof:

From theorem 1 and (9), a sufficient condition for the ILC to converge is $\left\|\left(I+\Delta_P WT\right)^{-1}\Delta_P WS\right\| < 1$. Note that

$$\left\|\left(I+\Delta_P WT\right)^{-1}\Delta_P WS\right\| < \left\|\left(I+\Delta_P WT\right)^{-1}\right\|\left\|\Delta_P\right\|\left\|WS\right\| \tag{25}$$

Because of the fact that (24) implies $\|WT\| < 1$, from $\left\|\Delta_P\right\| \le 1$ we have $\left\|\Delta_P WT\right\| \le \left\|\Delta_P\right\|\left\|WT\right\| < 1$. Therefore,

$$\left\|\left(I+\Delta_P WT\right)^{-1}\right\| < \frac{1}{1-\left\|\Delta_P WT\right\|} < \frac{1}{1-\|WT\|} \tag{26}$$

Combining (25) and condition (24) yields

$$\left\|\left(I+\Delta_P WT\right)^{-1}\Delta_P WS\right\| < \frac{\|WS\|}{1-\|WT\|} < 1 \tag{27}$$

Hence the ILC system is convergent. ■

Remark 3. Note that condition (24) has the similar form as the robust performance condition for the uncertain plant $\tilde{P}$ and the controller C. Thus theorem 2 has shown that the robust ILC system for the uncertain plant can be designed in two steps: At first, synthesize the feedback controller achieving robust performance according to the condition (24) independently; secondly, design the feedforward controller based on the gradient method.

Remark 4. Due to the fact that

$$\|W\| = \|W(S+T)\| \le \|WS\| + \|WT\|$$

a necessary condition for (24) is $\|W\| < 1$.

4. ROBUST ILC ALGORITHM BASED ON H_∞ OPTIMAL APPROACH

The aforementioned analysis is established in the general linear operator space. The practical computational algorithm depends on the form of systems dynamics in detail.

Suppose input space U is $L_2^m[0,T]$ and output space Y is $L_2^n[0,T]$, the inner products in U and Y are defined as

$$\langle u_1, u_2\rangle_u = \langle u_1, u_2\rangle_{2T} = \int_0^T u_1^T(t)\, u_2(t)\, dt$$

$$\langle y_1, y_2\rangle_y = \langle y_1, y_2\rangle_{2T} = \int_0^T y_1^T(t)\, y_2(t)\, dt$$

the induced norm of plant P is denoted $\|P\|$. In the meantime, suppose the extended input space $\bar{U}$ is $L_2^m[0,+\infty)$ and the extended output space $\bar{Y}$ is $L_2^n[0,+\infty)$, the inner products in $\bar{U}$ and $\bar{Y}$ are defined as

$$\langle u_1, u_2\rangle_{\bar{u}} = \langle u_1, u_2\rangle_2 = \int_0^{+\infty} u_1^T(t)\, u_2(t)\, dt$$

$$\langle y_1, y_2\rangle_{\bar{y}} = \langle y_1, y_2\rangle_2 = \int_0^{+\infty} y_1^T(t)\, y_2(t)\, dt$$

the extended induced norm of plant P is denoted $\|P\|_\infty$, i.e., H_∞ norm. The following lemma shows the relationship between $\|P\|_\infty$ and $\|P\|$.

Lemma 1. Consider the plant P as either an operator from $L_2^m[0,T]$ to $L_2^n[0,T]$, or an operator from $L_2^m[0,+\infty)$ to $L_2^n[0,+\infty)$. The corresponding induced norm and the extended induced norm of P

are $\|P\|$ and $\|P\|_\infty$, respectively, then

$$\|P\| \le \|P\|_\infty \tag{28}$$

The following theorem is derived from lemma 1 and theorem 2 immediately.

Theorem 3. Consider the uncertain system $\tilde{P}$ described as (1) and (2), regarded as an operator from $L_2^m[0,T]$ to $L_2^n[0,T]$. The ILC updating law is described as (11) and (18). If the condition

$$\|WT\|_\infty + \|WS\|_\infty < 1 \tag{29}$$

is satisfied, then the ILC system is convergent.

It has been clearly shown from (29) that a more conservative sufficient condition ensuring the ILC robustly converge is $\|WT\|_\infty < \frac{1}{2}$ and $\|WS\|_\infty < \frac{1}{2}$, or

$$\left\| \begin{bmatrix} WS \\ WT \end{bmatrix} \right\|_\infty < \frac{1}{2} \tag{30}$$

Hence, the feedback controller C can be obtained by H_∞ optimal approach. According to the linear fractional transformation (LFT) and

$$\begin{bmatrix} WS \\ WT \end{bmatrix} = \begin{bmatrix} W \\ 0 \end{bmatrix} + \begin{bmatrix} -W \\ W \end{bmatrix} C\left(I+PC\right)^{-1} P \tag{31}$$

the "augmented plant" P_H in the standard H_∞ control problem is

$$P_H = \begin{bmatrix} W & -W \\ 0 & W \\ P & -P \end{bmatrix} \tag{32}$$

On the other hand, for linear system, if the feedback controller C is given, the adjoint system of the closed-loop system G can be synthesized directly. Suppose that G has m inputs and n outputs with dynamics described by a linear continuous state-space form as

$$\left.\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)x(t) \end{aligned}\right\} \tag{33}$$

where $x(t)$, $u(t)$ and $y(t)$ represent the state, the input and the output, respectively. The initial state of system is

$$x(t_0) = x_0 \tag{34}$$

In this case, the adjoint system G^* of system G can be obtained by

$$\left.\begin{aligned} \dot{g}(t) &= -A^T(t)g(t) - C^T(t)w(t) \\ z(t) &= B^T(t)g(t) \end{aligned}\right\} \tag{35}$$

the corresponding terminal boundary condition of G^* is relation to practical performance index. For example, the terminal boundary condition corresponding to performance index (3) is

$$g(T) = 0 \tag{36}$$

The computational procedure for robust ILC algorithm based on the H_∞ optimal approach and the gradient method is summarized as follows:

1) For the uncertain system $\tilde{P}$ described as (1) and (2), design the feedback controller C by solving the conventional H_∞ optimal control problem (Zhou, *et al.*, 1996) according to the augmented plant (32), and examine the condition (29) and (30).
2) Fix the feedback controller C. Get the closed-loop system G of the nominal plant P and transform G to its minimal realization as (33) and (34), then solve the adjoint system G^* of G according to (35) and (36).
3) Get the ILC updating law according to (11) and (18).

5. SIMULATION EXAMPLE

To illustrate the robustness of the algorithm, the simulation results for the same plant used by Moon, *et al.* (1998) is given in this section. The nominal plant and the uncertainty weighting function are

$$P(s) = \frac{25s+80}{s^2+24s+370}, \quad W(s) = 0.5\frac{s+10}{s+100}$$

where $|W(s)|$ is an increasing function of frequency, the initial state is $x(0)=0$. The uncertain system model is described as (2). The reference trajectory to be tracked contains frequency components of up to 8 Hz and is composed of a series of harmonious sinusoidal signals, i.e.,

$$\begin{aligned} r(t) &= 4\sin(2\pi t) + \sin(4\pi t) \\ &\quad + 0.75\sin(8\pi t) + 0.5\sin(16\pi t) \end{aligned}$$

where $t \in [0,1]$.

Following the design procedure in Section 4, according to (28) the H_∞ augmented plant is

$$G_H(s) = \begin{bmatrix} 0.5\frac{s+10}{s+100} & -0.5\frac{s+10}{s+100} \\ 0 & 0.5\frac{s+10}{s+100} \\ \frac{25s+80}{s^2+24s+370} & -\frac{25s+80}{s^2+24s+370} \end{bmatrix}$$

The feedback controller C was obtained by solving the conventional H_∞ optimal control approach, i.e.,

$$A_c = \begin{bmatrix} -6.9 & 3.1 & -10.4 & -0.8 \\ -32.7 & -112.4 & -156.7 & -29.2 \\ -159.4 & -21 & -523.3 & -61.2 \\ -376.6 & 38.5 & -1131 & -210.5 \end{bmatrix},$$

$$B_c = \begin{bmatrix} 4.369 & -32.82 & -183 & -423.8 \end{bmatrix}^T,$$

$$C_c = \begin{bmatrix} -0.6046 & 3.482 & 2.406 & 2.591 \end{bmatrix},$$

$$D_c = [0]$$

In this case, $\left\| \begin{bmatrix} WS \\ WT \end{bmatrix} \right\|_\infty < 0.5039$, which does not satisfy the condition (30). Notice the fact that $\|WS\|_\infty = 0.5006$ and $\|WT\|_\infty = 0.2782$, however, we have $\|WS\|_\infty + \|WT\|_\infty = 0.7788 < 1$. It satisfies the condition (29), this shows that the algorithm proposed in this paper is applicable to the uncertain system in this example.

We applied the ILC updating law obtained from the design procedure to three representative plants in the set of uncertain systems for comparison:

Plant 1. $\Delta_p = 1.0$

Plant 2. $\Delta_p = 0$ (the nominal plant)

Plant 3. $\Delta_p = -1.0$

Fig. 2 shows the tracking output of the plant 1 for 40 iterations. Fig. 3 shows the root mean square (rms) values of the tracking error versus the iteration numbers. It is clearly seen the algorithm converges in similar speed for those representative plants. This indicates the robustness of the proposed algorithm against the plant uncertainty.

Compare the tracking control performances of the proposed algorithm with those of Moon's learning algorithm in (Moon, *et al.*, 1998). While the simulation result in (Moon, *et al.*, 1998) shows that the tracking error converges after 10th trial and the rms value of the ultimate error is about 0.5, the simulation results of the proposed algorithm show that the tracking error seems to converge after the 40th trial and the rms value of the ultimate error is less than 0.1 for every representative plant. It can be observed from fig. 3 that the rms value of the tracking error is less than 0.32 after 10th trail. This verifies the benefit of the proposed algorithm.

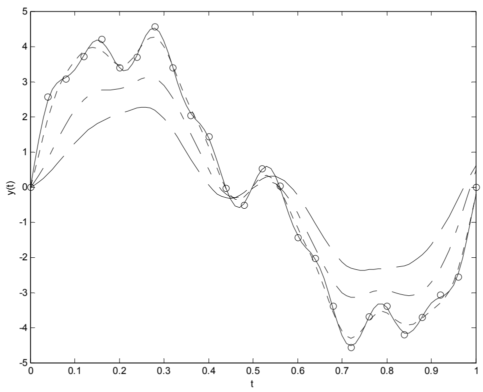

Fig. 2. Output and reference of the plant 1
Legend: dashed line (---) trail 1; dashdot line (-·-) trail 2; dotted line (···) trail 10; solid line (—) trail 40; "○" reference

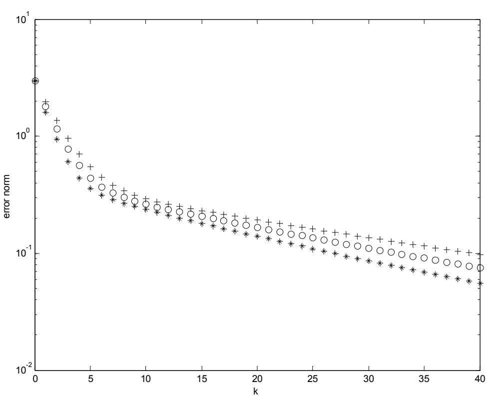

Fig. 3. RMS values of tracking errors versus iteration number
Legend: "+" $\Delta_p = -1.0$; "○" $\Delta_p = 0$; "*" $\Delta_p = 1.0$

6. CONCLUSIONS

In this paper, a gradient-type ILC design method using both feedback and feedforward actions is proposed for a class of uncertain linear systems. In order to avoid the drawback of the ILC method designed in frequency domain purely that the convergence analysis is made in the infinite time range, a sufficient condition for convergence of the iterative process in the presence of plant uncertainty is derived from the operator theory. The feedforward action is obtained by the gradient method and the feedback action is synthesized by the standard H_∞ optimal approach. Based on the derived sufficient condition, a two-step procedure of designing the robust ILC algorithm is suggested. It is shown that the feedforward action has relation to the adjoint system of the closed nominal system. The simulation result demonstrates the effectiveness and robustness of the proposed ILC algorithm against the plant uncertainty.

REFERENCES

Amann N., D. H. Owens and E. Rogers (1996a). Iterative learning control using optimal feedback and feedforward actions. *International Journal of Control*, **65**(2), 277-293.

Amann N., D. H. Owens, E. Rogers and A. Wahl (1996b). An H_∞ approach to linear iterative learning control design. *International Journal of Adaptive Control and Signal Processing*, **10**(6), 767-781.

Arimoto, S., S. Kawamura and F. Miyazaki (1984). Bettering operation of robots by learning. *Journal of Robotic Systems*, **1**(2), 123-140.

Furuta, K. and M. Yamakita (1987). The design of a learning control system for multivariable systems. In: *Proceedings of the 1987 IEEE International Symposium on Intelligent Control*, 371-376, Philadelphia, Pennsylvania, USA.

Heinzinger G., D. Fenwick, B. Paden and F. Miyazaki (1989). Robust leaning control. In: *Proceedings of the 28th IEEE Conference on Decision and Control*, 436-440, Tampa, FL.

Moon J. H., T. Y. Doh, and M. J. Chung (1998). A robust approach to iterative learning control design for uncertain systems. *Automatica*, **34**(8), 1001-1004.

Moore, K. L. (1998). Iterative learning control - an expository overview. *Applied and Computational Controls, Signal Processing and Circuits*, **1**, 151-214.

Padieu F. and R. Su (1990). An H_∞ approach to learning control system. *International Journal of Adaptive Control and Signal Processing*, **4**, 465-474.

Zhou, K., J. C. Doyle and K. Glover (1996). *Robust and Optimal Control*. Prentice-Hall, Englewood Cliffs.