12th IFAC International Symposium on
Advanced Control of Chemical Processes
July 14-17, 2024. Toronto, Canada

Multivariate singular spectrum analysis and detrended fluctuation analysis for plant-wide oscillations denoising

Wahiba Bounoua [*] **Muhammad Faisal Aftab** [*]

[*] *Department of Engineering Sciences, University of Agder, 4879 Grimstad, Norway*

Abstract: Oscillations are considered the most important indicator of poorly performing control loops. However, noise and other disturbances conceal these oscillations, thus making the detection task quite difficult. Furthermore, the efficiency of most detection and diagnosis techniques proposed in the literature is reduced considerably in the presence of noise. Therefore, denoising is recommended to make the detection task more straightforward. In this work, the multivariate singular spectrum analysis (MSSA) is employed to denoise the plant-wide oscillatory control loops. This approach stands in contrast to existing methods that typically focus on addressing noise in individual control loops. In order to improve the efficiency of MSSA, detrended fluctuation analysis (DFA) is incorporated to select only the significant components and eliminate the noise to provide a noise-free version of the multivariate data. The effectiveness of the proposed MSSA-DFA method has been verified using a numerical example and real industrial plant data.

Keywords: Denoising, plant-wide oscillations, industrial control loops, multivariate singular spectrum analysis, detrended fluctuation analysis

1. INTRODUCTION

In this era of data-driven analysis, control performance monitoring (CPM) relies heavily on the quality of the data being processed. However, plant data such as controllers outputs (OPs), process variables (PVs), and manipulated variables (MVs) derived from industrial control loops are typically corrupted by noise. Poorly performing controllers are usually characterised by the emergence of oscillations in these signals. Such oscillations are relatively simple to detect with visual inspection when the signal is purely sinusoidal and devoid of noise and other disturbances (Jelali and Huang, 2010). Noise is the undesirable signal produced internally by a process instrument or externally by unknown disturbances, and its presence is inevitable. Most of the CPM techniques found in the literature struggle to detect these oscillations in the presence of noise (Bounoua et al., 2022). Stiction is a common problem in control systems, and it can significantly impact controllers' performance. One approach to detecting stiction involves analysing the relationship between PV and OP using patterns generated by PV-OP plots. Several studies have demonstrated that the ability of PV-OP plots to identify stiction is restricted in noisy environments (Rengaswamy et al., 2020). Therefore, eliminating noise before processing the data is vital to capture the most accurate information relevant to CPM. Removing noise from signals can be challenging, especially in multivariate systems where several signals may be correlated or interdependent.

Recently, more attention has been paid to preprocessing methods applied in CPM in order to deal with noise and/or nonstationary trends. Empirical mode decomposition (EMD), used earlier as a decomposition technique to detect and diagnose oscillations in industrial control loops (Srinivasan et al., 2007; Aftab et al., 2017a,b, 2018b,a, 2019), has been employed as a denoising technique. In Lang et al. (2020), a novel oscillation denoising scheme by integrating the ensemble EMD (EEMD) with the canonical correlation analysis (CCA) was proposed. In Chen et al. (2021), EEMD was also used to decompose the single-loop oscillation data combined with the detrended fluctuation analysis (DFA) and CCA algorithm to extract the effective oscillation information. More recently, these works were extended to address the problem of detrending as it was demonstrated that the trends are severely compromised during the denoising via EEMD-CCA (Lang et al., 2022). These works have addressed the denoising problem in the single control loops only, and the plant-wide oscillations denoising problem still needs to be addressed.

Typically, plant-wide oscillations involve multivariate signals from multiple control loops, which makes their analysis and denoising challenging. To address this issue, this paper proposes a robust and effective solution to the problem of denoising plant-wide oscillations that combines multivariate singular spectrum analysis (MSSA) and detrended fluctuation analysis (DFA) to denoise multivariate/plant-wide data. The proposed framework is designed to identify and extract noisy components from the multivariate signals using DFA and reconstruct the noise-free time series using the significant components.

The effectiveness of the proposed framework is evaluated using both simulated data and real industrial case studies. The results show that the proposed framework is highly effective in denoising multivariate signals and plant-wide oscillations. Additionally, the accuracy of the framework in extracting relevant information is evaluated by considering high noise levels. The results demonstrate that the proposed framework is robust and accurate even in the presence of high levels of noise.

The remainder of this paper is organised as follows: Sections 2 and 3 introduce the different techniques and methodologies employed in the proposed framework. Section 4 describes the MSSA-DFA algorithm. Section 5 presents the application results using simulated and industrial case studies. Finally, a conclusion is reported in section 6.

2. DETRENDED FLUCTUATION ANALYSIS

Detrended fluctuation analysis (DFA) is a method for estimating the long-term correlation properties of a time series. It measures how the variance of the time series scales with time and can be used to detect the presence of long-term correlations or trends in the data (Bryce and Sprague, 2012). The input time series with N observations $\mathbf{b}(i) \in \Re^N$ is transformed into an unbounded process $\mathbf{y}(t)$ called the signal profile, which is an integrated signal with a subtracted offset as:

$$\mathbf{y}(t) = \sum_{i=1}^{t} [\mathbf{b}(i) - \langle \mathbf{b} \rangle] \tag{1}$$

where $\langle \mathbf{b} \rangle$ denotes the signal's mean:

$$\langle \mathbf{b} \rangle = \frac{1}{N} \left(\sum_{i=1}^{N} \mathbf{b}(i) \right) \tag{2}$$

$\mathbf{y}(t)$ is then divided into overlapping segments of size Δw (a set of segment sizes $\Xi = \{\Delta w : w = 1, 2, \dots, P\}$ is selected for the analysis).

The integrated signal is then locally fitted to a polynomial in each segment to calculate the local trend $\mathbf{y}_{\Delta w}(t)$. Following this, the detrended fluctuation $F(\Delta w)$ is calculated for each segment Δw by subtracting the local trend $\mathbf{y}_{\Delta w}$ from the integrated time series $\mathbf{y}$ and calculating the root-mean-square (RMS) fluctuation over the segment:

$$F(\Delta w) = \sqrt{\frac{1}{\Delta w} \sum_{t=1}^{\Delta w} \left(\mathbf{y}(t) - \mathbf{y}_{\Delta w}(t) \right)^2} \tag{3}$$

The procedure is repeated over the whole time series and for all segment sizes $\Delta w \in \Xi$. Finally, the DFA exponent value α is computed by fitting a power-law function to the detrended fluctuation $F(\Delta w)$ as a function of the segment size Δw:

$$F(\Delta w) \propto \Delta w^{\alpha}$$

The acquired pairs of Δw and $F(\Delta w)$ are plotted on a log-log graph. A line is fitted to the linear part of this graph, where the DFA value is calculated as the slope of this straight line.

The DFA method is a useful tool for analysing the scaling properties of a time series and can be used to detect long-term correlations or trends that are not apparent from other methods, such as autocorrelation analysis. Moreover, DFA is relatively simple to implement and does not require a priori assumptions about the underlying model or parameters. A detailed procedure for calculating DFA can be found in Bounoua et al. (2023).

3. SINGULAR SPECTRUM ANALYSIS

Singular spectrum analysis (SSA) is an advanced and effective approach to analysing vector stochastic processes (Elsner and Tsonis, 1996). The original vector can be decomposed into independent and interpretable modes representing the oscillatory components, noise, and non-stationary trends if applicable (Zhang, 2018). SSA generally includes performing three steps (Golyandina et al., 2018): (1) Embedding in which a trajectory matrix is constructed through time-lagged replication of the 1-D time series under analysis; (2) Decomposition of the trajectory matrix into a sum of matrices of rank 1, which can be achieved through the singular value decomposition (SVD); (3) Reconstruction step where the original form of the input is reconstructed from groups of the decomposition step; this gives the different components that construct the original signal.

The embedding step involves constructing a trajectory matrix $\mathbf{X}$ from the original time series $x_1, x_2, \dots, x_N$. The trajectory matrix $\mathbf{X}$ is formed by taking a window of length M and sliding it along the time series, resulting in $\widetilde{N} = N - M + 1$ columns of the trajectory matrix:

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & \dots & x_{\widetilde{N}} \\ x_2 & x_3 & \dots & x_{\widetilde{N}+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_M & x_{M+1} & \dots & x_N \end{bmatrix} \tag{4}$$

The decomposition step involves computing the SVD of the trajectory matrix $\mathbf{X}$:

$$\mathbf{X} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T \tag{5}$$

where $\mathbf{U}$ is an $M \times M$ orthogonal matrix of left singular vectors, $\mathbf{\Sigma}$ is an $M \times \widetilde{N}$ diagonal matrix of singular values, and $\mathbf{V}$ is a $\widetilde{N} \times \widetilde{N}$ orthogonal matrix of right singular vectors.

The first r columns of $\mathbf{U}$ and the first r rows of $\mathbf{V}$ are used to form empirical orthogonal functions (EOFs). These EOFs are linear combinations of the original time series, which capture the most significant temporal patterns or trends in the data. The reconstruction step involves projecting the original time series $x_1, x_2, \dots, x_N$ onto the EOFs to obtain a set of reconstructed time series $\hat{x}_1, \hat{x}_2, \dots, \hat{x}_N$.

4. MULTIVARIATE SINGULAR SPECTRUM ANALYSIS BASED ON DFA PROCEDURE

MSSA is the multivariate extension of SSA that allows for the analysis of spatial and temporal correlations among several time series (Gruszczynska et al., 2018).

Let $\mathbf{X}(n) = (\mathbf{x}_1(n), \mathbf{x}_2(n), \ldots, \mathbf{x}_L(n))$ be the input data matrix with L variables and N samples where $\mathbf{x}_l(n) = \{\mathbf{x}_l(n)\}_{n=1:N} \in \Re^N$. MSSA is then performed through the following steps:

Step 1: The data matrix is embedded to construct the multivariate trajectory matrix by forming M-lagged duplicate of each time series in $\mathbf{X}(n)$ as:

$$\widetilde{\mathbf{X}}_l = \begin{pmatrix} \mathbf{x}_l(1) & \mathbf{x}_l(2) & \cdots & \mathbf{x}_l(\widetilde{N}) \\ \mathbf{x}_l(2) & \mathbf{x}_l(3) & \cdots & \mathbf{x}_l(\widetilde{N}+1) \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{x}_l(M) & \mathbf{x}_l(M+1) & \cdots & \mathbf{x}_l(N) \end{pmatrix} \in \Re^{M \times \widetilde{N}}, \tag{6}$$

for $(l = 1, \ldots, L)$ where M is an integer called window length that is selected as $1 < M < N/2$

The trajectory matrix of the multivariate data matrix is then given as follows:

$$\widetilde{\mathbf{X}} = \left(\widetilde{\mathbf{X}}_1, \widetilde{\mathbf{X}}_2, \ldots, \widetilde{\mathbf{X}}_L\right)^T \in \Re^{LM \times \widetilde{N}} \tag{7}$$

Step 2: Estimate the covariance matrix of the trajectory matrix:

$$\widetilde{\mathbf{C}}_{\widetilde{\mathbf{X}}} = \frac{1}{\widetilde{N}} \widetilde{\mathbf{X}} \widetilde{\mathbf{X}}^T \in \Re^{LM \times LM} \tag{8}$$

Step 3: Similar to SSA, the eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{LM} \geq 0$ and eigenvectors $\left\{\mathbf{e}^k\right\}_{k=1,\ldots,LM}$ are computed through the eigendecomposition of the covariance matrix. Each eigenvector $\mathbf{e}^k$ consists of L blocks, each block consisting of M entries associated with each of the L time series in $\mathbf{X}$:

$$\mathbf{e}^k = \left(e^k_{11}, \ldots, e^k_{1M}, e^k_{21}, \ldots, e^k_{2M}, \ldots, e^k_{L1}, \ldots, e^k_{LM}\right)^T \tag{9}$$

Step 4: The principal components $\left\{\mathbf{p}^k\right\}_{k=1,\ldots,LM}$ are computed by projecting the trajectory matrix onto the eigenvectors as follows:

$$\mathbf{p}^k(n) = \left(\widetilde{\mathbf{X}}^{(n)}, \mathbf{e}^k\right) = \sum_{m=1}^{M} \sum_{l=1}^{L} \mathbf{x}_l(n+m-1) e^k_{lm} \tag{10}$$

where $n = 1, \ldots, N-M+1$

Step 5: Compute the reconstructed components (RCs). The k-th RC at n for variable l is defined as:

$$R^k_l(n) = \begin{cases} \dfrac{1}{M} \displaystyle\sum_{j=1}^{M} \mathbf{p}^k(n-j+1)\mathbf{e}^k_l(j) & \text{for} \quad M \leq n \leq N-M+1 \\ \dfrac{1}{n} \displaystyle\sum_{j=1}^{n} \mathbf{p}^k(n-j+1)\mathbf{e}^k_l(j) & \text{for} \quad 1 \leq n < M-1 \\ \dfrac{1}{N-n+1} \displaystyle\sum_{j=n-N+M}^{M} \mathbf{p}^k(n-j+1)\mathbf{e}^k_l(j) & \text{for} \quad N-M+2 \leq n \leq N \end{cases} \tag{11}$$

Step 6: At this stage, the DFA of the reconstructed components is computed individually in order to decide what RCs are kept for the denoising process. Only S RCs that are significant in terms of DFA are kept, i.e. with DFA$>$ 1. This is repeated for every variable $l \in \{1, 2, \cdots, L\}$.

Step 7: Finally, summing up the S reconstructed components gives the denoised version of each variable.

5. APPLICATION

In this section, a numerical example and a real industrial process will be used to demonstrate the efficiency of the suggested approach.

5.1 Numerical case study

A simulated multivariate signals case is used first to affirm the efficiency of the proposed framework. The data consists of three signals oscillating at multiple frequencies. The first signal oscillates at a single frequency of 0.1 rad/s, the second signal simulates oscillations due to nonlinearity in the process at a fundamental frequency of 0.1 rad/s and a harmonic at 0.3 rad/s, and the third signal contains oscillations at three frequencies 0.1, 0.3, and 0.4 rad/s. The three signals were corrupted by a coloured noise component generated by filtering white noise using a filter with a transfer function $H(z) = \frac{1-0.2z^1}{1-0.1z^1+0.8z^2}$.

The proposed method was implemented for this multivariate data set. The upper panel of Fig. 1 shows the original noisy signals where oscillations are obscured by noise, especially for the multiple frequencies case studies. First, a window length $M = 30$ was selected in this study to obtain the covariance matrix, eigenvectors, and PCs. For this window size, a total of 90 variables are now contained in the trajectory matrix $\widetilde{\mathbf{X}}$. A visual depiction of the covariance matrix computed through Eq.(5) for the augmented matrix is presented in Fig. 2.

Based on this covariance matrix, $LM = 90$ eigenvectors were extracted in order to compute the PCs. At this step, the number of PCs to keep for further analysis is selected using the correlation dimension proposed in Bounoua and Bakdi (2021). This is a nonlinear technique that can provide important information about the underlying structure of the data and extract the number of principal components. Amongst the 90 original PCs, only 16 PCs were chosen. This means ensuring that the residuals that reflect the noise are excluded and that only significant PCs are retained. The first four PCs that were selected are shown in Fig.3.

Accordingly, 16 RCs were obtained for each of the three variables. At this stage, DFA was employed to determine which components to keep so that only components having

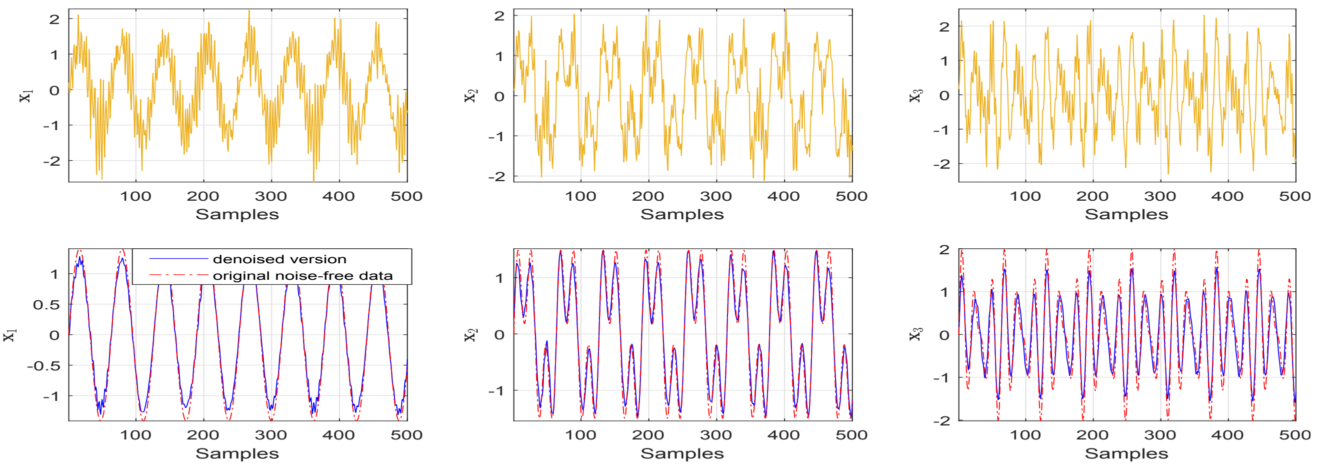

Fig. 1. Numerical case study: the multivariate original noisy data (upper panel) and the denoised version using MSSA-DFA compared with the original noise-free oscillatory signals (lower panel)

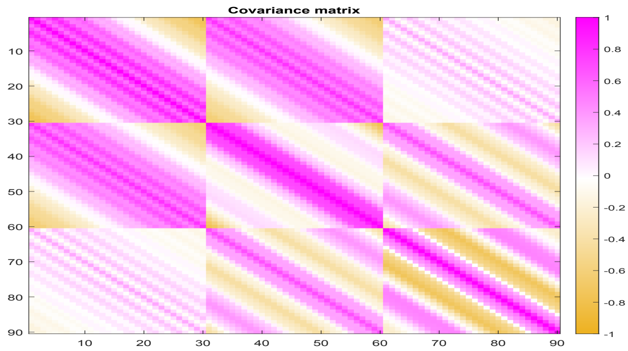

Fig. 2. The covariance matrix of the augmented data set of the numerical case study.

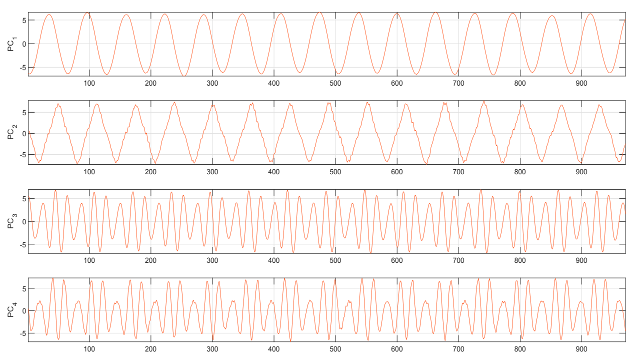

Fig. 3. The first four PCs derived from MSSA-DFA framework for the numerical case study.

significant variations are selected i.e., having DFA$>$ 1. From the 16 RCs, only 6 components are kept and summed up to reconstruct the denoised version of the signals under analysis. The lower panel of Fig.1 illustrates the results obtained from applying the MSSA-DFA proposed method. The results demonstrate the high proficiency of the suggested method in the batch processing of the multivariate data and extracting only the meaningful oscillatory data. This processing method performed adequately for all three types of oscillations, including linear and nonlinear process oscillations with multiple frequencies.

5.2 Industrial case study

To further confirm the validity of the proposed scheme, a multivariate data set from an industry case study will be employed. The data was collected from the Eastman Chemical Company plant, which is a well-known process consisting of five major units: three distillation columns and two decanters, as well as several recycle streams. The plant encompasses 15 control loops and 30 plant tags, making it a complex multivariate system.

For 48 operational hours, the data was sampled every 20 seconds, yielding 8640 samples per tag. The dataset contains information on various process variables, including temperature, pressure, flow rate, and level, as well as the corresponding controller outputs.

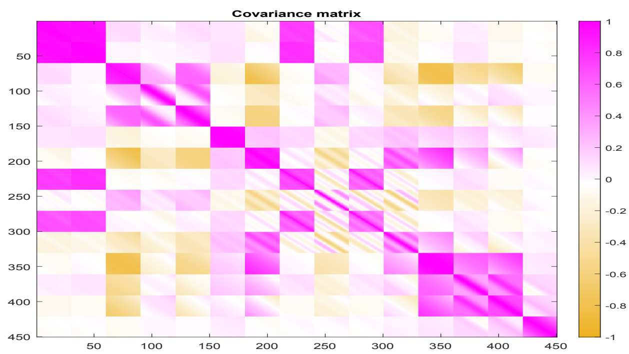

Fig. 4. The covariance matrix of the augmented data set for the industrial case study.

The Eastman Chemical Company dataset is a valuable resource for evaluating the efficacy of the proposed framework and validating its suitability for real-world industrial applications. The dataset exhibits various characteristics that are typical of industrial control systems, such as noisy signals, complex multivariate dynamics, and nonlinear behaviours, among others. The process is well described in Thornhill et al. (2003), and the data is publicly available in Bauer et al. (2018). The data set comprises PV, OP, set points, and error signals. PV and OP are mean-centred and normalised to unit standard deviation. In this study, the OP data set was used for the analysis, consisting of 15 signals.

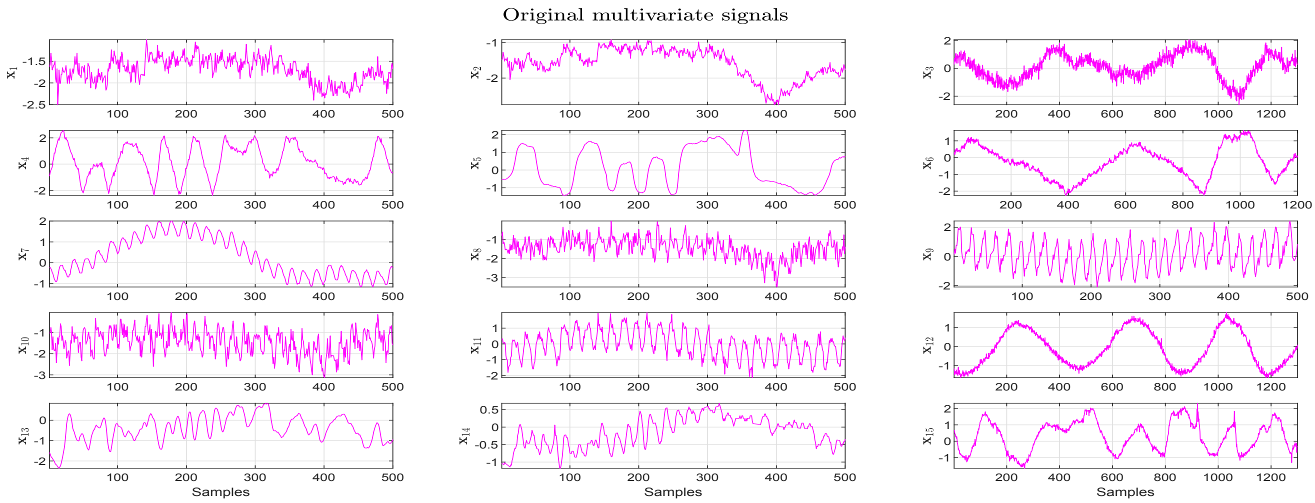

Fig. 5. The original multivariate control loops signals of the industrial data set.

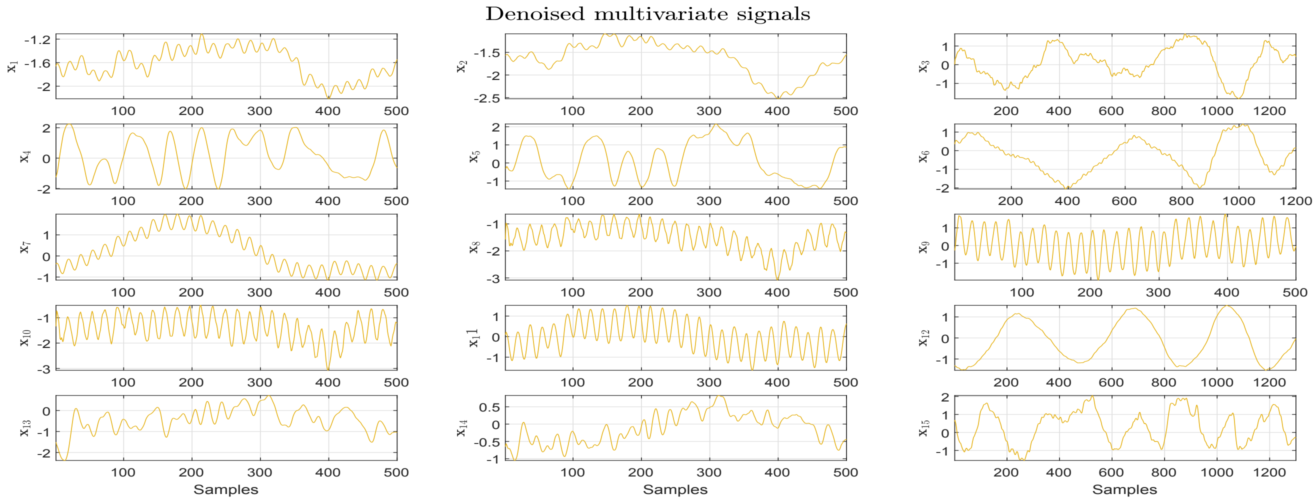

Fig. 6. The denoised version of the plant-wide industrial data set.

In the given data, it is evident that the process is undergoing a plant-wide oscillation. The PVs and OPs exhibit distinct patterns that reflect the oscillatory behaviour of the system. These patterns can be characterised by their frequency, amplitude, and phase, among other parameters. Therefore, it is essential to denoise in such cases to identify these patterns. Similar to the numerical case study, the procedure in Section 4. was applied to this data with the same parameters, including the window size and the threshold of DFA. In this case, $LM = 450$ variables are included in the trajectory matrix. Fig.4 illustrates the obtained covariance matrix.

The eigendecomposition of this matrix produced 450 eigenvectors and, therefore, 450 PCs were derived. As a result of applying the correlation dimension, only 91 PCs were used to obtain the reconstructed components for each of the 15 signals under investigation. Ensuing this, DFA values were calculated for the RCs, and 43 out of 91 RCs were used to obtain the noise-free signals. This study used the proposed framework to analyse a dataset comprising 8640 observations. The entire range of observations was used to implement the proposed framework, which affects the efficiency of advanced signal processing techniques to extract relevant information from the data and identify key features. However, for visualisation purposes, a small sample size will be demonstrated, as shown in Fig.5. The sample size was carefully selected to preserve the key features of the data. The resulting denoised version of this case study is depicted in Fig.6. As can be observed from the results, the oscillations in the signal are considerably more distinct, and the generated variables are far smoother than the original ones. This confirms the efficacy of the MSSA-DFA framework in dealing with different types of signals and varying levels of noise, regardless of the number of signals treated simultaneously. The MSSA-DFA framework is particularly useful in industrial control systems, where noisy signals are prevalent and can significantly impact the accuracy of control performance monitoring. This method can be compared to other methods, such as deep learning techniques, which could be done in the future.

6. CONCLUSIONS

The proposed strategy utilizes MSSA method to obtain a smoothed version of the multivariate data from industrial control loops. To enhance the denoising performance, DFA is incorporated to prevent the addition of any erratic components. DFA ensures that only significant reconstructed components are included in the computations of the reconstructed components.

The efficacy of the proposed method is demonstrated through a numerical case study that includes three signals exhibiting oscillations from a linear and a nonlinear process with multiple frequencies corrupted by coloured noise. The results show that the proposed MSSA-DFA approach can effectively reduce the impact of noise and extract relevant information from complex multivariate signals. Moreover, real multivariate data from the industrial Eastman Chemical plant is used to test the suggested MSSA-DFA approach in denoising the data under various conditions.

The proposed method can have considerable implications for industrial control systems, where the presence of noisy signals can significantly impact the accuracy of control performance monitoring techniques. By incorporating appropriate signal preprocessing techniques, such as the MSSA-DFA approach, it is possible to enhance data set quality and the accuracy and effectiveness of advanced processing tools, thus improving the performance of industrial control systems.

REFERENCES

Aftab, M.F., Hovd, M., and Sivalingam, S. (2017a). Convergent cross mapping (CCM) based approach for isolating the source of plant-wide disturbances. In *2017 IEEE Conference on Control Technology and Applications (CCTA)*, 1492–1498. IEEE, Mauna Lani Resort, HI, USA. doi:10.1109/CCTA.2017.8062669.

Aftab, M.F., Hovd, M., and Sivalingam, S. (2017b). Detecting non-linearity induced oscillations via the dyadic filter bank property of multivariate empirical mode decomposition. *Journal of Process Control*, 60, 68–81. doi:10.1016/j.jprocont.2017.08.005.

Aftab, M.F., Hovd, M., and Sivalingam, S. (2018a). Improved oscillation detection via noise-assisted data analysis. *Control Engineering Practice*, 81, 162–171. doi:10.1016/j.conengprac.2018.08.019.

Aftab, M.F., Hovd, M., and Sivalingam, S. (2018b). Plant-wide oscillation detection using multivariate empirical mode decomposition. *Computers & Chemical Engineering*, 117, 320–330. doi:10.1016/j.compchemeng.2018.06.007.

Aftab, M.F., Hovd, M., and Sivalingam, S. (2019). Diagnosis of plant-wide oscillations by combining multivariate empirical mode decomposition and delay vector variance. *Journal of Process Control*, 83, 177–186. doi:10.1016/j.jprocont.2019.01.001.

Bauer, M., Auret, L., Roux, D.l., and Aharonson, V. (2018). An Industrial PID Data Repository for Control Loop Performance Monitoring (CPM). *IFAC-PapersOnLine*, 51(4), 823–828. doi:10.1016/j.ifacol.2018.06.173. Number: 4.

Bounoua, W., Aftab, M.F., and Omlin, C.W.P. (2022). Controller Performance Monitoring: A Survey of Problems and a Review of Approaches from a Data-Driven Perspective with a Focus on Oscillations Detection and Diagnosis. *Industrial & Engineering Chemistry Research*, acs.iecr.2c02785. doi:10.1021/acs.iecr.2c02785.

Bounoua, W., Aftab, M.F., and Omlin, C.W.P. (2023). Online detrended fluctuation analysis and improved empirical wavelet transform for real-time oscillations detection in industrial control loops. *Computers & Chemical Engineering*, 108173. doi:10.1016/j.compchemeng.2023.108173.

Bounoua, W. and Bakdi, A. (2021). Fault detection and diagnosis of nonlinear dynamical processes through correlation dimension and fractal analysis based dynamic kernel PCA. *Chemical Engineering Science*, 229, 116099. doi:10.1016/j.ces.2020.116099.

Bryce, R.M. and Sprague, K.B. (2012). Revisiting detrended fluctuation analysis. *Scientific Reports*, 2(1), 315. doi:10.1038/srep00315.

Chen, C., He, B., Lang, X., Li, P., Zhang, Y., and Xie, L. (2021). Denoising of Industrial Oscillation Data Based on EEMD-DFA-CCA. In *2021 China Automation Congress (CAC)*, 4173–4178. IEEE, Beijing, China. doi:10.1109/CAC53003.2021.9727385.

Elsner, J.B. and Tsonis, A.A. (1996). *Singular Spectrum Analysis*. Springer US, Boston, MA. doi:10.1007/978-1-4757-2514-8.

Golyandina, N., Korobeynikov, A., and Zhigljavsky, A. (2018). *Singular Spectrum Analysis with R*. Use R! Springer Berlin Heidelberg, Berlin, Heidelberg. doi:10.1007/978-3-662-57380-8.

Gruszczynska, M., Rosat, S., Klos, A., Gruszczynski, M., and Bogusz, J. (2018). Multichannel Singular Spectrum Analysis in the Estimates of Common Environmental Effects Affecting GPS Observations. *Pure and Applied Geophysics*, 175(5), 1805–1822. doi:10.1007/s00024-018-1814-0.

Jelali, M. and Huang, B. (eds.) (2010). *Detection and Diagnosis of Stiction in Control Loops*. Advances in Industrial Control. Springer London, London. doi:10.1007/978-1-84882-775-2.

Lang, X., Liu, Y., Zhang, Y., Xie, L., Horch, A., and Su, H. (2020). Denoising of Industrial Oscillation Data Using EEMD with CCA. *IFAC-PapersOnLine*, 53(2), 11662–11668. doi:10.1016/j.ifacol.2020.12.655.

Lang, X., Zhang, Y., Xie, L., Li, P., Horch, A., and Su, H. (2022). Detrending and Denoising of Industrial Oscillation Data. *IEEE Transactions on Industrial Informatics*, 1–11. doi:10.1109/TII.2022.3188844.

Rengaswamy, R., Srinivasan, B., and Bhatt, N.P. (2020). *Process Control Fundamentals: Analysis, Design, Assessment, and Diagnosis*. CRC Press, First edition. | Boca Raton : CRC Press, 2020., 1 edition. doi:10.1201/9780367433437.

Srinivasan, R., Rengaswamy, R., and Miller, R. (2007). A modified empirical mode decomposition (EMD) process for oscillation characterization in control loops. *Control Engineering Practice*, 15(9), 1135–1148. doi:10.1016/j.conengprac.2007.01.014. Number: 9.

Thornhill, N.F., Cox, J.W., and Paulonis, M.A. (2003). Diagnosis of plant-wide oscillation through data-driven analysis and process understanding. *Control Engineering Practice*, 11(12), 1481–1490. doi:10.1016/S0967-0661(03)00108-4.

Zhang, Z. (2018). *Multivariate Time Series Analysis in Climate and Environmental Research*. Springer International Publishing, Cham. doi:10.1007/978-3-319-67340-0.