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Energy Efficient Temperature and Humidity Control in Building Climate Systems

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Abstract: Energy efficient building climate control involves maintaining thermal comfort across a wide range of environmental conditions while minimizing energy usage. However, the design of energy efficient control poses a significant challenge owing to the strong coupling between temperature and humidity. In this work, we present a control oriented model for the heating, ventilation and air conditioning (HVAC) system and provide a polytopic approximation of thermal comfort in terms of temperature and humidity ratio. A novel energy optimal control formulation based on generalized disjunctive programming is proposed to systematically account for the strong coupling effects and latent heat consideration. An extensive simulation study is performed to validate the efficcacy of the proposed control strategy across a wide range of operational and weather conditions.

Keywords: HVAC Control, Generalized Disjunctive Programming, Hybrid MPC, Energy Efficiency.

1. INTRODUCTION

Energy efficient building climate control centers on the task of maintaining thermal comfort while utilizing the minimum amount of energy in response to time varying heat load and weather conditions. While thermal comfort depends on several factors, temperature and humidity play a key role in maintaining the comfort and well-being of the occupants (Ramspeck et al. (2004)). HVAC systems are usually employed to maintain the indoor environment to ensure the occupant's thermal comfort. HVAC systems are inherently complex, nonlinear, and over-actuated in nature, necessitating the use of advanced control strategies for their reliable and energy-efficient operation.

In recent years, MPC has emerged as an effective control mechanism to enable energy-efficient temperature control of buildings (see Kelman and Borrelli (2011); Ma et al. (2012); Ghawash et al. (2022)). MPC provides a systematic framework to handle complex nonlinear system dynamics and incorporate time varying energy prices and weather predictions. However, the predominant focus of the existing research pertains to temperature control with limited attention directed towards incorporating humidity dynamics and latent heat consideration within the control framework. The inability to explicitly incorporate humidity within the problem formulation can adversely affect the occupant's thermal comfort. The inadequate moisture regulation may foster mold growth and related concerns. Additionally, the failure to account for the latent heat can lead to suboptimal operation of HVAC equipment, thereby increasing the actual operational cost of the overall system (Raman et al. (2019)).

The strong coupling between temperature and humidity poses a significant challenge in achieving energy optimal control using conventional control approaches. Among the limited works that factor in humidity, the temperature and humidity deficit control for the greenhouse have been studied for maintaining optimum conditions for plant growth by Ito and Tabei (2021). The work mainly focuses on humidity deficit control and does not account for the dehumidification process which significantly simplifies the problem formulation. The work involving the empirical model of the cooling coil based on the data obtained from EnergyPlus has also been used to design the energy optimal control for HVAC system (Raman et al. (2019)). The validity of the empirical model in a small operational region and the requirement of additional sensors restrict its usability in practical applications. Similarly, an identified state space model with on-off actuators has also been investigated for the control design of the climate chamber (Dostál and Ferkl (2014)). Moreover, temperature and humidity control have also been investigated in the literature with different simplifying assumptions on the relative humidity of the conditioned air as in Goyal and Barooah (2013); Goyal et al. (2013). Despite these recent advancements in temperature and humidity control, a control framework is required to systematically account for the coupling effects and latent heat considerations. Such a control framework must exhibit the necessary flexibility to be adopted for a broad range of climate conditions with readily available sensor measurements.

In this work, we present a control oriented model for zone dynamics and different components of the HVAC system. A steady state hybrid model for the cooling coil is proposed which accounts for the coupling between temperature and humidity. An energy optimal control framework based on generalized disjunctive programming (GDP) is presented to handle the hybrid nature of the problem. The proposed framework incorporates the power consumption model of different actuators into the objective function while also accounting for the latent heat considerations. Simulations are provided under different weather conditions to evaluate the effectiveness of the proposed approach across a wide range of operating conditions.

The rest of the paper is organized as follows: Section 2 provides a brief overview of the working principle of the HVAC system and details the temperature and humidity modeling of the zone and different components of the HVAC system. Section 3 discusses the polytopic approximation of thermal comfort and presents the proposed energy optimal formulation based on the GDP framework. Section 4 provides the simulation results for four different operating regions to show the efficacy of the proposed approach. Section 5 concludes the paper and highlights the future directions.

2. HEATING VENTILATION AND AIR CONDITIONING (HVAC) SYSTEM

HVAC systems are widely employed to maintain the occupant's thermal comfort within single and multi-zone setups. An Air Handling Unit (AHU) is a key component of an HVAC system. The AHU is responsible for treating and conditioning the air before it is distributed throughout the space/zone. Constant Air Volume (CAV) and Variable Air Volume (VAV) are among the popular AHU configurations. CAV AHUs keep the mass flow rate constant and mainly rely on conditioning the air whereas the VAV AHUs vary the mass flow rate and condition the supply air to achieve the desired thermal comfort. For simplicity of exposition, we primarily focus on the VAV systems with a single-zone setup, although the developments can be extended to multi-zone setups. An AHU configuration is considered that can perform several functions like mixing outdoor and recirculated air, cooling, cooling and dehumidification, humidification, and heating to maintain the occupant's thermal comfort. The main components of an AHU include dampers (outside, return air and exhaust air), a filter, a mixing chamber, a cooling coil, a steam humidifier, a heater, and a fan as illustrated in Fig.1. In some AHU designs a preheating coil is also installed before the cooling coil which primarily provides protection against frost.

The AHU draws in the ambient air from the outside or recirculates indoor air using the outside and return air dampers. A portion of the recirculated indoor air may be mixed with fresh outdoor air which helps maintain the desired thermal comfort in an energy efficient manner. A minimum fresh outdoor air supply is always maintained to dilute indoor pollutants and maintain oxygen levels. The mixed air passes through a filter that removes dust, pollen, allergens, and other particles. The mixed air may need to be either cooled or heated depending on the desired indoor temperature requirement. This is achieved through the use of cooling and heating coils. The AHU can also adjust the humidity levels in the air. In humidification, moisture

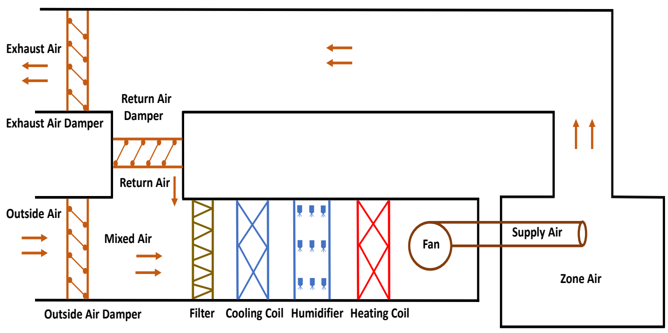


Fig. 1. A generic AHU configuration used for maintaining the temperature and humidity of the zone.

is added to the dry air, while in dehumidification, excess moisture is removed from the humid air. This process is crucial for maintaining a comfortable thermal environment and preventing issues like mold growth. It must be noted that dehumidification is achieved by cooling the air below the dewpoint using the cooling coil. When dehumidification occurs, the cooled air usually needs to be reheated to avoid thermal discomfort. The AHU also houses a fan that pushes the supply/conditioned air through the ducts. The fan's speed can be adjusted to control the airflow rate.

2.1 Modelling of Zone and Air Handling Unit

Next, we present control orientated models of the temperature and humidity of the zone and different components of an AHU based on mass and energy balances.

Zone Model: The zone air (ZA) temperature and humidity primarily depend on the temperature and humidity of the supply air (SA) from AHU and sensible and latent heat load. The heat transfer taking place through external and internal walls as well as the solar thermal gain affects the ZA temperature. The continuously operated electrical equipment contributes a constant sensible heat gain, whereas the occupants, on-off equipment, and infiltration losses result in time varying sensible and latent heat load. Assuming the air in the zone is fully mixed and the density of the air remains constant, the zone energy balance leads to (1). Similar models have also been reported by Ma et al. (2012); Kelman and Borrelli (2011); Raman et al. (2019).

$$C_z \frac{dT_{za}}{dt} = \dot{m}_{sa} C_{pa} T_{sa} - \dot{m}_{sa} C_{pa} T_{za} + \alpha(T_{oa} - T_{za}) + s(t) + A_e \eta_{sol}(t) \tag{1}$$

Here T_{za}, T_{sa}, T_{oa} are the zone, supply and outside air temperature respectively. C_{pa} is the specific heat capacity of air, α is the heat transfer coefficient and $s(t)$ represents the time varying sensible heat load. $\eta_{sol}(t)$ is the solar thermal gain and A_e represents the effective area of the zone. Similarly, the humidity ratio dynamics can be written as follows:

$$C_h \frac{dW_{za}}{dt} = \dot{m}_{sa} W_{sa} - \dot{m}_{sa} W_{za} + h(t). \tag{2}$$

Here, $h(t)$ represents the time varying latent heat load. W_{za}, W_{sa}, W_{oa} are the zone, supply, and outside air humidity ratios respectively. C_z and C_h are the empirical parameters of the model. Zone air and return air (RA) are used interchangeably throughout the description.

Dampers and Mixing Chamber Model: The mixing of the outside air (OA) and RA is achieved by modulating the openings of AHU dampers. The mixing of the OA and RA streams in correct proportions is necessary to regulate the mixed air temperature and humidity. An appropriate OA/RA mixing ratio can reduce or completely eliminate the active use of other actuators resulting in considerable reduction in the operating cost of the AHU. Under steady conditions, matched damper configuration and adiabatic mixing with no condensation, the mixed air (MA) temperature is given by:

$$T_{ma} = \frac{\Phi(d_{oa})\dot{m}_{sa}T_{oa} + (1-\Phi(d_{oa}))\dot{m}_{sa}T_{ra}}{\dot{m}_{sa}}. \tag{3}$$

Similarly, the MA humidity ratio can be rewritten as follows:

$$W_{ma} = \frac{\Phi(d_{oa})\dot{m}_{sa}W_{oa} + (1-\Phi(d_{oa}))\dot{m}_{sa}W_{ra}}{\dot{m}_{sa}} \tag{4}$$

where $\Phi(d_{oa})$ represents the installed flow characteristics and d_{oa} is the opening of the OA dampers. Note that Φ and $1-\Phi$ are the fraction of the mass flow rate of SA contributed by the OA and RA dampers respectively. For simplicity of exposition, we consider the case of linear flow characteristics (installed) of the dampers (i.e. $\Phi(d_{oa}) = d_{oa}$). More details on the assumptions and the case for the nonlinear flow characteristics can be found in the work of Ghawash et al. (2022). Finally, we note that the power consumption associated with dampers' motors is negligible as compared to power consumption of other components of AHU.

Cooling/Heating Coil Model: Several models with different levels of complexity are available in the literature to model the behavior of cooling and heating coils (see Zhou and Braun (2007a,b); Crawley et al. (2000)). For control purposes, a simplified static model based on inequality constraints can be adopted that ensures the cooled air (CA) temperature lies within the range of the incoming air temperature and a minimum achievable temperature (representing the cooling coil's capacity).

$$T_{ca}^{min} \leq T_{ca} \leq T_{ma} \tag{5}$$

Here, T_{ca}^{min} is the minimum achievable cooled air temperature. The cooling coil can be operated in cooling only and cooling and dehumidification modes depending on the CA temperature and dewpoint point temperature of the MA. When the CA temperature is above the dewpoint temperature of the MA, the humidity ratio remains unaffected. However, when the CA temperature goes below the dewpoint temperature of the MA, dehumidification takes place. In order to compute the humidity ratio of the dehumidified air, first we consider the relationship between humidity ratio ($W_{dehum}[kg/kg]$) and specific humidity ($q_{dehum}[kg/kg]$) which can be expressed as follows [1]:

$$W_{dehum} = \frac{q_{dehum}}{1-q_{dehum}} \tag{6}$$

where the q_{dehum} can be obtained using the relationship described in Hanzer et al. (2018).

$$q_{dehum} = \frac{0.622p_s(T_{ca})}{p - 0.378p_s(T_{ca})} \tag{7}$$

[1] Note that $W = \frac{m_w}{m_d}$ and $q = \frac{m_w}{m_d+m_w}$. m_w and m_d represents mass of the water and dry air respectively.

where $p_s(T_{ca}) = 6.112 \times e^{\frac{17.67T_{ca}}{T_{ca}+243.5}}$ is the saturation vapour pressure and $p = 101.325$ kPa is the atomospheric pressure. It must be noted that the temperature and humidity are strongly coupled in the cooling and dehumidification mode and can not be chosen independently. Finally, the model of the cooling coil contains both operational modes with a conditional relationship between cooled and dewpoint temperatures governing the activation of these modes. Such a hybrid model of the cooling coil can be written as follows.

$$W_{ca} = \begin{cases} W_{ma} & T_{ca} \geq T_{dp} \\ W_{dehum} & T_{ca} \leq T_{dp} \end{cases} \tag{8}$$

where T_{dp} represents the air dewpoint temperature. The model ensures that humidity remains unaffected when the cooled air temperature is above the dew point temperature. However, when the cooled air temperature becomes lower than the dew point, the humidity is updated using (6). The power consumption associated with the cooling coil is given by

$$P_{cc} = \frac{\dot{m}_{sa}C_{pa}(T_{ma}-T_{ca})}{\eta_{cc}COP_{cc}} + \frac{\dot{m}_{sa}h_{wv}(W_{ca}-W_{ma})}{\eta_{cc}COP_{cc}} \tag{9}$$

where η_{cc} is the efficiency of the cooling coil and COP_{cc} represents the coefficient of performance of the cooling plant. Moreover, h_{wv} represents the latent heat of the vaporization of water. Finally, we note that dew point air temperature can be easily calculated as a function of temperature and relative humidity of the mixed air ($T_{dp} = \gamma(T_{ma}, \psi_{ma})$) as discussed in Lawrence (2005). Similarly, a simple model for the heating coil is given as follows:

$$T_{ca} \leq T_{ha} \leq T_{ha}^{max} \tag{10}$$

Here, T_{ha}^{max} is the maximum achievable heated air temperature. Note that the humidity ratio remains unaffected by the increase in air temperature (i.e $W_{ha} = W_{ca}$). Moreover, the power consumption associated with the heating coil is given by:

$$P_{hc} = \frac{\dot{m}_{sa}C_{pa}(T_{ha}-T_{ca})}{\eta_{hc}COP_{hc}} \tag{11}$$

where η_{hc} is the efficiency of the heating coil and COP_{hc} represents the coefficient of performance of the heating plant. The heating coil is primarily used to regulate the temperature of zone air. However, when dehumidification occurs, the cooled air is usually required to be reheated to avoid thermal discomfort.

Steam Humidification: Steam humidification systems operate by heating water to produce steam, which is subsequently introduced into the air using dispersion nozzles. The steam quickly vaporizes upon contact with the surrounding air, raising the humidity levels. One advantage of steam humidification is its ability to provide precise humidity control, as it can add moisture to the air without causing significant temperature fluctuations (see Jo et al. (2017)). Hence, a simplified model for the isothermal process of steam humidification is defined based on inequality constraints.

$$W_{ca} \leq W_{hum} \leq W_{hum}^{max}. \tag{12}$$

Here, W_{hum}^{max} is the maximum achievable humidification level of the air. The power consumption associated with the humidification process is approximately given by:

$$P_{hum} = \frac{\dot{m}_{sa} h_{wv} (W_{hum} - W_{ca})}{\eta_{hum}} \quad (13)$$

where η_{hum} is the efficiency of the humidification process.

Fan Heat Loss Model: As the air passes through the fan motor, a slight increase in the temperature takes place mainly due to the heat losses (β) of the fan motor ($T_{sa} = T_{ha} + \frac{\beta P_f(m_{sa})}{\dot{m}_{sa} C_{pa}}$). On the other hand, the air humidity ratio remains unchanged (i.e $W_{sa} = W_{hum}$). Finally, a fairly accurate approximation of fan power consumption can be obtained using a quadratic polynomial.

$$P_f(m_{sa}) = k_0 m_{sa}^2 + k_1 m_{sa} + k_2 \quad (14)$$

3. ENERGY OPTIMAL CONTROL FORMULATION

3.1 Objectives and Constraints

Our main objective is to maintain thermal comfort while minimizing the total energy usage of the actuators. Thermal comfort is the state in which individuals feel neither excessively hot nor overly cold. Although thermal comfort depends on several factors, temperature and humidity regulation plays a key role in maintaining the comfort and occupants' well-being. The thermal comfort is typically described in terms of acceptable temperature and relative humidity ranges (e.g $\Gamma = \{T_{za}^{min} \leq T_{za} \leq T_{za}^{max}, \psi_{za}^{min} \leq \psi_{za} \leq \psi_{za}^{max}\}$). A polytopic approximation of the thermal comfort envelope in terms of temperature and humidity ratio can be obtained as follows:

$$\begin{aligned} &T_{za}^{min} \leq T_{za} \leq T_{za}^{max} \\ &W_{za} \geq \gamma_l (T_{za} - T_{za}^{min}) + \mathcal{F}(T_{za}^{min}, \psi_{za}^{min}) \\ &W_{za} \leq \gamma_u (T_{za} - T_{za}^{min}) + \mathcal{F}(T_{za}^{max}, \psi_{za}^{max}). \end{aligned} \quad (15)$$

Here $\gamma_l = \frac{\mathcal{F}(T_{za}^{max}, \psi_{za}^{min}) - \mathcal{F}(T_{za}^{min}, \psi_{za}^{min})}{T_{za}^{max} - T_{za}^{min}}$ and $\gamma_u = \frac{\mathcal{F}(T_{za}^{max}, \psi_{za}^{max}) - \mathcal{F}(T_{za}^{min}, \psi_{za}^{max})}{T_{za}^{max} - T_{za}^{min}}$. $\mathcal{F}(.,.)$ represents the transformation between relative humidity and humidity ratio. Fig. 2 shows the approximate polytopic thermal comfort envelope on the psychometric chart.

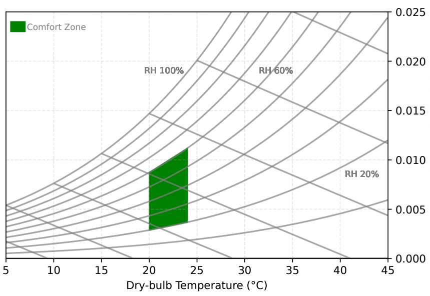


Fig. 2. Polytopic approximation of the thermal comfort envelope $\Gamma = \{20°C \leq T_{za} \leq 24°C,\ 20\% \leq \psi_{za} \leq 60\%\}$.

In addition to maintaining thermal comfort, the control strategy must adhere to the physical and operational constraints. A minimum fraction of the OA must be maintained to ensure good indoor air quality. The OA is typically fresher and cleaner than the ZA and helps to reduce the concentration of pollutants (volatile organic compounds, carbon monoxide, and nitrogen dioxide). The maximum and minimum ventilation rate constraints must also be respected. Moreover, the physical limits of different actuators must be respected.

3.2 Generlaized Disjunctive Programming (GDP)

In order to handle the hybrid nature of the cooling coil, GDP is used to formulate the energy optimal control for the HVAC system. GDP offers a high-level modeling framework for problems involving continuous and discrete decision decisions (see Grossmann and Trespalacios (2013)). In GDP, a logic representation is adopted to represent the problem where mixed integer logic is expressed through disjunctions and integer logic through propositions. Such a logical structure of the problem can be represented as follows:

$$\min_{\xi} \quad J(\theta) \quad (16a)$$

$$\text{s.t. } g(\theta) \leq 0 \quad (16b)$$

$$\veebar\, i \in \Sigma_j \begin{bmatrix} Y_j^i \\ z_j^i(\theta) \leq 0 \end{bmatrix}, \ \forall j \in \mathcal{P} \quad (16c)$$

$$\Omega(Y) = True \quad (16d)$$

$$\theta \in \mathbb{R}^n, Y_j^i \in \{True, False\}. \quad (16e)$$

Here, θ and Y_j^i represent the continuous and boolean variables respectively. $J(\theta)$ represents the objective function and $g(\theta) \leq 0$ represents the global constraints that must hold true irrespective of the discrete decisions. In the continuous space, the logical structure is represented through a set of disjunctions $j \in \mathcal{P}$, where each disjunction comprises $|\Sigma_j|$ terms linked by the XOR operator ($\veebar$). Each term in the disjunction consists of a boolean variable, associated with a set of inequalities. When a term is activated within a disjunction ($Y_j^i = True$), the corresponding inequalities are enforced. Conversely, when a term is inactive ($Y_j^i = False$), the corresponding constraints are disregarded. The XOR operator ensures that only one term is activated at any given time instance. The symbolic equation ($\Omega(Y) = True$) denotes the logical propositions that establish the relationship among the boolean variables.

To represent the energy optimal control in terms of GDP, the objective function must encompass the information on the power consumption of different actuators.

$$\begin{aligned} P_{ahu} = \sum_{k=1}^{N} r^k (P_f(m_{sa}^{k-1}) + P_{cc}(T_{ca}^k, W_{ca}^k) \\ + P_{hum}(W_{sa}^k) + P_{hc}(T_{ha}^k)) \end{aligned} \quad (17)$$

where r^k represents the time varying electrical energy prices. The global constraints include the zone's temperature and humidity dynamics and simplified models for different AHU components. The global constraints also include the physical limits of actuators. For all $k \in \{1, \cdots, N\}$,

$$T_{ma}^k = f_{ma}^{temp}(T_{za}^{k-1}, T_{oa}^{k-1}, d_{oa}^{k-1}) \quad (18a)$$

$$W_{ma}^k = f_{ma}^{hum}(W_{za}^{k-1}, W_{oa}^{k-1}, d_{oa}^{k-1}) \quad (18b)$$

$$T_{ca}^{min} \leq T_{ca}^k \leq T_{ma}^k,\ T_{ca}^k \leq T_{ha}^k \leq T_{ha}^{max} \quad (18c)$$

$$T_{sa}^k = f^{fan}(T_{ha}^k),\ T_{sa}^{min} \leq T_{sa}^k \leq T_{sa}^{max} \quad (18d)$$

$$W_{ca}^k \leq W_{sa}^k \leq W_{hum}^{max} \quad (18e)$$

$$T_{za}^k = f_{za}^{temp}(T_{za}^{k-1}, T_{sa}^k, T_{oa}^{k-1}, m_{sa}^{k-1}) \quad (18f)$$

$$W_{za}^k = f_{za}^{hum}(W_{za}^{k-1}, W_{hum}^k, m_{sa}^{k-1}) \quad (18g)$$

$$d_{oa}^{min} \leq d_{oa}^{k-1} \leq d_{oa}^{max},\ m_{sa}^{min} \leq m_{sa}^{k-1} \leq m_{sa}^{max} \quad (18h)$$

Here $f_{ma}^{temp}, f_{ma}^{hum}, f_{za}^{temp}, f_{za}^{hum}$ represent the temperature and humidity models for the mixing chamber and zone respectively whereas f^{fan} is the model for the fan heat gain. Furthermore, the hybrid nature of the cooling coil model can be efficiently captured using a disjunction of the form:

$$\begin{bmatrix} Y_1^k \\ W_{ca}^k = W_{ma}^k \\ T_{ca}^k \geq T_{dp}^k \end{bmatrix} \veebar \begin{bmatrix} Y_2^k \\ W_{ca}^k = W_{dehum}^k \\ T_{ca}^k \leq T_{dp}^k \end{bmatrix}. \tag{19}$$

Here note that when $Y_1^k = True$ then $T_{ca} \geq T_{dp}$, the mixed air temperature is only cooled and the humidity ratio remains unchanged. Conversely, when $Y_2^k = True$, cooling and dehumidification of the mixed air temperature take place resulting in the desired changes in both temperature and humidity. The XOR operator ($\veebar$) ensures exclusive activation of either cooling or cooling and dehumidification operational mode [2]. The solution to the GDP problems can be obtained by reformulating the problem as mixed integer programs. Two main approaches are usually employed to convert a GDP into an MIP: *i*) Big-M reformulation *ii*) Hull Relaxation. Although the hull relaxation approach provides a tighter reformulation, the Big-M technique is usually employed owing to the advantage of simplicity. We note that the objective function as well as the global constraints can be directly used, however, the disjunction containing the hybrid cooling coil dynamics must be converted into mixed integer inequalities. To accomplish this, binary variables $b_i^k \in \{0,1\}$ are introduced establishing a direct mapping to the boolean variables (e.g. $Y_i^k = True$ is transformed as $b_i^k = 1$, and $Y_i^k = False$ is transformed as $b_i^k = 0$). Using these boolean variables the disjunction can be modelled as follows:

Proposition 1. The disjunction constraints in GDP are enforced $\begin{bmatrix} Y_1^k \\ W_{ca}^k = W_{ma}^k \\ T_{ca}^k \geq T_k^{dp} \end{bmatrix} \veebar \begin{bmatrix} Y_2^k \\ W_{ca}^k = W_{dehum}^k \\ T_{ca}^k \leq T_{dp}^k \end{bmatrix}$ iff the constraints (20a) to (20d) hold:

$$m_{ca}(1 - b_1^k) \leq W_{ca}^k - W_{ma}^k \leq M_{ca}(1 - b_1^k) \tag{20a}$$

$$m_{ca}(1 - b_2^k) \leq W_{ca}^k - W_{dehum}^k \leq M_{ca}(1 - b_2^k) \tag{20b}$$

$$T_{ca}^k \leq T_{dp}^k + M_{dp}^1 b_1^k,\ T_{ca}^k \geq T_{dp}^k - M_{dp}^2 b_2^k \tag{20c}$$

$$\sum_{j \in \{1,2\}} b_j^k = 1, \forall k \in \{1, \cdots, N\} \tag{20d}$$

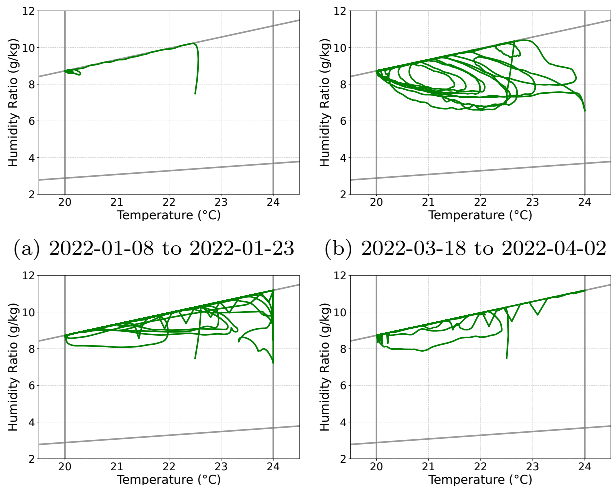
The proof of proposition 1 follows a similar argument as in Grossmann and Trespalacios (2013). Note that the condition in (20d) requires that only one binary variable must be turned on at each time instant. When $T_{ca} \leq T_{dp}$ then the feasibility of the constraints requires setting the $b_2^k = 1$ and hence the constraints in (20b) are binding and ensures that the state evolution takes place using (6). Similarly, when $T_{ca} \geq T_{dp}$, then the feasibility of the constraints requires setting $b_1^k = 1$, which ensures that the cooled air humidity remains unaffected. It must be noted that the values of the Big-M variables must be chosen appropriately to avoid infeasibility and weak relaxations. Finally, the energy optimal control problem that systematically accounts for the temperature humidity coupling and latent heat consideration is formulated as a mixed integer nonlinear program (MINLP) given as follows:

$$\begin{aligned} \min_{\theta} \quad & P_{ahu} + \lambda \Delta\theta \\ \text{s.t.} \quad & (18a) - (18h), (20a) - (20d) \end{aligned} \tag{21}$$

where $\theta = \{\mathbf{d}_{oa}, \mathbf{m}_{sa}, \mathbf{T}_{ma}, \mathbf{T}_{ca}, \mathbf{T}_{ha}, \mathbf{T}_{sa}, \mathbf{W}_{ma}, \mathbf{W}_{sa}\}$ are the decision variables and λ is the penalty term on rate constraint of different variables ($\Delta\theta$). It is important to note that such a formulation acts as the supervisory layer responsible for determining the set points of the local controller for different components of AHU. These setpoints include: mass flow rate of air (m_{sa}), OA damper opening (d_{oa}), CA temperature (T_{ca}), and SA temperature (T_{sa}) and humidity ratio (W_{sa}). The sensor measurements corresponding to these setpoints are easily accessible in most AHU configurations.

4. SIMULATION RESULTS

In this section, we compare the efficacy of the proposed control strategy to regulate the temperature and humidity within the desired thermal comfort envelope. The AHU is equipped with motorized opposed blade dampers, a cooling coil, a heating coil, and a fan driven by a variable frequency drive. A lower bound of 17°C must be maintained on the supply air temperature. The zone is subjected to a constant sensible and latent heat load. The prediction horizon is set to 30 minutes with the discretization step of 3 minutes. Bonmin is used to solve the MINLP (Saltzman (2002)), whereas CasADI (Andersson et al. (2019)) is used as the high-level interface. The simulation setup considers the dynamic model for the zone, whereas steady state models are adopted for AHU components. Four regions (15 days) are chosen to evaluate the performance during summer, winter, and transition regions between seasons. Fig. 3 shows the evolution of temperature and humidity under the influence of the proposed control strategy in different regions.



(a) 2022-01-08 to 2022-01-23 (b) 2022-03-18 to 2022-04-02

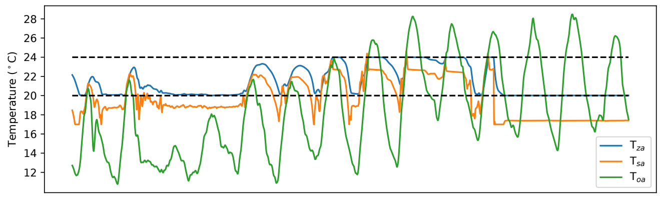
(c) 2022-04-29 to 2022-05-14 (d) 2022-09-16 to 2022-10-01

Fig. 3. The proposed control strategy maintains the temperature and humidity within the desired thermal comfort envelope.

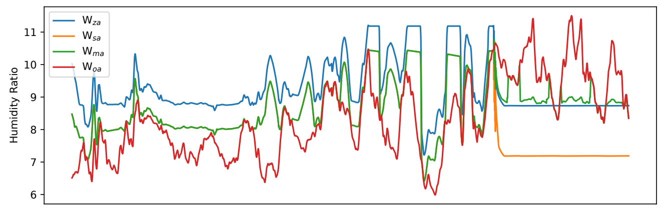
Fig.4 shows the detailed performance of the control strategy in region-3. During cold and dry weather conditions

[2] Note that when the disjunction consists of only two terms Y_2^k can be replaced with $= 1 - Y_1^k$. However, Y_2^k is used for the clarity of the presentation.

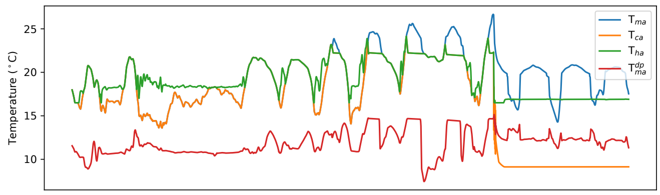
(roughly the first seven days), the dampers are adjusted to achieve a mixed air ratio that reduces the need for the active use of additional equipment. There are instances when heating the MA is necessary to ensure thermal comfort requirements. The period of hot and dry weather conditions between day 7 and day 10 requires the cooling of the MA to maintain the desired thermal comfort. Finally, when the weather conditions become hot and humid (roughly between day 11 and day 14), the mixed air is cooled below the dewpoint temperature to achieve the necessary dehumidification. Subsequently, the mixed air is reheated to avoid thermal discomfort to the occupants. A significant amount of energy is spent in this region to maintain the thermal comfort requirement.



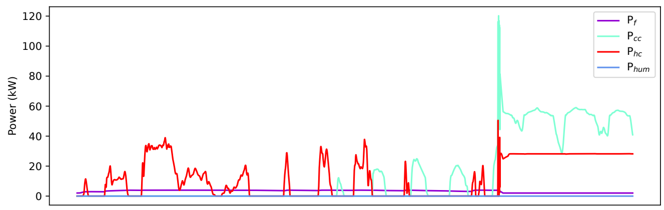
(a) Evolution of ZA, SA, OA temperatures. ZA temperature is regulated within the desired temperature range (20 - 24° C).



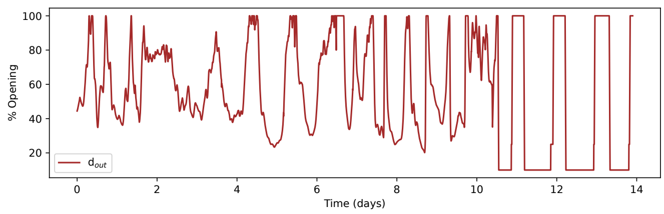
(b) Evolution of humidity ratio.



(c) Air temperature at different stages of the AHU.



(d) Power consumption of different actuators.



(e) Outside air damper opening.

Fig. 4. Performance of the proposed control strategy in region 3.

5. CONCLUSIONS

In this paper, we studied the problem of energy efficient temperature and humidity control design to maintain the thermal comfort requirements. A novel control approach based on generalized disjunctive programming was employed to systematically account for the hybrid cooling coil model and latent heat considerations. The performance of the control strategy was validated across a wide range of weather conditions. In the future, we plan to compare the energy optimal formulation with a suboptimal NLP formulation that can significantly reduce the computational burden of solving a MINLP.

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