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A Feasibility Condition for the Governor-based Tuning of Explicit MPC: Application to a Hydraulic Plant ⋆

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Abstract: We propose a feasibility condition for reference governors that are used to tune the behavior of a reference-tracking explicit model predictive controller. Governors allow for a simple tuning of existing controllers without modifying them. However, since properties like feasibility are often not accounted for by the governors, their applicability may be limited. We prove that feasibility can be guaranteed using information about the explicit solution, and demonstrate the resulting algorithm by controlling the flow rate in a laboratory-scale hydraulic plant.

Keywords: controller tuning, model predictive control, governor, feasibility, hydraulic plant

1. INTRODUCTION

Whenever existing controllers cannot be altered, governors provide a means to achieve a desired control behavior without modifying the existing controller. In, e.g., Bemporad (1998); Gilbert and Kolmanovsky (2002), certain forms of reference governors are used to ensure constraint satisfaction for controllers that do not take constraints into account. Klaučo and Kvasnica (2019) use governors for providing reference values to existing low-level controllers that are optimal with respect to a high-level optimization-based controller.

Apart from extending controllers by constraint satisfaction or optimizing the reference inputs, governors are used to directly tune the control behavior of an existing controller towards a more conservative or more aggressive response. A recent overview on the governor-based tuning of a wide range of controller structures with respect to the control response can be found in Fikar et al. (2023). An example of a practical implementation of a reference governor to tune the cascaded PID controllers of two different unmanned aerial vehicles (UAVs) is given in Dyrska et al. (2023). Here, not only the dependent controller structure itself but also further restrictions such as fundamentally different hard- and software frameworks of the UAVs motivated the high-level tuning using reference governors.

Another promising field for tuning the control behavior using governors is explicit model predictive control (EMPC). In contrast to classic MPC, finding the optimal input signal using EMPC is reduced to locating and evaluating the optimal piece-wise affine feedback law for the current system state (Bemporad et al., 2002; Seron et al., 2003). However, the control response of the offline-determined EMPC solution cannot be tuned without a recomputation of the explicit solution, which can be computationally restrictive.

To avoid a modification of the EMPC during the design phase or in the case that the EMPC already exists, it is possible to tune the existing EMPC using governors as presented in Fikar et al. (2023). However, when applied to EMPC problems, governors can lead to parameters that are no longer part of the explicit solution. In this case, no feasible solution exists for the tuned parameter, and postprocessing of the parameter vector or the input signal would be necessary.

In this paper, we present a feasibility condition based on the convex hull of the explicit solution, allowing an online adjustment of the tuning factor used within the governor. Only in time steps in which the desired tuning factor would lead to infeasible parameters, the tuning factor forwarded to the governor is modified to keep the parameters within the feasible set. We introduce the feasibility condition for a reference-tracking EMPC, however, we claim that the basic idea can be applied also to other cases.

We implemented and tested the presented approach for the reference tracking of the flow rate of a laboratory-scale hydraulic plant. This application demonstrates our approach can cope with the typical challenges arising in process control applications, such as having to re-tune existing controllers to compensate for fouling and aging, without recomputing complex explicit solutions.

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We introduce the class of reference-tracking EMPC problems tuned in this paper in Section 2. Section 3 summarizes the governor-based tuning of the EMPC followed by the main result, i.e., a feasibility condition and the resulting algorithm for a safe application of the tuning. In Section 4, we present experimental results for the hydraulic plant, before concluding the paper in Section 5.

2. PROBLEM STATEMENT

We consider the control of linear discrete-time systems of the form

$$x(k+1) = Ax(k) + Bu(k), \quad y(k) = Cx(k), \tag{1}$$

with states $x(k) \in \mathbb{R}^n$, inputs $u(k) \in \mathbb{R}^m$, and outputs $y(k) \in \mathbb{R}^\ell$, and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $C \in \mathbb{R}^{\ell\times n}$ describing the state, input, and output matrix, respectively. For simplicity, we assume $m = \ell = 1$ in the remainder. However, the tuning method can be generalized to systems with multiple inputs and outputs. We further assume system (1) to be observable and the output $y(k)$ to equal the first system state, which can be achieved by rearranging the state equations and transforming (1) into the observer normal form if necessary.

To regulate the output value $y(k)$ in (1) to a reference value $y_\mathrm{r}(k)$, we introduce the optimization problem

$$\min_{y(\cdot),u(\cdot)} \sum_{k=0}^{N-1} \left(\|e(k)\|_Q^2 + \|u(k)\|_{R_\mathrm{u}}^2 + \|\Delta u(k)\|_{R_{\Delta\mathrm{u}}}^2\right) \tag{2a}$$

$$\text{s.t.}\quad x(k+1) = Ax(k) + Bu(k),\ k = 0,...,N-1, \tag{2b}$$

$$y(k) = Cx(k),\ k = 0,...,N-1, \tag{2c}$$

$$y_\mathrm{min} \le y(k) \le y_\mathrm{max},\ k = 0,...,N-1, \tag{2d}$$

$$u_\mathrm{min} \le u(k) \le u_\mathrm{max},\ k = 0,...,N-1, \tag{2e}$$

$$e(k) = y_\mathrm{r}(k) - y(k),\ k = 0,...,N-1, \tag{2f}$$

$$\Delta u(k) = u(k) - u(k-1),\ k = 0,...,N-1, \tag{2g}$$

$$x(0) = x(k). \tag{2h}$$

For a horizon N, the objective function (2a) quadratically penalizes the error between output and reference value $e(k) = y_\mathrm{r}(k) - y(k)$, the use of the input $u(k)$, and the rate of change of the input value between two subsequent time steps $\Delta u(k) = u(k) - u(k-1)$ by $\|v(k)\|_P^2 = v^\top Pv$ using weighting matrices $Q \succ 0$, $R_\mathrm{u} \succ 0$, and $R_{\Delta\mathrm{u}} \succ 0$ of the obvious dimensions. Note that, while the penalization of the input $u(k)$ will lead to an offset in the tracking result, we included it here to reduce the usage of the motor rate for the application introduced in Section 4.1. For simplicity, we assume lower and upper bounds on outputs (2d) and inputs (2e) apply. We further assume $y_\mathrm{r}(k)$ to be bounded by the constraints specified in (2d), i.e., $y_\mathrm{min} \le y_\mathrm{r}(k) \le y_\mathrm{max}$ for all k. These bounds are automatically taken care of by tools such as the Multi-Parametric Toolbox (Herceg et al., 2013) when calculating the explicit solution introduced below. Problem (2) is initialized by the current state $x(0) = x(k)$, the input signal of the previous time step $u(k-1)$, and the reference value $y_\mathrm{r}(k)$. We introduce the parameter vector $z(k) \in \mathbb{R}^p$

$$z(k) = \left(x(k)^\top, u(k-1)^\top, y_\mathrm{r}(k)^\top\right)^\top \tag{3}$$

with $p = n + m + \ell$ to collect the initial parameters of problem (2).

In classic MPC, problem (2) is solved in every time step resulting in an optimal input signal $U^\star(k) = (u^{\star\top}(0), ..., u^{\star\top}(N-1))^\top$ along horizon N, and the first input $u^\star(0)$ is applied to the system. Depending on the complexity of problem (2), the computational effort required for solving the optimization problem online, i.e., during runtime of the controller within the designated sampling time, can be restrictive.

It is well known, however, that problem (2) can be solved explicitly (Bemporad et al., 2002; Seron et al., 2003). In this case, a feasible set $\mathcal{F}$ and affine control laws $L_i z + c_i$ exist, where $\mathcal{F}$ is composed of polytopes $\mathcal{P}_i$, $i \in \mathcal{D} = \{1, ..., d\}$ with pairwise disjoint interiors and $\bigcup_{i\in\mathcal{D}} \mathcal{P}_i = \mathcal{F}$, such that the optimal solution to (2), $U^\star(z(k))$, is given by

$$U^\star(z(k)) = \begin{cases} L_1 z(k) + c_1 & \text{if } z(k) \in \mathcal{P}_1 \\ \vdots & \\ L_d z(k) + c_d & \text{if } z(k) \in \mathcal{P}_d \end{cases} \tag{4}$$

Instead of solving problem (2) online, i.e., in every time step, the EMPC solution is computed once and offline, while the online control reduces to finding the matching polytope $\mathcal{P}_i$ for parameter vector $z(k)$ and applying the corresponding affine control law $L_i z(k) + c_i$. We call a parameter vector $z(k)$ feasible if $z(k) \in \mathcal{F}$.

It is a drawback of EMPC, however, that any change of the MPC tuning parameters requires the computation of a new explicit solution. Therefore, the reference governor introduced in the following section aims at tuning the EMPC without recomputing it.

3. GOVERNOR-BASED TUNING OF EXPLICIT MPC WITH FEASIBILITY GUARANTEES

We briefly summarize the idea of tuning existing controllers using governors (see also Fikar et al. (2023)), and describe in more detail how the control behavior of an EMPC as in (4) can be tuned by using a reference governor. Subsequently, we present a feasibility condition that guarantees a robust application of the reference governor to an existing EMPC.

3.1 Tuning of explicit MPC by governors

The main motivation for using a governor is to tune the control behavior of an existing controller without modifying it. This can be achieved by, e.g., manipulating the control error that is forwarded to the controller as

$$\bar{e}(k) = Ke(k) = K(y_\mathrm{r}(k) - y(k)), \tag{5}$$

with scalar tuning factor $K \ge 0$ and $\bar{e}(k)$ the tuned control error scaled by K.

As stated in Section 2, the EMPC is evaluated for the parameter vector $z(k)$ consisting of states $x(k)$, the input signal of the previous time step $u(k-1)$, and the reference value $y_\mathrm{r}(k)$. Thus, the control error $e(k) = y_\mathrm{r}(k) - y(k)$ is not directly tunable without adapting the EMPC as it is computed within the EMPC internally. However, it is possible to achieve the desired tuning of $e(k)$ manipulating the reference value $y_\mathrm{r}(k)$ only, by reformulating the *error governor* introduced by (5) into the so-called *reference governor*

$$\bar{y}_\mathrm{r}(k) = Ky_\mathrm{r}(k) + (1-K)y(k). \tag{6}$$

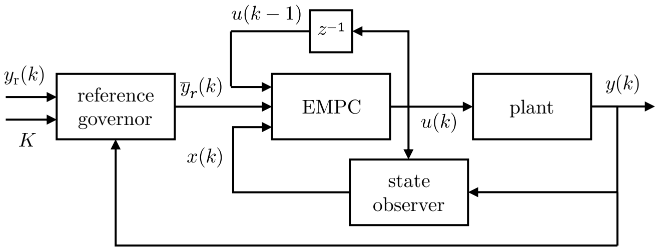

Fig. 1. EMPC extended by the reference governor.

Here, a manipulated reference value $\bar{y}_\mathrm{r}(k)$ results depending on the original reference value $y_\mathrm{r}(k)$, the current output value $y(k)$, and the tuning factor K.

Figure 1 depicts the integration of the reference governor into a general EMPC setup. The control signal $u(k)$ is determined by the EMPC and forwarded to the plant. The EMPC is evaluated for the current state $x(k)$, the input of the previous time step $u(k-1)$, and the reference value $y_\mathrm{r}(k)$. We use a standard observer, based on the model (2b)-(2c), to reconstruct the unmeasured state $x(k)$. The previous input signal $u(k-1)$ results from a unit delay, indicated by z^{-1}. The measured output value $y(k)$ as well as the actual reference value $y_\mathrm{r}(k)$ and the tuning factor K serve as an input to the reference governor. The reference governor forwards the tuned reference value $\bar{y}_\mathrm{r}(k)$ resulting from (6) to the EMPC. The block diagram in Figure 1 emphasizes that the original EMPC controller remains unchanged by adding the reference governor. However, the results in Section 4 will demonstrate that the desired tuning in the control behavior is nevertheless achieved by the proposed framework.

3.2 Feasibility guarantees for reference governors

When the reference governor (6) is combined with an EMPC law $U^\star(z(k))$ from (4), the EMPC law is not evaluated for

$$z(k) = \left(x(k)^\top, u(k-1)^\top, y_\mathrm{r}(k)^\top\right)^\top \tag{7}$$

from (3) (repeated here for convenience) but for

$$\bar{z}(k) = \left(x(k)^\top, u(k-1)^\top, \bar{y}_\mathrm{r}(k)^\top\right)^\top \tag{8}$$

where $y_\mathrm{r}(k)$ has been replaced by $\bar{y}_\mathrm{r}(k)$ from (6).

For $0 \le K < 1$, the controller that results from combining EMPC with the reference governor in this fashion becomes more conservative, since the amplitude of the tuned control error $|\bar{e}(k)|$ is artificially reduced compared to the original error $|e(k)|$ (see (5)).

For the convex class of MPC problems introduced in Section 2, such a more conservative tuning always leads to a feasible reference value $\bar{y}_\mathrm{r}(k)$ the EMPC can be evaluated for. We state this more precisely in Proposition 1, which also covers $K = 1$. Recall $\mathcal{F}$ refers to the feasible set of the explicit control law (4).

Proposition 1. Let $z(k)$ and $\bar{z}(k)$ be as in (7) and (8), respectively. If $z(k) \in \mathcal{F}$ and $K \in [0, 1]$, then $\bar{z}(k) \in \mathcal{F}$.

Proof. The tuned reference value $\bar{y}_\mathrm{r}(k)$ from (6) leading to $\bar{z}(k)$ as in (8) is a convex combination of $y(k)$ and $y_\mathrm{r}(k)$ for $0 \le K \le 1$. Since, by assumption, $z(k) \in \mathcal{F}$, with $z(k)$ containing $y(k)$ and $y_\mathrm{r}(k)$, and since the feasible set $\mathcal{F}$ is convex (Bemporad et al., 2002, Thm. 4), $\bar{z}(k) \in \mathcal{F}$. □

For $K > 1$, the resulting control behavior becomes more aggressive, since the amplitude of the tuned control error $|\bar{e}(k)|$ is artificially increased. While Proposition 1 exploits that, for $K < 1$, the tuned reference value $\bar{y}_\mathrm{r}(k)$ is located on the line segment that connects $y(k)$ and $y_\mathrm{r}(k)$, $\bar{y}_\mathrm{r}(k)$ does no longer lie in between these two points for $K > 1$. Consequently, $\bar{z}(k)$ resulting for $\bar{y}_\mathrm{r}(k)$ is no longer guaranteed to lie inside the convex feasible set $\mathcal{F}$.

However, it is possible to determine, depending on the current parameter vector $z(k)$, a maximum value of K such that feasibility of the tuned parameter vector $\bar{z}(k)$ resulting for $\bar{y}_\mathrm{r}(k)$ is guaranteed, i.e., $\bar{z}(k) \in \mathcal{F}$ holds. This feasibility condition is stated in Proposition 2. As a preparation we introduce

$$\tilde{z}(k) = (x(k)^\top, u(k-1)^\top)^\top \tag{9}$$

as the variant of parameter vector $z(k)$ in (7) without $y_\mathrm{r}(k)$. Further, since $\mathcal{F}$ is a convex set composed of convex polyhedrons according to Bemporad et al. (2002), Thm. 4 and the corresponding proof, we can describe $\mathcal{F}$ by q hyperplanes

$$Cz(k) \le d,\ C \in \mathbb{R}^{q\times p},\ d \in \mathbb{R}^q.$$

For a certain hyperplane $C_i z(k) \le d_i$ denoted by index $i \in \mathcal{Q}$, with $\mathcal{Q} = \{1, ..., q\}$, we introduce

$$C_{i,\mathrm{z}}\tilde{z}(k) + C_{i,\mathrm{r}}y_\mathrm{r}(k) \le d_i \tag{10}$$

as a notation separating the reference value in $z(k)$ from the remaining parameters $\tilde{z}(k)$. We define the subset $\mathcal{Q}_\mathrm{r} \subset \mathcal{Q}$ containing all q_r indices i such that $C_{i,\mathrm{r}} \neq 0$, i.e., indices corresponding to hyperplanes that can be reached by varying $y_\mathrm{r}(k)$.

Proposition 2. Let $z(k)$, $\bar{z}(k)$, and $\tilde{z}(k)$ be as in (7), (8), and (9), and let $y(k)$ and $y_\mathrm{r}(k)$ be the current output and reference value, respectively. If $z(k) \in \mathcal{F}$, then

$$\hat{K}^\star(k) = \min\left(\hat{K}_i(k)\right),\ \ \forall\ \hat{K}_i(k) \ge 1,\ \ i \in \mathcal{Q}_\mathrm{r} \tag{11}$$

with

$$\hat{K}_i(k) = \frac{d_i - C_{i,\mathrm{z}}\tilde{z}(k) - C_{i,\mathrm{r}}y(k)}{C_{i,\mathrm{r}}y_\mathrm{r}(k) - C_{i,\mathrm{r}}y(k)} \tag{12}$$

describes the upper bound on the tuning parameter K, i.e., $K \le \hat{K}^\star(k)$, such that $\bar{z}(k) \in \mathcal{F}$.

Proof. For a tuned reference value $\bar{y}_\mathrm{r}(k)$ resulting from applying the largest possible tuning factor $\hat{K}_i(k)$ such that $\bar{z}(k) \in \mathcal{F}$, relation (10) reads

$$C_{i,\mathrm{z}}\tilde{z}(k) + C_{i,\mathrm{r}}\bar{y}_\mathrm{r}(k) = d_i \tag{13}$$

for all $i \in \mathcal{Q}_\mathrm{r}$. Substituting $\bar{y}_\mathrm{r}(k) = \hat{K}_i(k)y_\mathrm{r}(k) + (1 - \hat{K}_i(k))y(k)$, (13) can be reformulated to

$$\hat{K}_i(k)(C_{i,\mathrm{r}}y_\mathrm{r}(k) - C_{i,\mathrm{r}}y(k)) = d_i - C_{i,\mathrm{z}}\tilde{z}(k) - C_{i,\mathrm{r}}y(k). \tag{14}$$

By solving (14) for $\hat{K}_i(k)$, (12) results. Since $\bar{y}_\mathrm{r}(k) = y_\mathrm{r}(k)$ for $K = 1$, only values $\hat{K}^\star(k) \ge 1$ are relevant. Further, since $\hat{K}^\star(k) = \min(\hat{K}_i(k))$ defines the upper bound on K for the current time step k and for all $i \in \mathcal{Q}_\mathrm{r}$, the maximum tuning factor $\hat{K}^\star(k)$ results from the first violation of one of the hyperplanes defining $\mathcal{F}$. □

We note that (11) is the most general formulation for calculating an upper bound on K. The tuning factor needs to be bounded with (11) if, e.g., a terminal constraint applies to $e(k)$ and $\mathcal{F}$ in the dimension of $y_\mathrm{r}(k)$ is also affected by the system dynamics. It is evident from (2), however, that $\mathcal{F}$ in the dimension of $y_\mathrm{r}(k)$ will often be bounded by the output constraints only. In this case, $C_{i,\mathrm{z}}$ is a vector of zeros, and $C_{i,\mathrm{r}} = 1$ or $C_{i,\mathrm{r}} = -1$ in (10). Then the computation of $\hat{K}^\star(k)$ reduces to the simpler evaluations of

$$\hat{K}^\star(k) = \min\Big(\hat{K}_i(k)\Big), \quad \forall \ \ \hat{K}_i(k) \geq 1, \quad i \in \{1, 2\} \qquad (15)$$

with

$$\hat{K}_1(k) = \frac{y_\mathrm{min} - y(k)}{y_\mathrm{r}(k) - y(k)} \text{ and } \hat{K}_2(k) = \frac{y_\mathrm{max} - y(k)}{y_\mathrm{r}(k) - y(k)}$$

for the lower and upper bound, respectively, which is a direct result of (11).

Remark 3. In case $\ell > 1$, i.e., there exist multiple outputs to be tuned, two cases need to be distinguished for applying the feasibility condition. If an individual $K_j, j = \{1, ..., \ell\}$ should be used for each of the ℓ output values, Proposition 2 applies analogously for each K_j. If the same tuning factor K is used for all references, $\hat{K}_j^\star(k), j = \{1, ..., \ell\}$ according to Proposition 2 has to be determined for each of the ℓ output values, and $\hat{K}^\star(k) = \min(\hat{K}_j^\star(k)), j = \{1, ..., \ell\}$ defines the upper bound for K.

Since an adjustment on the tuning factor may be necessary, we need to distinguish between the desired tuning factor $K^\star$ we want to use for a tuning of the EMPC, and the tuning factor K we actually forward to the reference governor (6). Algorithm 1 describes the implementation of the framework shown in Figure 1 for a more aggressive tuning including the feasibility check described by Proposition 2. Before evaluating the EMPC, the current state is observed and a maximum feasible value $\hat{K}^\star$ is determined using (11) (or, if applicable, (15)) in lines 2 and 3. The value of K used in (6) is updated by comparing the maximum feasible tuning factor $\hat{K}^\star$ with the desired tuning factor $K^\star$. In case $K^\star$ is no longer feasible, i.e., $\hat{K}^\star < K^\star$, K in (6) is set to $K = \hat{K}^\star$ (lines 4-5). Otherwise, K in (6) remains equal to $K^\star$ (lines 6-7). The remaining lines of Algorithm 1 are straightforward and consist of computing the tuned reference value $\bar{y}_\mathrm{r}$, evaluating the EMPC for the tuned parameter vector $\bar{z}$, and applying the resulting optimal control input $u^\star$ to the plant.

Following Algorithm 1, the desired tuning factor $K^\star$ is used whenever it is feasible. If a correction of K towards a smaller value $\hat{K}^\star(k)$ was necessary, the desired tuning factor $K^\star$ will be used again as soon as the tuned parameter vector $\bar{z}(k)$ will allow the application of $K^\star$ without violating the constraints according to the feasibility condition in Proposition 2.

Algorithm 1 Explicit MPC control tuned by reference governor with guaranteed feasibility

1: **Input:** Measured output y, reference y_r, previous input u^-, desired tuning factor $K^\star$
2: Estimate current state x with state observer
3: Determine maximum feasible $\hat{K}^\star$ acc. to Prop. 2
4: **if** $\hat{K}^\star < K$ **then**
5:     Set $K \leftarrow \hat{K}^\star$ in (6)
6: **else**
7:     Set $K \leftarrow K^\star$ in (6)
8: **end if**
9: Determine tuned $\bar{y}_\mathrm{r}$ according to (6)
10: Evaluate EMPC (4) for $\bar{z} = (x^\top, u^{-\top}, \bar{y}_\mathrm{r}^\top)^\top$
11: Apply optimal input signal $u^\star$
12: **Output:** Updated u^-

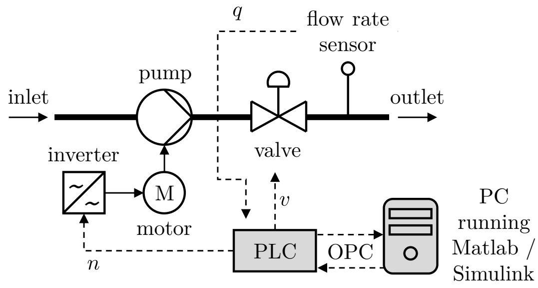

Fig. 2. Sketch of the hydraulic plant.

4. EXPERIMENTS ON A HYDRAULIC PLANT

We applied the tuning procedure to an EMPC designed for controlling the flow rate of a laboratory-scale hydraulic plant. The plant includes an industrial centrifugal pump with variable speed drive, a controllable discharge valve, and a flow rate sensor. Control and operation are performed by a PLC with connection to a Matlab / Simulink PC for advanced control tasks. The general plant layout is depicted in Figure 2. We first summarize the model, followed by the design of the EMPC controller. Subsequently, we present experimental results from applying a reference governor for tuning and discuss the closed-loop behavior both qualitatively and quantitatively.

4.1 System modeling

We modeled the plant component-wise, where pump and valve were both captured by nonlinear-static models, while the flow rate sensor incorporates all dynamic components in a linear-dynamic model. The pump model was derived from a grey-box approach (Leonow et al., 2017, 2023) using generic centrifugal pump characteristics and hydraulic affinity laws, resulting in

$$p = c_2 \cdot \varphi^2 + c_1 \cdot \varphi \cdot n + c_0 \cdot n^{1.7} + p_0 \qquad (16)$$

with rotational speed n in min^{-1}, pump discharge pressure p in bar, steady state flow rate φ in $\mathrm{L\,min}^{-1}$, and coefficients $c_2 = -2.94{\cdot}10^{-8}$, $c_1 = -6.77{\cdot}10^{-9}$, $c_0 = 2.022{\cdot}10^{-6}$. The pump inlet pressure p_0 is constant. The discharge valve is located downstream of the pump and reduces the pump pressure p back down to the pump inlet pressure p_0. The valve model follows from the generic orifice model (Gülich, 2010, p. 36)

$$\varphi = \sqrt{(p - p_0) \cdot \zeta(v)} \ , \qquad (17)$$

where the friction coefficient $\zeta(v)$ depends on the percentage of the valve opening v.

The flow rate sensor model results from a step response identification as

$$1.75 \cdot \ddot{q}(t) + 2.91 \cdot \dot{q}(t) + q(t) = \varphi(t) \ , \qquad (18)$$

with flow rate $q(t)$ measured in $\mathrm{L\,min}^{-1}$. Inserting (17) in (18) and substituting p by (16) yields the nonlinear-dynamic plant model with input $n(t)$ and output $q(t)$.

We linearized the resulting model around a steady state operating point with $n_0 = 1050\ \mathrm{min}^{-1}$, $v_0 = 50\ \%$, $q_0 = 42.47\ \mathrm{L\,min}^{-1}$, $\dot{q}_0 = \ddot{q}_0 = 0$, discretized it with $T_\mathrm{S} = 0.25$ s, and adjusted the system gain using measurement data, resulting in a linear model (1) with matrices

$$A = \begin{pmatrix} 0.98 & 0.20 \\ -0.12 & 0.65 \end{pmatrix},\quad B = \begin{pmatrix} 7.50 \cdot 10^{-4} \\ 5.59 \cdot 10^{-3} \end{pmatrix},\quad C = (1\ \ 0)\,,$$

for use in EMPC.

4.2 Explicit MPC design

We designed a reference tracking MPC as described in (4) for controlling the flow rate $q(k)$ of the plant towards certain reference values. The constraints read

$$\begin{aligned} 600 \leq\, & u(k) \leq 1500, \\ 20 \leq\, & y(k) \leq 60 \end{aligned}$$

for all k. Note that the whole range of possible pump speeds $u(k) = n(k)$ is admitted. The cost uses the weighting matrices

$$Q = \begin{pmatrix} 2000 & 0 \\ 0 & 2000 \end{pmatrix},\quad R_\mathrm{u} = 0.01,\quad R_{\Delta\mathrm{u}} = 1.$$

Note that the difference between Q and R_u is chosen not only to weight the objective, but also to compensate for the different ranges of flow rate $y(k)$ and pump speed $u(k)$. We used the Multi-Parametric Toolbox (Herceg et al., 2013) to compute the explicit solution. For a horizon of $N = 10$, the EMPC solution consists of $d = 433$ regions.

The EMPC is evaluated for a vector $z(k) = (x(k)^\top, u(k-1)^\top, y_\mathrm{r}(k)^\top)^\top$, i.e., the system state needs to be known. Since only the system output, i.e., the flow rate $q(k)$, is measured, we used a Luenberger observer of the form

$$\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k) + Bu(k) + L(y(k) - \hat{y}(k)) \\ \hat{y}(k) &= C\hat{x}(k), \end{aligned}$$

with $L = (0.13\ \ -0.08)$ and observed signals labeled by a hat to estimate the system state.

4.3 Experimental results

To evaluate the effect of the governor-based tuning and to present the adjustments on the tuning factor K to preserve feasibility, we performed several step changes in the reference value $y_\mathrm{r}(k)$.

Starting with a reference value for the flow rate of $y_\mathrm{r}(k) = 40\,\mathrm{L\,min}^{-1}$, we applied two steps with a return to the starting point, i.e., one aiming at a lower flow rate by $40\,\mathrm{L\,min}^{-1} \to 32\,\mathrm{L\,min}^{-1} \to 40\,\mathrm{L\,min}^{-1}$, and an increase by $40\,\mathrm{L\,min}^{-1} \to 50\,\mathrm{L\,min}^{-1} \to 40\,\mathrm{L\,min}^{-1}$. Step changes were applied every 45 s, i.e., after every 180 time steps.

The experiments were performed for $K^\star = 1$ as a benchmark, a more conservative tuning with $K^\star = 0.5$, and a more aggressive tuning choosing a desired tuning factor of $K^\star = 3$ for all cases. Figure 3 shows the resulting trajectories of the output value, the applied input signals, the reference values entering the EMPC, and the applied tuning factors. The desired behavior can be observed for all step changes.

Independently of K, all results indicate a stable behavior and convergence to the new reference value. Regarding the output value shown in the top plot, the benchmark resulting for $K^\star = 1$ (blue) has a small overshoot in all steps. As expected, the overshooting increases for a more aggressive tuning using a tuning factor of $K^\star = 3$ (orange). In contrast, no overshoot is visible for the more conservative tuning realized by $K^\star = 0.5$ (green). The second plot shows that, in contrast to the other variants, the more aggressive controller (orange) reaches the lower and upper input bounds for all reference changes. The reference values forwarded to the EMPC are shown in the third plot and explain the trajectories of the output value in the top plot. Note that, while choosing the tuning factor too low or too high might decrease the control performance, we chose a strong tuning in both directions to demonstrate the effect of the tuning more thoroughly. As shown in Figure 3 and the analysis below, the tuned controllers show the desired behavior.

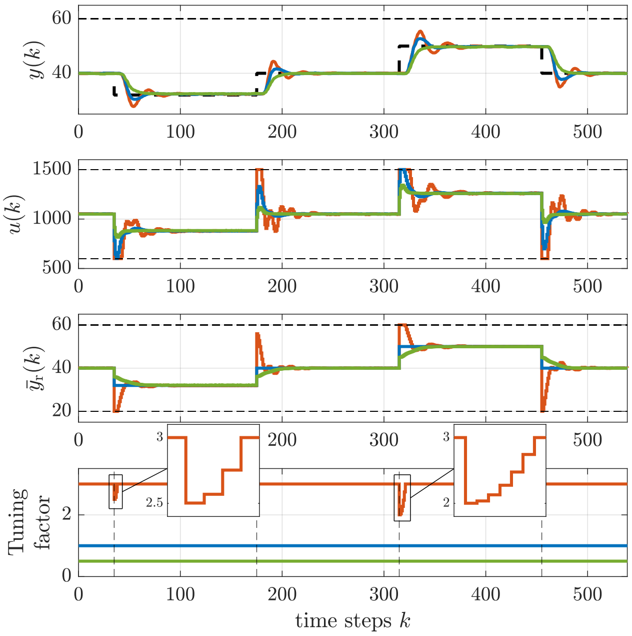

Fig. 3. Measured output value (top), applied input signal (second plot), tuned reference value (third plot), and applied tuning factor (bottom) over time steps k for experiments with a benchmark tuning (blue), a more conservative tuning (green), and a more aggressive tuning (orange). In the top plot, original reference values $y_\mathrm{r}(k)$ are sketched by the dashed black line.

The bottom plot shows the tuning factors applied during the experiments, complemented by dashed lines distinguishing time steps in which a reference step change occurred. For the more aggressive tuning, the desired tuning factor $K^\star = 3$ would have led to a constraint violation by $\bar{y}_\mathrm{r}$ for step changes $40\,\mathrm{L\,min}^{-1} \to 32\,\mathrm{L\,min}^{-1}$ and $40\,\mathrm{L\,min}^{-1} \to 50\,\mathrm{L\,min}^{-1}$. In the first case, K was reduced from $K^\star = 3$ down to a value of $\hat{K}^\star(k) = 2.73$ by Algorithm 1, and increased until reaching $K^\star$ again in the following time steps. For the latter case, K was reduced more significantly down to a value of $\hat{K}^\star(k) = 2.01$, before gradually returning to the original $K^\star$ in the subsequent time steps. Both variations in the tuning factor are shown in more detail by the boxes within the bottom plot in Figure 3.

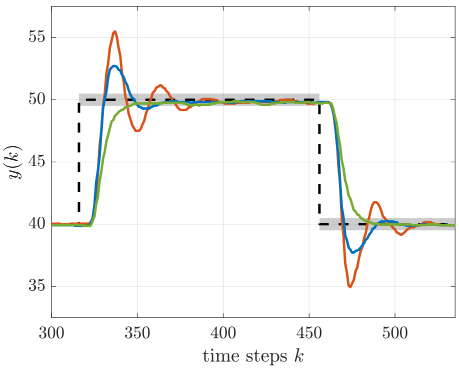

Fig. 4. Detailed output trajectory measured for reference step changes $40\,\mathrm{L\,min^{-1}} \rightarrow 50\,\mathrm{L\,min^{-1}}$ and $50\,\mathrm{L\,min^{-1}} \rightarrow 40\,\mathrm{L\,min^{-1}}$. The colors carry over from Fig. 3, the gray areas show the 5% tolerance band.

In summary, Algorithm 1 resulting from the feasibility condition stated in Proposition 2 was able to preserve feasibility. As expected, tuning factors $K^\star = 1$ and $K^\star = 0.5$ remained unchanged, which corroborates Proposition 1.

To confirm the observations made for Figure 3, we analyzed two reference step changes in more detail using well-established parameters. Figure 4 shows in detail the steps $40\,\mathrm{L\,min^{-1}} \rightarrow 50\,\mathrm{L\,min^{-1}}$ and $50\,\mathrm{L\,min^{-1}} \rightarrow 40\,\mathrm{L\,min^{-1}}$. For both steps, we measured the rise time t_r, the settling time t_s, and the maximum overshoot e_max. To minimize the influence of disturbances, we used the tolerance band of the settling time also for the rise time, i.e., for t_r we measured the time between changing the reference value and entering the tolerance band for the first time. The procedure is identical to the one we performed in Dyrska et al. (2023). Here, we chose a tolerance band of 5 % of the reference step change in $y_\mathrm{r}(k)$ above and below the new reference value ($\pm 0.5\,\mathrm{L\,min^{-1}}$, light gray areas in Fig. 4). The results of the quantitative analysis are summarized in Table 1 and underline the observations above. For the first reference step change, i.e., $40\,\mathrm{L\,min^{-1}} \rightarrow 50\,\mathrm{L\,min^{-1}}$, the more conservative tuning eliminated the overshoot such that $t_\mathrm{r} = t_\mathrm{s}$. Compared to the benchmark tuning, the rise time increased by around 107%, while the settling time decreased by around 32.14%. Due to saturation, only a slight reduction of the rise time could be achieved by applying the more aggressive tuning. Here, the rise time decreased by around 1.68%. In contrast, both, the settling time and the maximum overshoot increased by around 50.96% and 102.55%, respectively.

As expected from Figure 4, similar behavior was measured for the step $50\,\mathrm{L\,min^{-1}} \rightarrow 40\,\mathrm{L\,min^{-1}}$. Rise and settling time increased by 70.94% and decreased by 29.41%, respectively, using a more conservative tuning of $K = 0.5$. While the rise time was again slightly reduced by applying a more aggressive tuning (around 7.41%), the settling time increased in contrast to the benchmark measurement by around 65.76%. This is due to the large increase in the overshooting of around 121.9%.

Table 1. Results for the step changes in Fig. 4.

reference step change	des. tuning factor $K^\star$	rise time t_r	settling time t_s	max. overshoot e_max
$40\,\mathrm{L\,min^{-1}}$	3	3.51 s	16.44 s	54.87 %
↓	1	3.57 s	10.89 s	27.09 %
$50\,\mathrm{L\,min^{-1}}$	0.5	7.39 s	7.39 s	0 %
$50\,\mathrm{L\,min^{-1}}$	3	3.25 s	14.09 s	50.57 %
↓	1	3.51 s	8.5 s	22.79 %
$40\,\mathrm{L\,min^{-1}}$	0.5	6 s	6 s	0 %

5. CONCLUSION

We presented a governor-based tuning approach for explicit MPC extended by a feasibility condition that prevents infeasible tuning results. The approach is based on a geometric analysis of the hyperplanes defining the feasible set to determine maximum values for the tuning parameter. We applied a tracking EMPC tuned by a reference governor for the flow rate control of a laboratory-scale hydraulic plant. Experiments on the plant corroborate the reliability and applicability of the proposed approach.

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