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New Adaptive ESO Based Data-Driven Anti-Disturbance Control for Nonlinear Systems with Convergence Guarantee ⋆

Shoulin Hao [*,**] **Yihui Gong** [*,**] **Naseem Ahmad** [*,**]
Shuhao Yue [*,**] **Tao Liu** [*,**]

[*] *Key Laboratory of Intelligent Control and Optimization for Industrial Equipment of Minstry of Education, Dalian University of Technology, Dalian, 116024, China.*
[**] *Institute for Advanced Control Technology, Dalian University of Technology, Dalian, 116024, China. (e-mail: slhao@dlut.edu.cn (S. Hao); tliu@dlut.edu.cn (T. Liu)).*

Abstract: In this paper, a new adaptive extended state observer based data-driven anti-disturbance control (AESO-DDADC) design is proposed for industrial nonlinear systems with unknown dynamics subject to external disturbances. By reformulating such system description into a compact-form dynamic linearization model with a residual term, a new AESO is firstly constructed to estimate the residual term using the partial derivative (PD) estimation from the previous time step, such that the residual term could be proactively counteracted by the feedback control law, in contrast to the existing data-driven ESO where the residual term in the PD estimation is absolutely neglected to facilitate the convergence analysis. Then, the bounded convergence of PD estimation and AESO is jointly analyzed by the Gerschgorin disk theorem, followed by robust convergence analysis of the established closed-loop system. Moreover, another AESO-DDADC scheme is developed using a partial-form dynamic linearization model of the system, along with rigorous robust convergence analysis. Finally, an illustrative example is shown to confirm the efficacy and advantages of the proposed designs.

Keywords: Nonlinear systems, data driven, adaptive extended state observer, anti-disturbance control, convergence analysis.

1. INTRODUCTION

Owing to the large scale and high complexity of modern industrial processes, it is challenging and cumbersome to accurately model a real process using the first principle mechanism or system identification. Sometimes, even though a mathematical model of the process could be acquired, it may be too complex (e.g., strong nonlinearity, high orders, time-varying parameters, etc.) to be used for control design and stability analysis. Control system design directly from the process data bypassing the system modeling or identification has therefore received increasing attentions in the past years, see the survey papers (Markovsky and Dörfler (2021); Hou and Wang (2013)) and the references therein. Among various data-driven control schemes, the model-free adaptive control (MFAC) methodology has made great progress in both theory and application, owing to its capability of controlling nonaffine nonlinear systems, see e.g., Hou and Jin (2013) and Hou and Xiong (2019).

It is well known that multifarious disturbances are pervasive in industrial manufacturing systems, inevitably jeopardizing the control performance of such a closed-loop system. How to reject/attenuate the adverse effect of the unknown disturbance is a vital issue in the process control field, and has received considerable attentions over the past decades. To date, a number of anti-disturbance control techniques have been developed, e.g., disturbance observer based control, active disturbance rejection control (ADRC), equivalent-input-disturbance based control, etc., as surveyed in Chen et al. (2016). However, most of the existing anti-disturbance control methods depend on the process model more or less, and therefore are difficult even cannot be applied to complex industrial processes.

Recently, by incorporating the disturbance estimation technique into the data-driven control methodology, some data-driven anti-disturbance control (DDADC) methods have been explored in the literature. Two discrete-time ESO based MFAC schemes were proposed in Chi et al. (2020) in the light of local compact-form dynamic linearization (CFDL) and partial-form dynamic linearization (PFDL) models, respectively. However, the convergence analysis was not fully addressed therein. In Chi et al. (2021), a data-driven ADRC scheme with a rigorous stability proof was designed for both SISO and MIMO nonlinear systems under a constrained input variation rate, followed by a similar design based on a PFDL data model (Chi et al. (2023)). To achieve higher tracking accuracy of nonlinear systems with unknown disturbance, Huang et al.

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(2022) studied an ESO based model-free adaptive sliding mode control with prescribed tracking performance. In Shen et al. (2023), an extended disturbance observer-based data-driven control was developed for networked nonlinear systems with event-triggered output subject to unknown disturbance. Note that the residual term estimation was overlooked in the partial derivative/Jacobian matrix estimation algorithm in the aforementioned methods, so as to facilitate the convergence analysis. This may lead to biased parameter estimation in the above data model for control design, and thus degrade the system performance. As far as we are concerned, it remains open as yet to estimate the residual term and use it for designing a DDADC scheme along with rigorous convergence analysis, which motivates this study.

In this paper, two new AESO-based DDADC schemes are proposed for nonlinear systems with unknown dynamics and disturbance based on the CFDL and PFDL data models, respectively. Rigorous convergence analysis is presented for the parameter estimation algorithm, AESO and the closed-loop tracking error. The efficacy of the proposed schemes is demonstrated by an illustrative example.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider a nonlinear system with unknown dynamics and external disturbance, generally formulated by

$$\begin{aligned} y(t+1) = f\big(&y(t), \ldots, y(t-n_y), \\ &u(t), \ldots, u(t-n_u)\big) + d(t), \end{aligned} \tag{1}$$

where $u(t)$ and $y(t)$ are the system input and measurable output, respectively; $d(t)$ represents an unknown yet bounded disturbance; $f(\cdot)$ is an unknown nonlinear function with input order n_u and output order n_y.

The control objective in this paper is to develop a new data-driven anti-disturbance control scheme with convergence guarantee for a nonlinear system described by (1), such that the adverse effect of the unknown disturbance could be proactively attenuated.

For analysis, the following assumptions commonly used in the literature are made in this study.

Assumption 1. (Chi et al. (2021)) The partial derivatives of $f(\cdot)$ with respect to its arguments exist. Moreover, the sign of $\partial f(\cdot)/\partial u(t)$ is unchanged in the system behavior.

Assumption 2. (Chi et al. (2020, 2021)) The function $f(\cdot)$ in (1) satisfies the globally Lipschitz condition, i.e.

$$\begin{aligned} &|f(z_1(t_1), \ldots, z_N(t_1)) - f(z_1(t_2), \ldots, z_N(t_2))| \\ &\quad \le L_1|z_1(t_1) - z_1(t_2)| + \cdots + L_N|z_N(t_1) - z_N(t_2)|, \end{aligned} \tag{2}$$

where $N \in \mathbb{Z}_+$, $L_i > 0, i = 1, \ldots, N$ is the finite constant.

Assumption 3. The variation rate of control input satisfies $|\Delta u(t)| \le \beta_{\Delta u}$, $\forall t \in \mathbb{Z}_+$, where $\beta_{\Delta u} \in (0, 1)$ and $\Delta u(t) = u(t) - u(t-1)$.

Under Assumptions 1 and 2, the nonlinear system in (1) could be equivalently converted into the following CFDL model with a residual term (Chi et al. (2020))

$$\Delta y(t+1) = \phi(t)\Delta u(t) + \xi(t), \tag{3}$$

where $\Delta y(t) = y(t) - y(t-1)$, $\phi(t)$ is an unknown partial derivative (PD) of $f(\cdot)$ with respect to $u(t)$, and $\xi(t)$ represents the residual uncertainties in the CFDL model. It follows from Assumption 2 that $\phi(t)$ is bounded and satisfies $|\phi(t)| \le L_{n_y+2}$ for any $t \in \mathbb{Z}_+$. Moreover, by Assumptions 1-3 along with the bounded disturbance, the boundedness of $\xi(t)$ is guaranteed and assumed to satisfy $|\xi(t)| \le \beta_\xi$ for any $t \in \mathbb{Z}_+$, where β_ξ is a finite constant. Different from the conventional MFAC in Hou and Jin (2013) where the uncertainty term $\xi(t)$ in (3) is lumped into $\phi(t)$ to deal with, $\phi(t)$ and $\xi(t)$ are separately estimated in this study to facilitate the control design for improving system performance.

3. AESO-DDADC DESIGN AND CONVERGENCE ANALYSIS

This section details the proposed AESO-DDADC scheme, and robust convergence analysis of the PD estimation algorithm, AESO and the resulting tracking error.

To begin with, the following cost function is defined

$$J(u(t)) = |e(t+1)|^2 + \lambda|u(t) - u(t-1)|^2, \tag{4}$$

where $e(t) = y_{\mathrm{d}}(t) - y(t)$, $y_{\mathrm{d}}(t)$ is a reference trajectory (or zero for regulation) satisfying $|y_{\mathrm{d}}(t)| \le \beta_{\mathrm{d}}$ with β_{d} being a finite constant, and $\lambda > 0$ is a user-specified tuning parameter to evaluate the impact of input variation. Note that the constraint on the input variation in Assumption 3 may be satisfied by taking a larger λ at the expense of possibly slow convergence speed.

Taking the derivative of $J(u(t))$ with respect to $u(t)$ and equaling the result to zero yield

$$u(t) = u(t-1) + \frac{\rho\phi(t)\left(y_{\mathrm{d}}(t+1) - y(t) - \xi(t)\right)}{\lambda + \phi^2(t)}, \tag{5}$$

where $\rho > 0$ is a tuning parameter to make the control law more flexible. Owing to the unavailable $\phi(t)$ and $\xi(t)$, the control law in (5) cannot be practically implemented. To cope with this issue, we move on to design iterative algorithms for estimating $\phi(t)$ and $\xi(t)$, respectively.

Define another cost function associated with $\phi(t)$ as

$$\begin{aligned} J\left(\phi(t)\right) = &|\Delta y(t) - \phi(t)\Delta u(t-1) - \xi(t-1)|^2 \\ &+ \mu|\phi(t) - \hat{\phi}(t-1)|^2, \end{aligned} \tag{6}$$

where $\mu > 0$ is a user-specified tuning parameter to mitigate the large variation of $\phi(t)$ estimation. Similar to the establishment of the control law in (5), the PD estimation algorithm could be derived as

$$\begin{aligned} \hat{\phi}(t) = &\hat{\phi}(t-1) + \eta\Delta u(t-1)/(\mu + \Delta u^2(t-1)) \\ &\times \left[\Delta y(t) - \hat{\phi}(t-1)\Delta u(t-1) - \xi(t-1)\right], \end{aligned} \tag{7}$$

where $\eta \in (0, 2)$ is a tuning parameter to provide a flexible parameter estimation algorithm. Also, the unavailable residual term $\xi(t-1)$ still hinders the execution of the PD estimation algorithm in (7).

In order to estimate $\xi(t)$, the following augmented data model is constructed

$$\begin{cases} x(t+1) = Ax(t) + B_1\phi(t)\Delta u(t) + B_2\Delta\xi(t), \\ \quad\; y(t) = Cx(t), \end{cases} \tag{8}$$

where $C = [1 \;\; 0]$ and

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ x(t) = \begin{bmatrix} y(t) \\ \xi(t-1) \end{bmatrix}.$$

According to the augmented system in (8), a new data-driven AESO is proposed as below

$$\hat{x}(t+1) = A\hat{x}(t)+B_1\hat{\phi}(t-1)\Delta u(t)+L\left[y(t) - C\hat{x}(t)\right], \quad (9)$$

where L is the observer gain to be determined, $\hat{x}(t) = [\hat{y}(t), \hat{\xi}(t-1)]^\top$, $\hat{y}(t)$ and $\hat{\xi}(t-1)$ are the respective estimations of $x(t)$, $y(t)$ and $\xi(t-1)$.

Remark 4. Differing from the recently developed data-driven ESO designs (Chi et al. (2020, 2021, 2023); Chen et al. (2022)) where the PD estimation $\hat{\phi}(t)$ is directly adopted to construct AESO, the PD estimation at the previous time instant, i.e., $\hat{\phi}(t-1)$, is used in the new AESO to make the rigorous convergence analysis possible.

In summary, the proposed AESO-DDADC scheme is detailed as below:

Algorithm 1 (AESO-DDADC)

Input: Initial values of PD estimation $\hat{\phi}(0)$, control input $u(0)$ and process output $y(0)$, reference trajectory $y_{\mathrm{d}}(t)$, upper bound of input variation rate $\beta_{\Delta u}$,tuning parameters η, μ, ρ, λ, threshold ε, AESO gain L and maximum time step $t_{\max}$.
Output: Process output $y(t)$.
while $t \le t_{\max}$ **do**
  Update the PD estimation $\hat{\phi}(t)$ by (7) with $\xi(t-1)$ replaced by its estimation $\hat{\xi}(t-1)$;
  if $|\hat{\phi}(t)| \le \varepsilon$ or $|\Delta u(t-1)| \le \varepsilon$ or $\mathrm{sign}(\hat{\phi}(t)) \ne \mathrm{sign}(\hat{\phi}(0))$ **then**
    $\hat{\phi}(t) = \hat{\phi}(0)$;
  end if
  Update the AESO by (9);
  Compute $\zeta(t) = \frac{\rho\hat{\phi}(t)\left(y_{\mathrm{d}}(t+1)-y(t)-\hat{\xi}(t)\right)}{\lambda+\hat{\phi}^2(t)}$;
  if $|\zeta(t)| \le \beta_{\Delta u}$ **then**
    Update the control input by $u(t) = u(t-1)+\zeta(t)$;
  else if $|\zeta(t)| > \beta_{\Delta u}$ **then**
    Update the control input by $u(t) = u(t-1) + \beta_{\Delta u}\mathrm{sign}(\zeta(t))$;
  end if
  Apply the control input $u(t)$ to the system in (1);
end while

Remark 5. In contrast to the existing data-driven control methods (Chi et al. (2021, 2023); Chen et al. (2022)) where the uncertainty estimation $\hat{\xi}(t-1)$ is overlooked in the PD estimation algorithm to facilitate the boundedness proof of $\hat{\phi}(t)$, $\hat{\xi}(t-1)$ is incorporated in the PD estimation in Algorithm 1 to further improve the estimation accuracy.

For the convenience of convergence analysis, the following lemmas are imposed.

Lemma 6. (Bell (1965)) Let $A = [a_{ij}]_{n\times n}$ be a complex matrix and R_i be the sum of the moduli of the off-diagonal elements in the i-th row. Then, each eigenvalue of A lies in the unions of the circle

$$|z - a_{ii}| \le R_i = \sum_{j=1, j\ne i} |a_{ij}|,\ i \in \{1, \dots, n\}.$$

Lemma 7. (Huang (1984)) Let $A \in \mathbb{R}^{n\times n}$. For any $\delta > 0$, there exists a proper matrix norm $\|\cdot\|$ such that $\|A\| < s(A) + \delta$, where $s(A)$ is the spectral radius of matrix A.

Define $\tilde{x}(t) \triangleq x(t)–\hat{x}(t) = [\tilde{y}(t), \tilde{\xi}(t–1)]^\top$, and the evolution of AESO estimation error can be derived as

$$\tilde{x}(t+1) = (A-LC)\tilde{x}(t)+B_1\tilde{\phi}(t-1)\Delta u(t)+\Omega(t), \quad (10)$$

where $\Omega(t) = B_1\Delta\phi(t)\Delta u(t) + B_2\Delta\xi(t)$. It is not difficult to verify that $\Omega(t)$ is bounded and satisfies $\|\Omega(t)\|_2 \le 2\beta_\phi\beta_{\Delta u} + 2\sqrt{2}\beta_\xi \triangleq \beta_\Omega < \infty$ for any $t \in \mathbb{Z}_+$.

The following theorem establishes sufficient conditions to ensure the boundedness of $\hat{\phi}(t)$ and $\hat{\xi}(t)$ for any $t \in \mathbb{Z}_+$.

Theorem 8. Consider the controlled system described by (1) satisfying Assumptions 1-3, where the PD estimation and AESO in Algorithm 1 is applied. If the following two conditions are satisfied: (i) the parameters $\eta > 0, \mu > 0$ and $\gamma_1 \in (0, 1)$ are selected to satisfy $\eta < 2\sqrt{\mu}\gamma_1$; (ii) the AESO gain L is taken such that $s(A - LC) < 1 - \gamma_3$ with $\beta_{\Delta u} < \gamma_3 \in (0, 1)$; then the boundedness of $\hat{\phi}(t)$ and $\hat{\xi}(t)$ is ensured, i.e., $|\hat{\phi}(t)| \le \beta_{\hat{\phi}}$, $|\hat{\xi}(t)| \le \beta_{\hat{\xi}}, \forall t \in \mathbb{Z}_+$, where $\beta_{\hat{\phi}} > 0$ and $\beta_{\hat{\xi}} > 0$ are two finite constants.

Proof. It follows from Algorithm 1 that the dynamics of PD estimation error can be derived by

$$\begin{aligned}\tilde{\phi}(t) = \phi(t) - \hat{\phi}(t) = \left[1 - \frac{\eta\Delta u^2(t-1)}{\mu + \Delta u^2(t-1)}\right]\tilde{\phi}(t-1) + \Delta\phi(t) \\ - \frac{\eta\Delta u(t-1)}{\mu + \Delta u^2(t-1)}[0 \ \ 1]\tilde{x}(t).\end{aligned} \quad (11)$$

Taking the norm on both sides of (10) and (11) gives

$$\|\tilde{x}(t+1)\|_2 \le s(A-LC)\|\tilde{x}(t)\|_2+\beta_{\Delta u}|\tilde{\phi}(t-1)|+\beta_\Omega, \quad (12)$$

$$\begin{aligned}|\tilde{\phi}(t)| \le \left|1 - \frac{\eta\Delta u^2(t-1)}{\mu + \Delta u^2(t-1)}\right| |\tilde{\phi}(t-1)| + 2\beta_\phi \\ + \left|\frac{\eta\Delta u(t-1)}{\mu + \Delta u^2(t-1)}\right| \|\tilde{x}(t)\|_2.\end{aligned} \quad (13)$$

Since $\eta \in (0, 2)$, there is a constant $\gamma_1 \in (0, 1)$ such that

$$\left|1 - \frac{\eta\Delta u^2(t-1)}{\mu + \Delta u^2(t-1)}\right| < 1 - \gamma_1.$$

Under the condition (i), it follows that

$$\left|\frac{\eta\Delta u(t-1)}{\mu + \Delta u^2(t-1)}\right| \le \frac{\eta}{2\sqrt{\mu}} \triangleq \gamma_2 < \gamma_1.$$

By selecting the observer gain L appropriately, there is a constant $\gamma_3 \in (0, 1)$ such that $s(A - LC) \le 1 - \gamma_3$. Combining (12) with (13) yields

$$\tilde{\Theta}(t) \le \Gamma\tilde{\Theta}(t-1) + M, \quad (14)$$

where $M = [2\beta_\phi \ \ \beta_\Omega]^\top$ and

$$\Gamma = \begin{bmatrix}1-\gamma_1 & \gamma_2 \\ \beta_{\Delta u} & 1-\gamma_3\end{bmatrix},\ \tilde{\Theta}(t) = \begin{bmatrix}|\tilde{\phi}(t)| \\ \|\tilde{x}(t+1)\|_2\end{bmatrix}.$$

By Lemma 6 together with the conditions (i) and (ii), it is computed that the Gerschgorin disks of matrix Γ satisfy

$$S_1 = \left\{z_1 \middle| |z_1 - (1-\gamma_1)| < \gamma_2\right\} \subset \left\{z_1 \middle| |z_1| < 1-\gamma_1+\gamma_2 < 1\right\},$$
$$S_2 = \left\{z_2 \middle| |z_2 - (1-\gamma_3)| < \beta_{\Delta u}\right\} \subset \left\{z_2 \middle| |z_2| < 1-\gamma_3+\beta_{\Delta u} < 1\right\},$$

implying that the spectral radius of Γ is less than one strictly, i.e., $s(\Gamma) = \max_{i=1,2}\{|z_i|\} < 1$. Thus, it follows from Lemma 7 that there exists a small constant ϱ such that $\|\Gamma\| \le s(\Gamma) + \varrho \triangleq \bar{\varrho} < 1$.

Taking a proper norm on both sides of (14) gives

$$\begin{aligned}\|\tilde{\Theta}(t)\| &\le \|\Gamma\|\|\tilde{\Theta}(t-1)\| + \|M\| \\ &\le \bar{\varrho}\|\tilde{\Theta}(t-1)\| + \|M\| \le \cdots \le \bar{\varrho}^t\|\tilde{\Theta}(0)\| + \frac{\|M\|}{1-\bar{\varrho}}.\end{aligned}$$

Therefore, it is concluded that $\|\tilde{\Theta}(t)\|$ is boundedly convergent under a bounded initial estimation error $\tilde{\Theta}(0)$. This indicates that both $\tilde{\xi}(t)$ and $\tilde{\phi}(t)$ are bounded for any $t \in \mathbb{Z}_+$. According to the boundedness of $\phi(t)$ and $\xi(t)$, the conclusion of this theorem follows immediately. This completes the proof. ■

Given the boundedness of $\hat{\phi}(t)$ and $\hat{\xi}(t)$ established in Theorem 8, robust convergence analysis of the tracking error under the proposed AESO-DDADC scheme in Algorithm 1 is presented below.

Theorem 9. Consider the controlled system described by (1) satisfying Assumptions 1-3, where Algorithm 1 is applied. If the parameters ρ and λ are taken such that

$$\lambda > \rho^2 L^2_{n_y+2}/4, \tag{15}$$

the tracking error $e(t)$ converges to a bound of $\frac{\beta_q}{1-c}$ as $t \to \infty$, where $\beta_q = 2\beta_{\mathrm{d}} + \beta_\xi + \frac{\rho\beta_\phi}{2\sqrt{\lambda}}(2\beta_{\mathrm{d}} + \beta_{\hat{\xi}})$ and $c \in (0,1)$ is a constant. Particularly, β_q is reduced to $\beta_\xi + \frac{\rho\beta_\phi}{2\sqrt{\lambda}}\beta_{\hat{\xi}}$ when $y_{\mathrm{d}}(t)$ is a constant-type reference trajectory.

Proof. Let $h(t) = \Delta u(t)/\zeta(t)$. It follows that $h(t) = 1$ if $|\zeta(t)| \le \beta_{\Delta u}$ and $h(t) \in (0,1)$ if $|\zeta(t)| > \beta_{\Delta u}$. Using the CFDL model in (3) and the feedback control law in Algorithm 1, the tracking error dynamics is derived by

$$e(t{+}1) = \left[1 - \rho h(t)\phi(t)\hat{\phi}(t)/(\lambda + \hat{\phi}^2(t))\right] e(t){+}q(t), \tag{16}$$

where

$$q(t) = \Delta y_{\mathrm{d}}(t+1) - \frac{\rho\phi(t)\hat{\phi}(t)}{\lambda + \hat{\phi}^2(t)}\left[\Delta y_{\mathrm{d}}(t+1) - \hat{\xi}(t)\right] - \xi(t).$$

By virtue of the bounded $\phi(t)$ and $\xi(t)$ and their estimations, the boundedness of $q(t)$ is guaranteed, i.e., $|q(t)| \le \beta_q$ for any $t \in \mathbb{Z}_+$.

By Assumption 1 and the initial resetting condition of parameter estimation algorithm for $\phi(t)$, there stands $\phi(t)\hat{\phi}(t) > 0$. Thus, it follows from (15) that

$$0 < \rho h(t)\phi(t)\hat{\phi}(t)/(\lambda{+}\hat{\phi}^2(t)) < \rho L_{n_y+2}/(2\sqrt{\lambda}) < 1, \tag{17}$$

which indicates that there is a constant $c \in (0,1)$ satisfying

$$\left|1 - \rho h(t)\phi(t)\hat{\phi}(t)/(\lambda + \hat{\phi}^2(t))\right| < c < 1. \tag{18}$$

Thus, one has

$$|e(t+1)| \le c|e(t)| + \beta_q \le \cdots \le c^{t+1}|e(0)| + \beta_q/(1-c).$$

Due to the finite initial tracking error $e(0)$, the conclusion holds. This completes the proof. ■

4. PFDL-BASED AESO-DDADC DESIGN

Consider another equivalent PFDL data model of the nonlinear system in (1) as follows

$$\Delta y(t+1) = \mathbf{\Phi}_H^\top(t)\Delta \boldsymbol{u}_H(t) + \vartheta(t), \tag{19}$$

where $\Delta \boldsymbol{u}_H(t) = [\Delta u(t), \ldots, \Delta u(t-H+1)]^\top$, $H > 0$ is a positive integer representing the linearization length, $\mathbf{\Phi}_H(t) = [\phi_1(t), \ldots, \phi_H(t)]^\top$ is the gradient vector of $f(\cdot)$ with respect to $\boldsymbol{u}_H(t)$, $\vartheta(t)$ is a nonlinear residual term, see Chi et al. (2020) for more details. Based on Assumptions 1-3 and the boundedness of disturbance, both $\mathbf{\Phi}_H(t)$ and $\vartheta(t)$ are bounded and assumed to satisfy $\|\mathbf{\Phi}_H(t)\| \le \beta_{\mathbf{\Phi}}$ and $|\vartheta(t)| < \beta_\vartheta$ for any $t \in \mathbb{Z}_+$, where $\beta_{\mathbf{\Phi}}$ and β_ϑ are two finite constants.

By optimizing the cost function in (4), the control law under the PFDL model in (19) can be derived as

$$\begin{aligned} u(t) = u(t-1) &+ \frac{\rho\phi_1(t)(y_{\mathrm{d}}(t+1) - y(t) - \vartheta(t))}{\lambda + \phi_1^2(t)} \\ &- \frac{\rho\phi_1(t)\sum_{i=2}^H \phi_i(t)\Delta u(t-i+1)}{\lambda + \phi_1^2(t)}, \end{aligned} \tag{20}$$

where ρ is the same as that in (5). Analogous to the PD estimation algorithm in Section 3, minimizing the following cost function

$$\begin{aligned} \bar{J}\left(\mathbf{\Phi}_H(t)\right) = &\left|\Delta y(t) - \mathbf{\Phi}_H^\top(t)\Delta \boldsymbol{u}_H(t{-}1) - \vartheta(t-1)\right|^2 \\ &+ \mu\|\mathbf{\Phi}_H(t) - \hat{\mathbf{\Phi}}_H(t-1)\|_2^2 \end{aligned} \tag{21}$$

yields the gradient estimation algorithm described as

$$\begin{aligned} \hat{\mathbf{\Phi}}_H(t) = \hat{\mathbf{\Phi}}_H(t-1) &+ \frac{\eta\Delta \boldsymbol{u}_H(t-1)}{\mu + \|\Delta \boldsymbol{u}_H(t-1)\|_2^2} \\ &\times \left[\Delta y(t) - \hat{\mathbf{\Phi}}_H^\top(t{-}1)\Delta \boldsymbol{u}_H(t{-}1) - \vartheta(t{-}1)\right], \end{aligned} \tag{22}$$

where η and μ are the same as those in (7), and $\hat{\mathbf{\Phi}}_H(t) = [\hat{\phi}_1(t), \ldots, \hat{\phi}_H(t)]^\top$ is the estimation of $\mathbf{\Phi}_H(t)$.

By augmenting $\vartheta(t)$ as a new state, the PFDL model in (19) is transformed into

$$\begin{cases} \bar{x}(t+1) = A\bar{x}(t) + B_1\mathbf{\Phi}_H^\top(t)\Delta \boldsymbol{u}_H(t) + B_2\Delta\vartheta(t), \\ \quad y(t) = C\bar{x}(t), \end{cases} \tag{23}$$

where $\bar{x}(t) = [y(t), \vartheta(t-1)]^\top$, A, B_1, B_2 and C are defined in (8). Correspondingly, the new AESO is designed as

$$\hat{\bar{x}}(t{+}1) = A\hat{\bar{x}}(t) + B_1\hat{\mathbf{\Phi}}_H^\top(t{-}1)\Delta \boldsymbol{u}_H(t) + L\left[y(t) - C\hat{\bar{x}}(t)\right], \tag{24}$$

where L is the observer gain, $\hat{\bar{x}}(t) = [\hat{y}(t), \hat{\vartheta}(t-1)]^\top$ and $\hat{\vartheta}(t-1)$ are the estimations of $\bar{x}(t)$ and $\vartheta(t-1)$, respectively.

To sum up, the proposed another PFLD-based AESO-DDADC scheme is given as below:

Theorem 10. Consider the controlled system described by (1) satisfying Assumptions 1-3, where the gradient estimation and AESO in Algorithm 2 are applied. If the following two conditions are satisfied: (i) the parameters $\eta > 0, \mu > 0$ and $\bar{\gamma}_1 \in (0,1)$ are selected to satisfy $\eta < 2\sqrt{\mu}\bar{\gamma}_1$; (ii) the observer gain L is taken such that $s(A - LC) < 1 - \bar{\gamma}_3$ with $\sqrt{H}\beta_{\Delta u} < \bar{\gamma}_3 \in (0,1)$; then, the boundedness of $\hat{\mathbf{\Phi}}_H(t)$ and $\hat{\vartheta}(t)$ is simultaneously guaranteed, i.e., $\|\hat{\mathbf{\Phi}}_H(t)\|_2 \le \beta_{\hat{\mathbf{\Phi}}}$, $|\hat{\vartheta}(t)| \le \beta_{\hat{\vartheta}}, \forall t \in \mathbb{Z}_+$, where $\beta_{\hat{\mathbf{\Phi}}} > 0$ and $\beta_{\hat{\vartheta}} > 0$ are two finite constants.

Proof. The proof of this theorem is similar to that of Theorem 8 and thus is omitted owing to the page limit. ■

Before presenting the bounded convergence of the tracking error $e(t)$ under the proposed PFDL-based AESO-DDADC scheme in Algorithm 2, the following useful lemma is introduced.

Lemma 11. (Jury (1964)) Let

$$A = \begin{bmatrix} a_1 & a_2 & \ldots & a_n \\ 1 & 0 & \ldots & 0 \\ & \ddots & \ddots & \vdots \\ & & 1 & 0 \end{bmatrix}.$$

If $\sum_{i=1}^n |a_i| < 1$, then $s(A) < 1$.

Algorithm 2 (PFDL-based AESO-DDADC)

Input: Initial values of gradient estimation $\hat{\mathbf{\Phi}}_H(0)$, control input $u(0)$ and process output $y(0)$, reference trajectory $y_\mathrm{d}(t)$, upper bound of input variation rate $\beta_{\Delta u}$, tuning parameters η, μ, ρ, λ, threshold ε, AESO gain L, linearization length H and maximum time step $t_{\max}$.
Output: Process output $y(t)$.
while $t \leq t_{\max}$ **do**
Update the gradient estimation $\hat{\mathbf{\Phi}}_H(t)$ by (22) with $\vartheta(t-1)$ replaced by its estimation $\hat{\vartheta}(t-1)$;
if $\|\hat{\mathbf{\Phi}}_H(t)\|_2 \leq \varepsilon$ or $\|\Delta \boldsymbol{u}_H(t-1)\|_2 \leq \varepsilon$ or $\mathrm{sign}(\hat{\mathbf{\Phi}}_H(t)) \neq \mathrm{sign}(\hat{\mathbf{\Phi}}_H(0))$ **then**
$\hat{\mathbf{\Phi}}_H(t) = \hat{\mathbf{\Phi}}_H(0)$;
end if
Update the AESO by (24);
Compute

$$\bar{\zeta}(t) = \rho\hat{\phi}_1(t)(y_\mathrm{d}(t+1) - y(t) - \hat{\vartheta}(t))/(\lambda + \hat{\phi}_1^2(t)) - \rho\hat{\phi}_1(t)\sum_{i=2}^{H}\hat{\phi}_i(t)\Delta u(t-i+1)/(\lambda + \hat{\phi}_1^2(t));$$

if $|\bar{\zeta}(t)| \leq \beta_{\Delta u}$ **then**
Update the control input by $u(t) = u(t-1) + \bar{\zeta}(t)$;
else if $|\bar{\zeta}(t)| > \beta_{\Delta u}$ **then**
Update the control input by $u(t) = u(t-1) + \beta_{\Delta u}\mathrm{sign}(\bar{\zeta}(t))$;
end if
Apply the control input $u(t)$ to the system in (1);
end while

Theorem 12. Consider the nonlinear system described by (1) under Assumptions 1-3, where the PFDL-based AESO-DDADC scheme in Algorithm 2 is applied. If the parameter ρ and λ are properly chosen, then the tracking error $e(t)$ converges to a finite bound as $t \to \infty$.

Proof. By letting $\bar{h}(t) = \frac{\Delta u(t)}{\bar{\zeta}(t)}$, it follows that $\bar{h}(t) = 1$ if $|\bar{\zeta}(t)| \leq \beta_{\Delta u}$ and $\bar{h}(t) \in (0,1)$ if $|\bar{\zeta}(t)| > \beta_{\Delta u}$. According to the control law in Algorithm 2, one has

$$\Delta \boldsymbol{u}_H(t) = \mathscr{A}(t)\Delta \boldsymbol{u}_H(t-1) + \boldsymbol{N}(t), \tag{25}$$

where $\boldsymbol{N}(t) = [N_1(t)\ \ 0\dots 0]^\top$, $\kappa_i(t) = -\frac{\rho\bar{h}(t)\hat{\phi}_1(t)\hat{\phi}_i(t)}{\lambda+\hat{\phi}_1^2(t)}, i = 2,\dots,H$, $N_1(t) = \frac{\rho\bar{h}(t)\hat{\phi}_1(t)(y_\mathrm{d}(t+1)-y(t)-\hat{\vartheta}(t))}{\lambda+\hat{\phi}_1^2(t)}$ and

$$\mathscr{A}(t) = \begin{bmatrix} \kappa_2(t) & \kappa_3(t) & \cdots & \kappa_H(t) & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}.$$

Owing to the boundedness of $\mathbf{\Phi}_H(t)$ and $\hat{\mathbf{\Phi}}_H(t)$, there exist finite constants $M_i, i = 2,3,4$ and a proper λ such that

$$\begin{aligned} &\left|\bar{h}(t)\hat{\phi}_1(t)/(\lambda + \hat{\phi}_1^2(t))\right| \leq \frac{1}{2\sqrt{\lambda}} \triangleq M_1 < \frac{0.5}{\beta_{\mathbf{\Phi}}}, \\ &0 < M_2 \leq \left|\frac{\bar{h}(t)\phi_1(t)\hat{\phi}_i(t)}{\lambda + \hat{\phi}_1^2(t)}\right| \leq \frac{\beta_{\mathbf{\Phi}}}{2\sqrt{\lambda}} < 0.5, \\ &M_1\|\mathbf{\Phi}_H(t)\| \leq M_3 < 0.5,\ M_2 + M_3 < 1, \\ &\left(\sum_{i=2}^{H}\left|\bar{h}(t)\hat{\phi}_1(t)\hat{\phi}_i(t)/(\lambda + \hat{\phi}_1^2(t))\right|\right)^{\frac{1}{H-1}} \leq M_4, \end{aligned} \tag{26}$$

By properly taking the parameter ρ, it follows that

$$\sum_{i=2}^{H}\rho\left|\bar{h}(t)\hat{\phi}_1(t)\hat{\phi}_i(t)/(\lambda + \hat{\phi}_1^2(t))\right| \leq \rho M_4^{H-1} \triangleq M_5 < 1,$$

which, by Lemma 11 and Theorem 4.3 in Hou and Jin (2013), gives $\|\mathscr{A}(t)\| < s(\mathscr{A}) + \epsilon \leq \rho^{\frac{1}{H-1}}M_4 + \epsilon < 1$, where ϵ is a sufficiently small positive constant.

Taking the norm on both sides of (25) gives

$$\|\Delta \boldsymbol{u}_H(t)\| \leq \|\mathscr{A}(t)\|\|\Delta \boldsymbol{u}_H(t-1)\| + \rho M_1\Big(2\beta_\mathrm{d} + \beta_{\hat{\vartheta}} + |e(t)|\Big) \leq \rho M_1\sum_{i=1}^{t}\sigma_1^{t-i}|e(i)| + c_1,$$

where $\sigma_1 \triangleq \rho^{\frac{1}{H-1}}M_4 + \epsilon$, $c_1 = \frac{\rho M_1}{1-\sigma_1}(2\beta_\mathrm{d} + \beta_{\hat{\vartheta}})$ and $\Delta \boldsymbol{u}_H(0)$ is set as zero.

Moreover, it follows from (19) and (25) that

$$e(t+1) = \left[1 - \frac{\rho\bar{h}(t)\phi_1(t)\hat{\phi}_1(t)}{\lambda + \hat{\phi}_1^2(t)}\right]e(t) + \Delta y_\mathrm{d}(t+1) - \vartheta(t) - \frac{\rho\bar{h}(t)\phi_1(t)\hat{\phi}_1(t)}{\lambda + \hat{\phi}_1^2(t)}(\Delta y_\mathrm{d}(t+1) - \hat{\vartheta}(t)) - \mathbf{\Phi}_H^\top(t)\mathscr{A}(t)\Delta \boldsymbol{u}_H(t-1).$$

By properly taking $\rho \in (0,1)$, it follows from (26) that

$$\left|1 - \rho\bar{h}(t)\phi_1(t)\hat{\phi}_1(t)/(\lambda + \hat{\phi}_1^2(t))\right| \leq 1 - \rho M_2 \triangleq \sigma_2 < 1.$$

Thus, one has

$$|e(t+1)| \leq g(t+1) + c_2/(1-\sigma_2), \tag{27}$$

where $c_2 \triangleq 2\beta_\mathrm{d} + \beta_\vartheta + 0.5\rho(2\beta_\mathrm{d} + \beta_{\hat{\vartheta}}) + \beta_{\mathbf{\Phi}}\sigma_1 c_1$, $g(t+1) \triangleq \sigma_2^t|e(1)| + \sigma_3\sum_{j=1}^{t-1}\sigma_2^{t-1-j}\sum_{i=1}^{j}\sigma_1^{j-i+1}|e(i)|$ with $g(2) = \sigma_2|e(1)|$ and $\sigma_3 \triangleq \rho M_3$.

Consequently, it can be deduced that

$$\begin{aligned} g(t+2) &= \sigma_2^{t+1}|e(1)| + \sigma_3\sum_{j=1}^{t}\sigma_2^{t-j}\sum_{i=1}^{j}\sigma_1^{j-i+1}|e(i)| \\ &\leq \sigma_2 g(t+1) + p(t) + c_3, \end{aligned} \tag{28}$$

where $p(t) = \sigma_3\sigma_1^t|e(1)| + \cdots + \sigma_3\sigma_1^2|e(t-1)| + \sigma_3\sigma_1 g(t)$ and $c_3 \triangleq \frac{\sigma_3\sigma_1 c_2}{1-\sigma_2}$. Since $M_2 + M_3 < 1$, one has $\sigma_2 = 1 - \rho M_2 > \rho(M_2 + M_3) - \rho M_2 = \sigma_3$, which implies that

$$\begin{aligned} p(t) &< \sigma_3\sigma_1^t|e(1)| + \cdots + \sigma_3\sigma_1^2|e(t-1)| + \sigma_2\sigma_1 g(t) \\ &= \sigma_1\left[\sigma_2^t|e(1)| + \sigma_3\sum_{j=1}^{t-1}\sigma_2^{t-1-i}\sum_{i=1}^{j}\sigma_1^{j-i+1}|e(i)|\right] = \sigma_1 g(t+1). \end{aligned}$$

Thus, it follows that

$$g(t+2) \leq (\sigma_1 + \sigma_2)g(t+1) + c_3. \tag{29}$$

By properly taking ρ such that $\rho^{\frac{1}{H-1}}M_4 < \rho M_2$, one has $0 < 1 - \rho M_2 + \rho^{\frac{1}{H-1}}M_4 < 1$. Therefore, there stands $\sigma_1 + \sigma_2 = 1 - \rho M_2 + \rho^{\frac{1}{H-1}}M_4 + \epsilon < 1$ with a sufficiently small ϵ. Therefore, it follows from (29) that

$$g(t+2) \leq (\sigma_1 + \sigma_2)^t g(2) + \frac{c_3}{1 - (\sigma_1 + \sigma_2)} \to \frac{c_3}{1 - (\sigma_1 + \sigma_2)},$$

as $t \to \infty$, which, together with (27), indicates the bounded convergence of tracking error $e(t)$. This completes the proof. ■

5. AN ILLUSTRATIVE EXAMPLE

Consider a nonlinear system studied in Hou and Jin (2013); Chi et al. (2020, 2021)

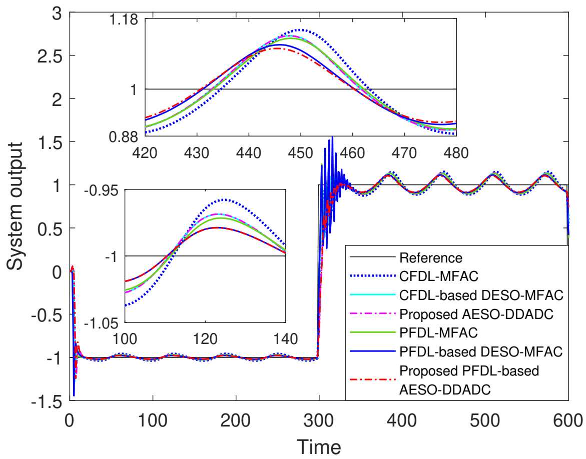

Fig. 1. Control results by different methods

$$y(t+1)=\begin{cases} y(t)/(1+y^2(t))+u^3(t)+d(t),\ t\le 300,\\ \dfrac{y(t)y(t-1)y(t-2)u(t-1)(y(t-2)-1)}{1+y^2(t-1)+y^2(t-2)}\\ +\dfrac{a(t)u(t)}{1+y^2(t-1)+y^2(t-2)}+d(t),\ t>300,\end{cases}$$

where $a(t)=\text{round}(t/500)$. The desired trajectory is set as $y_d(t)=(-1)^{\text{round}(t/300+0.5)}$ and the disturbance is taken as $d(t)=0.2\sin(t/10)$. For illustration, the parameters in Algorithm 1 are taken as $\mu=1$, $\lambda=0.1$, $\rho=0.3$, $\eta=0.1$, $\varepsilon=1\times10^{-5}$, $\beta_{\Delta u}=0.2$, $L=[0.8,0.2]^\top$, and the initial values are $u(0)=0$, $y(0)=0$, $\hat{x}(0)=[0,0]^\top$ and $\hat{\phi}(0)=1$. It can be verified that convergence conditions in Theorem 8 hold. For comparison, the existing CFDL-MFAC (Hou and Jin (2013)) and CFDL-based DESO-MFAC (Chi et al. (2020)) are also applied under the same parameter settings. The simulation results are shown in Fig. 1. It is seen that the proposed AESO-DDADC scheme exhibits an improved tracking performance compared with CFDL-based DESO-MFAC and CFDL-MFAC, respectively.

Moreover, the proposed PFDL-based AESO-DDADC design along with PFDL-MFAC (Hou and Jin (2013)) and PFDL-based DESO-MFAC (Chi et al. (2020)) are also performed by taking the linearization length as $H=2$ to satisfy the conditions in Theorem 10 and the initial value of gradient vector as $\hat{\mathbf{\Phi}}_H(0)=[0.5,0.1]^\top$. The rest parameters are chosen as the same with the above case. The corresponding simulation results are also shown in Fig. 1. It is seen that the steady-state tracking performance by the proposed PFDL-based AESO-DDADC scheme outperforms those by PFDL-MFAC and PFDL-based DESO-MFAC, while all the PFDL-based methods deliver improved steady-state tracking performance compared with the CFDL-based ones. Note that the output responses with smaller oscillation at the jumping point ($t=300$) are obtained by both of the proposed designs by taking into consideration of the constraint on input variation rate, compared to the severe oscillation by the PFDL-MFAC and PFDL-based DESO MFAC methods.

6. CONCLUSIONS

For nonlinear systems with unknown dynamics subject to external disturbance, this paper has proposed two new AESO-based DDADC schemes based on CFDL and PFDL data models, respectively. To estimate the residual term in the CFDL/PFDL model for active compensation in the parameter estimation algorithm and counteraction by the feedback control law, a new AESO is developed by using the PD/gradient estimation at the previous time step, rather than the current time step in the existing data-driven ESO (e.g., Chi et al. (2020, 2021, 2023); Chen et al. (2022); Shen et al. (2023)). Robust convergence of the proposed schemes has been analyzed rigorously. An illustrative example from the literature has well validated the efficacy and advantages of the proposed schemes.

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