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A Virtual Cycle-based Iterative learning Control Framework for Repetitive System with Randomly Varying Initial State

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Abstract: Iterative learning control (ILC) has been considered a powerful strategy for repetitive process control. However, a fundamental assumption of conventional ILC is that each cycle must start from a predetermined fixed initial state. This assumption can be strict and challenging to achieve in real-world industrial applications. To address the issues arising from varying initial states, we propose an ILC framework that learns from a virtual cycle generated using historical data. We establish three conditions for generating the virtual cycle, and theoretical results demonstrate guaranteed convergence. To ensure the practicality of our framework, we relax one of the conditions, enabling the virtual cycle to be generated by solving a convex optimization problem. The effectiveness of our framework in improving control performance is verified through an injection molding example.

Keywords: Iterative learning control, Repetitive process, Process control, Optimization, Chemical engineering

1. INTRODUCTION

Repetitive processes play a significant role in industrial manufacturing across various sectors, including specialty chemicals, pharmaceuticals, and polymers (Gao et al. (2021, 2024); Lu et al. (2019)). The use of Iterative Learning Control (ILC) has gained traction as an effective control strategy for repetitive processes due to its ability to leverage repeatability and address uncertainties according to Bristow et al. (2006) and Ahn et al. (2007). Consequently, since its initial development by Arimoto et al. (1984), ILC has garnered considerable attention and has been extensively studied in both theoretical and practical contexts (Bu et al. (2017); Son et al. (2013); Amann et al. (1996)).

In conventional ILC scheme, it is typically assumed that the repetitive task begins from a predetermined fixed initial state (Moore et al. (1992); Xu (2011)). However, in practical engineering applications, achieving this condition can often be challenging (Wei et al. (2023); Wei and Li (2017)). For instance, in the case of a robot manipulator, the precision of the actuator may prevent it from perfectly resetting to the desired initial state, leading to random variations in the initial state from one cycle to another (Tao et al. (2020)). Similarly, in chemical production processes, the initial temperature of a reactor can be influenced by external environmental factors. These practical requirements necessitate the development of ILC designs that can effectively handle systems with randomly varying initial states.

The utilization of average operator-based ILC has emerged as an effective approach for mitigating the challenges posed by randomly varying initial states (Li et al. (2013); Wei and Li (2017)). Pioneered by Park (2005), the average operator-based ILC was initially proposed as a means to address the issue of varying initial states. Subsequently, Li et al. (2013) conducted rigorous analyses, demonstrating the assured convergence of tracking error in terms of mathematical expectation. Furthermore, Li et al. (2015)

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further enhanced this method by incorporating only a limited number of recent cycles, successfully applying it to nonlinear systems. Wei and Li (2017), on the other hand, introduced a weighted sum approach instead of the average operator, thereby the control law is extended to a more general high-order ILC form. These existing approaches effectively alleviate the adverse effects of initial state variations. However, they lack an adjustment mechanism based on forthcoming cycle information. Consequently, by harnessing the upcoming cycle information, there exists an opportunity to further enhance controller performance.

In this paper, we propose a virtual cycle-based ILC framework for systems with randomly varying initial states. Motivated by the work of Wei and Li (2017), we introduce the concept of generating a virtual cycle using a linear combination of historical data. Notably, the manner in which the virtual cycle is formed is adjusted based on the upcoming cycle's initial state information. The contribution of this paper can be summarized as follows.

- We propose a novel ILC framework that learns from a generated virtual cycle. The virtual cycle is formed by a linear combination of historical data and the upcoming cycle's initial state information. This approach effectively addresses the issue of initial state variation, enhancing the effectiveness of iterative learning.
- We have refined three conditions necessary to create an ideal virtual cycle. Building upon these conditions, we provide a rigorous theoretical analysis that demonstrates the convergence of our framework.
- We provide a feasible algorithm for implementing the virtual cycle-based ILC framework. The effectiveness of the algorithm is verified through a numerical example.

The remainder of this paper is organized as follows: Section 2 formulates the problem and provides the necessary preliminaries. Section 3 presents the design of the ILC framework, along with theoretical analysis and a feasible algorithm for implementing our framework. The results of the numerical example are presented in Section 4, followed by the conclusion in Section 5.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1 Problem Formulation

Consider the following unknown linear system

$$\begin{aligned} x_k(t+1) &= Ax_k(t) + Bu_k(t), \\ y_k(t+1) &= Cx_k(t+1), \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$, $y \in \mathbb{R}^{n_y}$ are the system states, inputs, and outputs respectively. $k \in [1, +\infty)$ is cycle index, $t \in [0, t_N - 1]$ is the time index. A, B, and C are the system matrices with appropriate dimension, yet their precise values are not available.

In this paper, we consider a tracking task starting from a randomly varying initial state. Compared to the conventional ILC tracking task, the inital state $x_k(0)$ of our task would vary randomly from cycle to cycle. Without loss of generality, we have following assumptions on initial state.

Assumption 1. For $\forall k \in [1, +\infty)$, the randomly varying initial state $x_k(0)$ is bounded by a convex hull Ω.

Assumption 2. The randomly varying initial state $x_k(0)$ can't be manipulated, but can be measured.

The control objective is to steer the system output to a given reference $y_d(t) \in \mathbb{R}^{n_y}$, for $t \in [1, t_N]$. We assume that the reference is carefully designed so that it is always realizable. That is, for any initial state $x(0)$ satisfying Assumption 1-2, there exists an input sequence $u_{d|x(0)}(t)$, such that

$$\begin{aligned} x_d(t+1) &= Ax_d(t) + Bu_{d|x(0)}(t), \\ y_d(t+1) &= Cx_d(t+1), \end{aligned} \tag{2}$$

holds for any $t \in [0, t_N - 1]$, where

$$x_d(0) = x(0).$$

To quantify the control performance, we define the whole cycle control error

$$\mathbf{e}_k = [e_k^\top(1), e_k^\top(2), \ldots, e_k^\top(t_N)]^\top \in \mathbb{R}^{n_y t_N}, \tag{3}$$

where

$$e_k(t) = y_d(t) - y_k(t). \tag{4}$$

And our main task in this paper is to design an model-free ILC controller so that $\|\mathbf{e}_k\|_2$ can always converge with existence of the randomly varying initial state from cycle to cycle.

Before facilitating the ILC design, we make the following preparations. Define

$$\begin{aligned} \mathbf{u}_k &= [u_k^\top(0), u_k^\top(1), \ldots, u_k^\top(t_N - 1)]^\top \in \mathbb{R}^{n_u t_N}, \\ \mathbf{y}_k &= [y_k^\top(1), y_k^\top(2), \ldots, y_k^\top(t_N)]^\top \in \mathbb{R}^{n_y t_N}, \end{aligned}$$

then (1) can be reorganized into lifted form

$$\mathbf{y}_k = G\mathbf{u}_k + Hx_k(0), \tag{5}$$

where

$$G := \begin{bmatrix} CB & 0 & \ldots & 0 \\ CAB & CB & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{t_N-1}B & CA^{t_N-2}B & \ldots & CB \end{bmatrix} \in \mathbb{R}^{n_y t_N \times n_u t_N}$$

$$H := \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{t_N} \end{bmatrix} \in \mathbb{R}^{n_y t_N \times n_x}.$$

Similarly, define

$$\mathbf{y}_d = [y_d^\top(1), y_d^\top(2), \ldots, y_d^\top(t_N)]^\top \in \mathbb{R}^{n_y t_N},$$

and (4) can also be reorganized into lifted form

$$\mathbf{e}_k = \mathbf{y}_d - (G\mathbf{u}_k + Hx_k(0)). \tag{6}$$

In the following part, the design and analysis would based on the lifted form representation given by (5)-(6).

2.2 Preliminaries on ILC

The conventional ILC laws typically involve direct learning from the previous cycle, as described by Lee et al. (2000); Bristow et al. (2006). A common form of these laws can be expressed as follows:

$$\mathbf{u}_k = \mathbf{u}_{k-1} + L\mathbf{e}_{k-1}. \tag{7}$$

To ensure the effectiveness of the learning process, we make the following assumption regarding the learning gain L:

Assumption 3. A suitable learning gain L is available to us, such that the condition

$$\|(I - GL)\|_2 < 1$$

is satisfied, where I denotes the unit matrix of the corresponding dimension.

However, the effectiveness of the learning mechanism described in (7) diminishes when the system exhibits initial state variations, leading to the introduction of additional error.This occurs due to the relationship described by (6), which can be extended to:

$$\mathbf{e}_k - \mathbf{e}_{k-1} = -(G(\mathbf{u}_k - \mathbf{u}_{k-1}) + H(x_k(0) - x_{k-1}(0))). \tag{8}$$

By substituting (7) into (8), we obtain:

$$\mathbf{e}_k - \mathbf{e}_{k-1} = -GL\mathbf{e}_{k-1} - H(x_k(0) - x_{k-1}(0)). \tag{9}$$

After simple arrangement of (9), we have

$$\mathbf{e}_k = (I - GL)\mathbf{e}_{k-1} - H(x_k(0) - x_{k-1}(0)), \tag{10}$$

where the second term on the right-hand side represents the additional error caused by the variation in the initial state. The existence of this term implies that the control performance may deteriorate further after the iterative learning, thus indicating the ineffectiveness of the learning mechanism.

The objective of this paper is to put forward an innovative ILC framework, grounded in (7), with the purpose of accommodating the challenge posed by randomly varying initial states.

3. VIRTUAL-CYCLE LEARNING ILC FRAMEWORK

3.1 ILC design

From (10), it is clear that the initial state variation would weaken the effectiveness of ILC. Assuming the system is going to implement cycle k with an observed initial state $x_k(0)$, the key issue is how to establish the ILC control law in order to effectively mitigate the negative impact of initial state variation on control performance. Therefore, in this paper, we propose a novel ILC framework that incorporates a novel concept of learning from a virtual cycle. This virtual cycle is generated using historical input-output data, with the aim of addressing the challenge of randomly varying initial states. The virtual cycle is desgned to share the same initial state as the upcoming cycle k, allowing learning from such a virtual cycle to intrinsically avoid the issue of initial state variation. As a result, the virtual cycle shall satisfy the following condition:

$$x_k^\star(0) = \sum_{i=1}^{m} w_i x_{k-i}(0) = x_k(0). \tag{11}$$

Here, the superscript $\star$ indicates the generated virtual cycle. The window size m determines the number of historical cycles used to generate the virtual cycle. To achieve the desired virtual cycle, we introduce weights w_i that facilitate a linear combination of the historical data. These weights are designed to satisfy:

$$\sum_{i=1}^{m} w_i = 1. \tag{12}$$

By leveraging the virtual cycle created from (11), (12), our proposed ILC control law is in the form of

$$\mathbf{u}_k = \mathbf{u}_k^\star + L\mathbf{e}_k^\star, \tag{13}$$

where

$$\mathbf{u}_k^\star = \sum_{i=1}^{m} w_i \mathbf{u}_{k-i}, \tag{14}$$

$$\mathbf{e}_k^\star = \sum_{i=1}^{m} w_i \mathbf{e}_{k-i}. \tag{15}$$

To provide a comprehensive understanding of the control law presented in (13), we leverage Lemma 1 to demonstrate how (13) inherently learns from a virtual cycle.

Lemma 1. (Virtual cycle) For a virtual cycle, with the initial state $x_k^\star(0)$ satisfying (11) and (12), if we utilize $\mathbf{u}_k^\star$ from (14) as the control input for this virtual cycle, then the control error of this virtual cycle is $\mathbf{e}_k^\star$ as given in (15).

Proof. To complete the proof, we only need to prove

$$\mathbf{e}_k^\star = \mathbf{y}_d - (G\mathbf{u}_k^\star + Hx_k^\star(0)).$$

Through the lifted form system description in (6), it hold that

$$\mathbf{e}_{k-i} = \mathbf{y}_d - (G\mathbf{u}_{k-i} + Hx_{k-i}(0)). \tag{16}$$

for any $i \in [1, m]$. By applying the multiplication of w_i on both sides to each equation corresponding to $k - i$, and subsequently aggregating them through a weighted sum, we can obtain

$$\sum_{i=1}^{m} w_i \mathbf{e}_{k-i} = \sum_{i=1}^{m} w_i \mathbf{y}_d - (\sum_{i=1}^{m} w_i G\mathbf{u}_{k-i} + \sum_{i=1}^{m} w_i H x_{k-i}(0)). \tag{17}$$

With (12), we can obtain $\sum_{i=1}^{m} w_i \mathbf{y}_d = \mathbf{y}_d$, and (17) is intrisically

$$\mathbf{e}_k^\star = \mathbf{y}_d - (G\mathbf{u}_k^\star + Hx_k^\star(0)), \tag{18}$$

which completes the proof.

Based on Lemma 1, it can be inferred that a virtual cycle can be conceptualized as a previously implemented cycle, commencing from the present initial state $x_k(0)$. By employing iterative learning based on this virtual cycle, the issue brought by fluctuating initial states can be circumvented, effectively transforming the problem into a conventional ILC tracking problem.

Simultaneously, the fundamental concept of ILC revolves around learning from the best-performing data in historical data. However, even when subjected to the constraints (11) and (12), there exist numerous possible combinations for generating virtual cycles. Some of these virtual cycles may exhibit exceedingly large control error norms $\|\mathbf{e}_k^\star\|_2$. Learning from such cycles not only contradicts the underlying principle of ILC but also fails to guarantee stability. Hence, an additional condition is introduced to refine the generating manner of virtual cycles, as expressed by:

$$\|\mathbf{e}_k^\star\|_2 \leq \|\mathbf{e}_k^{m_{max}}\|_2, \tag{19}$$

where $\|\mathbf{e}_k^{m_{max}}\|_2$ corresponds to the maximum norm of control error observed within the past m cycles.

3.2 Theoretical analysis

In this part, we make theoretical analysis to the proposed virtual-cycle learning ILC framework.

Theorem 2. (Learning Improvement) Consider the system given in (1), with Assumption 1-3 holding. For a chosen m, if there exists a group of weights satisfying (11), (12), and (19), then by implementing the virtual cycle ILC control law as given in (13) for cycle k, it is guaranteed that

$$\|\mathbf{e}_k\|_2 < \|\mathbf{e}_k^\star\|_2.$$

Proof. By substituting (13) into (6), we have

$$\mathbf{e}_k = \mathbf{y}_d - (G(\mathbf{u}_k^\star + L\mathbf{e}_k^\star) + Hx_k(0)). \tag{20}$$

By Substracting (20) from (18), we can obtain

$$\mathbf{e}_k^\star - \mathbf{e}_k = G(\mathbf{u}_k^\star + L\mathbf{e}_k^\star) - G\mathbf{u}_k^\star = GL\mathbf{e}_k^\star.$$

Through arrangement, we have

$$\|\mathbf{e}_k\|_2 = \|I - GL\|_2\|\mathbf{e}_k^\star\|_2.$$

From Assumption 3 we know $\|I - GL\|_2 < 1$, hence $\|\mathbf{e}_k\|_2 < \|\mathbf{e}_k^\star\|_2$, which completes the proof.

Theorem 3. (Convergence) Consider the system given in (1), with Assumptions 1-3 holding. For a chosen m, if there exists a group of weights satisfying (11), (12), and (19), then by implementing the virtual cycle ILC control law as given in (13) for cycle k, it is guaranteed that

$$\lim_{k\to\infty} \|\mathbf{e}_k\|_2 = 0.$$

Proof. By combining the result of Theorem 2 and equation (19), we can establish the following inequalities:

$$\|\mathbf{e}_k\|_2 \le \|I - GL\|_2\|\mathbf{e}_k^{m_{max}}\|_2. \tag{21}$$

With (21), we can infer that $\|\mathbf{e}_k\|_2$ is smaller than the maximum value among sequence $\{\|\mathbf{e}_{k-m}\|_2, ..., \|\mathbf{e}_{k-1}\|_2\}$, and further obtain that for any k

$$\|\mathbf{e}_{k+1}^{m_{max}}\|_2 \le \|\mathbf{e}_k^{m_{max}}\|_2. \tag{22}$$

Then we observe the sequence $\{\|\mathbf{e}_k\|_2, ..., \|\mathbf{e}_{k+m-1}\|_2\}$, assume the maximum value is $\|\mathbf{e}_{k+j}\|_2$, then it has to be satisfied that

$$\|\mathbf{e}_{k+j}\|_2 \le \|I - GL\|_2\|\mathbf{e}_{k+j}^{m_{max}}\|_2 \le \|I - GL\|_2\|\mathbf{e}_k^{m_{max}}\|_2.$$

Since $\|\mathbf{e}_{k+m}^{m_{max}}\|_2 = \|\mathbf{e}_{k+j}\|_2$, we can obtain

$$\|\mathbf{e}_{k+m}^{m_{max}}\|_2 \le \|I - GL\|_2\|\mathbf{e}_k^{m_{max}}\|_2. \tag{23}$$

By the same logic of (23), for any positive integer n, we have

$$\|\mathbf{e}_{k+nm}^{m_{max}}\|_2 \le (\|I - GL\|_2)^n\|\mathbf{e}_k^{m_{max}}\|_2. \tag{24}$$

According to (24), for any k, we have

$$\lim_{n\to\infty} \|\mathbf{e}_{k+nm}^{m_{max}}\|_2 = 0.$$

Hence, we have

$$\lim_{k\to\infty} \|\mathbf{e}_k^{m_{max}}\|_2 = 0. \tag{25}$$

Finally, combine (21) and (25), we have $\lim_{k\to\infty} \|\mathbf{e}_k\|_2 = 0$, which completes the proof.

3.3 Derivation of the weight w_i

In this part, we would discuss how to solve the weights w_i. It is important to note that we cannot always guarantee the existence of a sequence of weights w_i that satisfies equations (11), (12), and (19). In certain cases, such as when the initial state of the upcoming cycle lies within the convex hull formed by the initial states of the past m cycles, any sequence satisfying equations (11) and (12) will automatically satisfy equation (19). However, the existence of such a sequence depends on the past initial states and error data.

To ensure the implementation of our proposed virtual-cycle learning ILC framework and find a suitable sequence of weights w_i, we relax the condition that needs to be satisfied. We establish equations (12) and (19) as hard constraints and aim to minimize the difference between the true upcoming initial state $x_k(0)$ and the virtual cycle weighted initial state $x_k^\star(0)$. This allows us to find the optimal sequence of weights for our ILC implementation. This relaxation of conditions can be interpreted as we can't generate a cycle with identical initial states, and we strive to generate a cycle with the most similar initial state for learning purposes.

Based on this concept, we can formulate a convex optimization problem as follows:

$$\begin{aligned} \min_{w_1,\dots,w_m} \; & \|x_k(0) - x_k^\star(0)\|_2 \\ s.t.\; & x_k^\star(0) = \sum_{i=1}^{m} w_i x_{k-i}(0), \\ & (12),\ (15),\ (19). \end{aligned} \tag{26}$$

The implementation of the ILC framework can be summarized as Alg. 1.

Algorithm 1 Virtual cycle learning ILC framework

1: **Initialization:** Initialize the ILC input $\mathbf{u}_1$, the window size m.
2: In the first cycle, implement the initial ILC input $\mathbf{u}_1$, record the input $\mathbf{u}_1$ and error $\mathbf{e}_1$.
3: **for** $k = 2 : \infty$ **do**
4: Measure the current cycle initial state $x_k(0)$.
5: Obtain the virtual cycle combination weights w_i by solving (26).
6: Generate the $\mathbf{u}_k$ via (13).
7: Online inplement $\mathbf{u}_k$ and record $\mathbf{u}_k$, $\mathbf{e}_k$ for the preparation of next iteration.
8: **end for**

4. NUMERICAL EXAMPLE

In this section, we employ a linearized injection molding process as a case study to demonstrate the efficacy of our proposed ILC framework. By showcasing this example, we aim to highlight the effectiveness of our approach in addressing the challenges posed by randomly varying initial states and enhancing overall system performance.

The injection molding process, considered as one of the typical repetitive processes, can be effectively divided into three distinct phases: filling, packing, and cooling. During the packing phase, the desired trajectory of the nozzle pressure is achieved by the manipulation of the hydraulic control valve opening. As stated by Zhou et al. (2022), through employing the least squares regression technique on historical data, it is possible to describe the nozzle pressure control process using the following state-space model.

$$\begin{aligned} x_k(t+1) &= \begin{bmatrix} 2.607 & 1 \\ -0.6086 & 0 \end{bmatrix} x_k(t) + \begin{bmatrix} 1.2390 \\ -0.9282 \end{bmatrix} u_k(t), \\ y_k(t+1) &= [1 \quad 0]x_k(t+1), \quad t \in [0, 60]. \end{aligned} \tag{27}$$

The desired trajectory of nozzle pressure is set as

$$y_d(t) = \begin{cases} 0, & 1 \le t \le 10, \\ 10(t-10), & 11 \le t \le 40, \\ 300, & 41 \le t \le 60, \end{cases} \quad (28)$$

and the length of a cycle $t_N = 60$. The ideal initial state of the system is $x(0) = [0, 0]^\top$. However, due to the uncertainty in the filling phase, the true initial state of packing phase would be randomly varying and satisfies $x_{i,k} \in [0, 0.5]^\top$, $i \in [1, 2]$.

In order to implement our proposed Algorithm 1, we set the window size as $m = 50$. The initial values for the ILC inputs are initialized as $u_1 = \mathbf{0}$, where $\mathbf{0}$ represents a zero vector of the appropriate dimension. To assess the control performance, we define two performance indexes. For the tracking performance at a specific time t, the performance measure is defined as

$$J_k(t) := \|e_k(t)\|_2. \quad (29)$$

For the tracking performance of a whole cycle, the performance measure is defined as

$$\mathbf{J}_k := \|\mathbf{e}_k\|_2. \quad (30)$$

To make a comparison, we compared the results of our proposed framework with the method given by Li et al. (2013). The results are shown in Fig. 1 and Fig. 2. Based on the observations from these figures, it can be concluded that our framework effectively addresses the challenges posed by randomly varying initial states, enabling iterative learning to enhance control performance as the cycle number increases. Ultimately, our framework demonstrates convergence and outperforms the method proposed by Li et al. (2013). The reason for this superior performance is that their method merely takes an average of the historical error. Although randomness can be mitigated, the impact of randomly varying initial states still affects the control performance. In contrast, our proposed method can create a virtual cycle that shares the most similar initial state as the upcoming cycle by utilizing historical data. This mechanism maximally avoids the issue of randomly varying initial states and ensures that the iterative learning mechanism remains effective, as it is cast into a fixed initial state system.

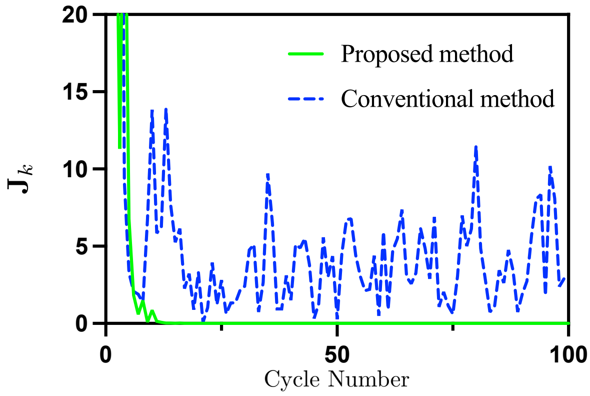

Fig. 1. Cycle-wise performance comparison between our proposed framework and the conventional method developed by Li et al. (2013).

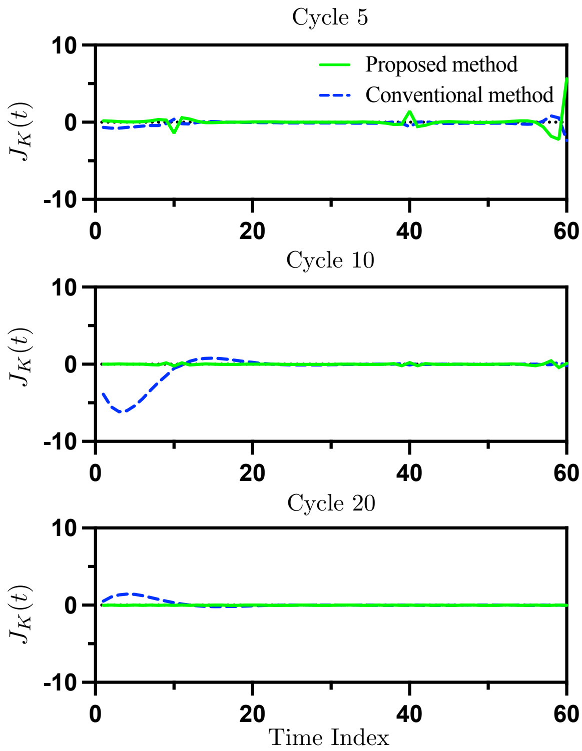

Fig. 2. Time-wise performance comparison between our proposed framework and the conventional method developed by Li et al. (2013).

It is noteworthy to mention that our proposed framework does not guarantee a monotonic decrease in $\|\mathbf{e}_k\|_2$ as the cycle number increases. Consequently, when plotting the control error of our method on a logarithmic scale, noticeable fluctuations can be observed. However, in accordance with Theorem 3, the upper bound of, which is $\|\mathbf{e}_k^{m_{max}}\|_2$ exhibits a monotonic decrease, which is confirmed by the results depicted in Fig. 3.

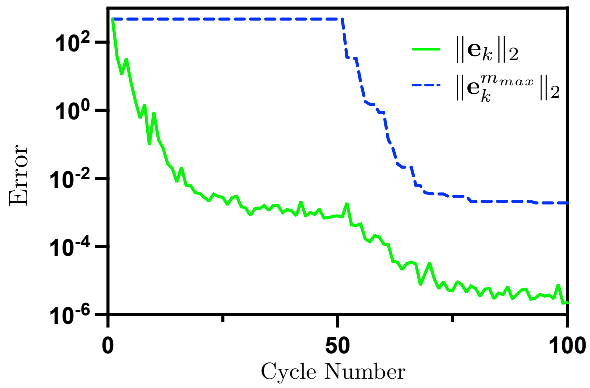

Fig. 3. Illustration of the fluctuation in $\|\mathbf{e}_k\|_2$ and the monotonic convergence of $\|\mathbf{e}_k^{m_{max}}\|_2$.

5. CONCLUSION

We propose a virtual cycle-based ILC framework for systems with randomly varying initial states. To avoid the negative impact brought by initial state variation, we create a linear combination of historical data and generate a virtual cycle with a similar initial state to the upcoming cycle. In terms of the linear combination manner, we establish three conditions that the weights shall satisfy in order to generate an ideal virtual cycle. Based on these three conditions, we provide a rigorous theoretical analysis to demonstrate convergence. However, since the ideal virtual cycle may not always exist, we relax one of the conditions into a soft constraint, allowing the weights to be solved through a convex optimization problem. The effectiveness and properties of our proposed framework are verified through an injection molding example.

Although the proposed framework is model-free, the design and analysis are based on the assumption of a linear system. In future research, we are considering the possibility of extending our framework to nonlinear systems.

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