PIO Based Data-Driven Iterative Learning Control for Nonlinear Batch Processes with Nonrepetitive Disturbances Subject to Input Constraints ⋆

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Abstract: This paper proposes a novel proportional-integral observer (PIO) based high-order data-driven iterative learning control (HODDILC) for nonlinear batch processes with nonrepetitive disturbances subject to input constraints. First, an equivalent dynamic linearization data model (DLDM) with an uncertainty term arising from the nonrepetitive disturbances is constructed in the batch direction. Based on the established DLDM along with a gradient estimation algorithm, a data-driven PIO is then designed to estimate the uncertainty term along the batch direction, followed by designing a HODDILC law in terms of the tracking errors over more than one previous batches and control inputs in the previous time instants of the current batch. Using the contraction mapping principle and matrix theory, rigorous convergence analysis is conducted in the iteration domain. A notable advantage of the proposed design is that only input and output measurement data are used rather than a process model. Finally, an illustrative example from the literature is given to demonstrate the effectiveness and merit of the proposed method.

Keywords: Nonlinear batch processes, nonrepetitive disturbances, input constraints, proportional-integral observer, data-driven iterative learning control.

1. INTRODUCTION

Batch processes have been widely applied in industry such as industrial injection molding (Gao et al. (2001)) and pharmaceutical crystallization (Nagy (2009)) in recent years due to its high flexibility and versatility. Over the past decades, tremendous control methods have been developed for such processes, see, e.g., Wang et al. (2009); Ahn et al. (2007) and the references therein. Among these methods, iterative learning control (ILC), as a class of intelligent control method, has been regarded as an effective control strategy to gradually improve the control performance by learning the repetitive information from the previous executions.

ILC has gained increasing attention in industry and academia since it was first proposed by Arimoto et al. (1984). It is known that the conventional ILC, also known as open-loop ILC, can realize perfect tracking for linear or nonlinear systems with completely repetitive features, and bounded tracking in the presence of nonrepetitive uncertainties (Meng and Moore (2017)). However, only error convergence along the batch direction was addressed while the system dynamics in the time domain was neglected. This may result in poor even unacceptable dynamic performance in the first few batches, especially for the open-loop unstable processes. To further improve the tracking performance, real-time feedback control was incorporated into the ILC design based on 2D/repetitive system theory (Shi et al. (2006); Hao et al. (2020); Rogers et al. (2005)). It should be noted that all the above mentioned ILC methods require some basic model information for the learning law design or convergence analysis.

Due to the development of modern industrial processes with large scale and high complexity, modeling a practical process accurately by the first principle or system identification is not an easy task. Even the mathematical model of a process can be established, it may suffer strong nonlinearity, time-varying parameters and high orders etc., hindering its application in control design and stability analysis. To circumvent this issue, data-driven control has

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attracted a lot of attention in recent years by using only the input and output measurement data (Hou and Jin (2013)). Meanwhile, some developments on data-driven ILC (DDILC) have been continuously reported, see, e.g., Chi et al. (2018a,b); Janssens et al. (2013) etc. In Chi et al. (2018a), a high-order DDILC (HODDILC) was proposed for nonlinear repetitive systems, where additional control input of previous time instants in the current batch was adopted for input update to improve the transient performance. To track the desired values at the endpoint of the batch operation, a HODDILC was developed to enhance the tracking performance with faster convergence (Chi et al. (2018b)). However, all nonlinear uncertainties are lumped into the so-call pseudo partial derivative or pseudo gradient, which may be too difficult to be well estimated if the considered systems are too complex. Recently, an extended state observer (ESO) based DDILC was developed in Hui et al. (2019) for a permanent magnet linear motor with initial shifts and disturbances. However, no effective guideline was given for the tuning of ESO gain. For nonlinear nonaffine systems with nonrepeatable uncertainties, Chi et al. (2020) proposed an observer-based DDILC such that the gradient and uncertainty term in DLDM could be separately estimated in the iteration domain. Moreover, input constraints are commonly unavoidable in practical applications due to the physical limitation of actuator. To the best of our knowledge, disturbance observer based HODDILC for unknown nonlinear batch processes with nonrepetitive disturbances and input constraints has not been fully explored, and therefore motivating this work.

In this paper, a proportional-integral observer (PIO) based HODDILC method is proposed for nonlinear nonaffine batch processes with nonrepetitive disturbances subject to input constraints by using tracking error over more than one batches and control inputs in previous time instants of the current batch. Convergence of tracking error and the boundedness of the estimated parameters along the batch direction are rigorously analyzed. The effectiveness of the proposed design is validated by an illustrative example.

Notations: $\mathbb{Z} = \{1, 2, \ldots\}$, $\mathbb{Z}_+ = \{0, 1, \ldots\}$, $\mathbb{Z}_N = \{0, 1, \ldots, N\}$ for any $N \in \mathbb{Z}_+$. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote n-dimensional Euclidean space and $n \times m$ real matrix space, respectively. I or 0 indicates the identity or zero matrix (vector) with appropriate dimensions. $\|\cdot\|$ is the consistent matrix norm. For a matrix A, $A^\top$ denotes its transpose. For any function $f_k(\cdot)$, denote by $\Delta f_k(\cdot) = f_k(\cdot) - f_{k-1}(\cdot)$ a difference function in the iteration domain.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider a discrete-time nonlinear batch process with nonrepetitive disturbances described as follows

$$y_k(t+1) = f(y_k(t), \ldots, y_k(t-n_y), u_k(t), \ldots, u_k(t-n_u), d_k(t)), \quad (1)$$

where $t \in \mathbb{Z}_{N-1}$ and $k \in \mathbb{Z}$ are the time and batch indices; N is the length of each batch; $y_k(t) \in \mathbb{R}$ and $u_k(t) \in \mathbb{R}$ are the system output and control input, respectively; n_y and n_u are two unknown system orders; $f(\cdot)$ is an unknown nonlinear nonaffine scalar function; $d_k(t)$ is the nonrepetitive and bounded disturbances satisfying $|d_k(t)| \leq \beta_d$ for any t and k. Without loss of generality, the initial conditions in (1) are assumed as $y_k(0) = y_0$ for all k, and $u_0(t) = 0$ for any t, where y_0 is a constant.

Following the similar way in Chi et al. (2020), the system output can be reformulated as

$$\begin{aligned}
y_k(1) &= f(y_k(0), u_k(0), d_k(0)) \triangleq g_0(y_k(0), u_k(0), d_k(0)),\\
y_k(2) &= f(y_k(1), y_k(0), u_k(1), u_k(0), d_k(1))\\
&= f\big(g_0(y_k(0), u_k(0), d_k(0)), y_k(0), u_k(1), u_k(0), d_k(1)\big)\\
&\triangleq g_1(y_k(0), u_k(1), u_k(0), d_k(1), d_k(0)),\\
&\ \ \vdots\\
y_k(t+1) &= f\big(y_k(t), \ldots, y_k(t-n_y), u_k(t), \ldots, u_k(t-n_u), d_k(t)\big)\\
&\triangleq g_t(y_k(0), u_k(t), \ldots, u_k(0), d_k(t), \ldots, d_k(0)),
\end{aligned}$$

where $g_t(\cdot)$ is a compound function of $f(\cdot)$.

Assumption 1. (Chi et al. (2018a)) The nonlinear function $g_t(\cdot)$ is continuously differentiable with respect to its arguments.

Assumption 2. (Chi et al. (2018a)) The vector-valued nonlinear function $\boldsymbol{g}(\cdot)$ is globally Lipschitz, i.e.,

$$\begin{aligned}
\|\boldsymbol{g}(y_1, \boldsymbol{u}_1, \boldsymbol{d}_1) - \boldsymbol{g}(y_2, \boldsymbol{u}_2, \boldsymbol{d}_2)\| &\leq L_y|y_1 - y_2|\\
+ L_u\|\boldsymbol{u}_1 - \boldsymbol{u}_2\| &+ L_d\|\boldsymbol{d}_1 - \boldsymbol{d}_2\|,
\end{aligned} \quad (2)$$

where $\boldsymbol{g}(\cdot) = [g_0(\cdot), \ldots, g_{N-2}(\cdot), g_{N-1}(\cdot)]^\top$, $0 < L_y < \infty$, $0 < L_u < \infty$ and $0 < L_d < \infty$ are three Lipschitz constants, respectively.

Assumption 3. The control input satisfies the following constraints

$$u_k(t) = \mathrm{sat}(v_k(t)) = \begin{cases} u_+, & v_k(t) > u_+,\\ v_k(t), & u_- \leq v_k(t) \leq u_+,\\ u_-, & v_k(t) < u_-, \end{cases} \quad (3)$$

where u_+ and u_- are two constants satisfying $u_- < u_+ < \infty$, and $v_k(t)$ is the unconstrained control input.

Based on the mean value theorem, it follows that

$$\boldsymbol{y}_i - \boldsymbol{y}_j = \boldsymbol{\Phi}_{i,j}^{N-1}(\boldsymbol{u}_i - \boldsymbol{u}_j) + \boldsymbol{\Psi}_{i,j}^{N-1}(\boldsymbol{d}_i - \boldsymbol{d}_j), \quad (4)$$

where $\boldsymbol{y}_i = [y_i(1), \ldots, y_i(N)]^\top$, $\boldsymbol{u}_i = [u_i(0), \ldots, u_i(N-1)]^\top$, $\boldsymbol{d}_i = [d_i(0), \ldots, d_i(N-1)]^\top$,

$$\boldsymbol{\Phi}_{i,j}^{N-1} = \begin{bmatrix} \phi_{i,j}^0(0) & 0 & \cdots & 0\\ \phi_{i,j}^1(0) & \phi_{i,j}^1(1) & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ \phi_{i,j}^{N-1}(0) & \phi_{i,j}^{N-1}(1) & \cdots & \phi_{i,j}^{N-1}(N-1) \end{bmatrix},$$

$$\boldsymbol{\Psi}_{i,j}^{N-1} = \begin{bmatrix} \psi_{i,j}^0(0) & 0 & \cdots & 0\\ \psi_{i,j}^1(0) & \psi_{i,j}^1(1) & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ \psi_{i,j}^{N-1}(0) & \psi_{i,j}^{N-1}(1) & \cdots & \psi_{i,j}^{N-1}(N-1) \end{bmatrix},$$

$\phi_{i,j}^l(m) = \partial g_l^* / \partial u_{ij}^*(m)$ and $\psi_{i,j}^l(m) = \partial g_l^* / \partial d_{ij}^*(m)$, $l, m \in \mathbb{Z}_{N-1}$ represent the proper partial derivatives of g_l with respect to $u_{ij}^*(m) \in [u_i(m), u_j(m)]$ and $d_{ij}^*(m) \in [d_i(m), d_j(m)]$, respectively.

Letting $i = k$ and $j = k-1$ in (4) yields the following dynamic linearization data model (DLDM) with a residual term

$$y_k(t+1) = y_{k-1}(t+1) + \Delta\boldsymbol{u}_k^\top(t)\boldsymbol{\phi}_{k,k-1}^t(t) + \xi_k(t), \quad (5)$$

where $\xi_k(t) = \Delta\boldsymbol{d}_k^\top(t)\boldsymbol{\psi}_{k,k-1}^t(t)$ and

$$
\begin{aligned}
\Delta \boldsymbol{d}_k(t) &= [\Delta d_k(0), \Delta d_k(1), \ldots, \Delta d_k(t)]^\top, \\
\Delta \boldsymbol{u}_k(t) &= [\Delta u_k(0), \Delta u_k(1), \ldots, \Delta u_k(t)]^\top, \\
\boldsymbol{\phi}^t_{k,k-1}(t) &= [\phi^t_{k,k-1}(0), \phi^t_{k,k-1}(1), \ldots, \phi^t_{k,k-1}(t)]^\top, \\
\boldsymbol{\psi}^t_{k,k-1}(t) &= [\psi^t_{k,k-1}(0), \psi^t_{k,k-1}(1), \ldots, \psi^t_{k,k-1}(t)]^\top.
\end{aligned} \tag{6}
$$

Based on Assumption (2), we have $\|\boldsymbol{\phi}^t_{k,k-1}(t)\| \le L_u$ and $\|\boldsymbol{\psi}^t_{k,k-1}(t)\| \le L_d$, which, together with the boundedness of $d_k(t)$, leads to the boundedness of $\xi_k(t)$ and satisfying $|\xi_k(t)| \le \beta_\xi < \infty$, where β_ξ is a constant. Subsequently, we rewrite $\boldsymbol{\phi}^t_{k,k-1}(t)$ as $\boldsymbol{\phi}_k(t) = [\phi_k(0), \phi_k(1), \ldots, \phi_k(t)]^\top$ for the notational simplicity.

3. PIO BASED HODDILC DESIGN

Motivated by the model-based proportional-integral observer (PIO) in Chang (2006) to estimate the system state and disturbance simultaneously, the following data-driven PIO in the iteration domain is proposed based on the established DLDM in (5)

$$
\begin{cases}
\hat{y}_k(t+1) = \hat{y}_{k-1}(t+1) + \Delta \boldsymbol{u}_k^\top(t)\hat{\boldsymbol{\phi}}_k(t) + \epsilon_{k-1}(t) + l_1\tilde{y}_{k-1}(t+1), \\
\quad \epsilon_k(t) = \epsilon_{k-1}(t) + l_2\tilde{y}_{k-1}(t+1), \\
\tilde{y}_k(t+1) = y_k(t+1) - \hat{y}_k(t+1),
\end{cases} \tag{7}
$$

where l_1 and l_2 are the observer gains to be determined, $\hat{y}_k(t)$, $\epsilon_{k-1}(t)$ and $\hat{\boldsymbol{\phi}}_k(t)$ are the respective estimates of $y_k(t)$, $\xi_k(t)$ and $\boldsymbol{\phi}_k(t)$. To make the PIO in (7) implementable, the following parameter estimation algorithm in Hou and Jin (2013) is adopted to obtain $\hat{\boldsymbol{\phi}}_k(t)$,

$$
\hat{\boldsymbol{\phi}}_k(t) = \hat{\boldsymbol{\phi}}_{k-1}(t) + \frac{\eta\Lambda_{k-1}(t+1)\Delta \boldsymbol{u}_{k-1}(t)}{\mu + \|\Delta \boldsymbol{u}_{k-1}(t)\|^2}, \tag{8}
$$

where $\Lambda_{k-1}(t+1) = \Delta y_{k-1}(t+1) - \Delta \boldsymbol{u}_{k-1}^\top(t)\hat{\boldsymbol{\phi}}_{k-1}(t)$, $\mu > 0$ is a weighting factor, and $\eta \in (0, 2]$ is the step factor.

To derive a HODDILC law, we first consider the following objective function with high-order error information as adopted in Chi et al. (2018a) by ignoring the constraints,

$$
J(u_k(t), \alpha) = \left(\sum_{i=1}^{q} \alpha_i e_{k-i+1}(t+1)\right)^2 + \lambda(u_k(t) - u_{k-1}(t))^2, \tag{9}
$$

where $e_k(t) \triangleq y_d(t) - y_k(t)$ is the tracking error, $\lambda > 0$ is another weighting factor and $\alpha = [\alpha_1, \ldots, \alpha_q]$ denotes the high-order factors satisfying $\sum_{i=1}^{q}\alpha_i = 1$, $\alpha_i \in [0, 1]$ and $\alpha_1 + \alpha_2 - \sum_{i=3}^{q}\alpha_i \triangleq \bar{\alpha} > 0$.

Reformulate (5) as

$$
\begin{aligned}
y_k(t+1) = y_{k-1}(t+1) &+ \Delta \boldsymbol{u}_k^\top(t-1)\boldsymbol{\phi}_k(t-1) \\
&+ \phi_k(t)\Delta u_k(t) + \xi_k(t).
\end{aligned} \tag{10}
$$

Then applying (10) to (9) leads to

$$
\begin{aligned}
&J(u_k(t), \alpha) \\
&= \Bigg(\alpha_1\Big(e_{k-1}(t+1) - \Delta \boldsymbol{u}_k^\top(t-1)\boldsymbol{\phi}_k(t-1) \\
&\quad - \phi_k(t)\Delta u_k(t) - \xi_k(t)\Big) + \sum_{i=2}^{q}\alpha_i e_{k-i+1}(t+1)\Bigg)^2 \\
&\quad + \lambda(u_k(t) - u_{k-1}(t))^2.
\end{aligned}
$$

Taking derivative of $J(u_k(t), \alpha)$ with respect to $u_k(t)$ and letting $\partial J(u_k(t), \alpha)/\partial u_k(t) = 0$, a learning law is derived as

$$
\begin{aligned}
u_k(t) = u_{k-1}(t) &- \frac{\rho\alpha_1^2\phi_k(t)(\sum_{i=0}^{t-1}\phi_k(i)\Delta u_k(i) + \xi_k(t))}{\lambda + \alpha_1^2\phi_k^2(t)} \\
&+ \frac{\rho\alpha_1\phi_k(t)\Big(\alpha_1 e_{k-1}(t+1) + \sum_{i=2}^{q}\alpha_i e_{k-i+1}(t+1)\Big)}{\lambda + \alpha_1^2\phi_k^2(t)}
\end{aligned} \tag{11}
$$

where $\rho > 0$ is a positive scalar. Also, to make the control law in (11) implementable, the unknown variables $\phi_k(t)$ and $\xi_k(t)$ are replaced by their estimates $\hat{\phi}_k(t)$ and $\epsilon_{k-1}(t)$.

To sum up, the overall PIO based HODDILC is constructed as follows by considering the input constraints

$$
\begin{cases}
\hat{y}_k(t+1) = \hat{y}_{k-1}(t+1) + \Delta \boldsymbol{u}_k^\top(t)\hat{\boldsymbol{\phi}}_k(t) + \epsilon_{k-1}(t) + l_1\tilde{y}_{k-1}(t+1), \\
\quad \epsilon_k(t) = \epsilon_{k-1}(t) + l_2\tilde{y}_{k-1}(t+1), \\
\tilde{y}_k(t+1) = y_k(t+1) - \hat{y}_k(t+1), \\
\quad \hat{\boldsymbol{\phi}}_k(t) = \hat{\boldsymbol{\phi}}_{k-1}(t) + \dfrac{\eta\Lambda_{k-1}(t+1)\Delta \boldsymbol{u}_{k-1}(t)}{\mu + \|\Delta \boldsymbol{u}_{k-1}(t)\|^2}, \\
\quad \hat{\boldsymbol{\phi}}_k(t) = \hat{\boldsymbol{\phi}}_0(t), \text{ if } \mathrm{sign}(\hat{\boldsymbol{\phi}}_k(t)) \ne \mathrm{sign}(\hat{\boldsymbol{\phi}}_0(t)) \text{ or } \|\hat{\boldsymbol{\phi}}_k(t)\| \le \varepsilon, \\
\quad v_k(t) = v_{k-1}(t) - \dfrac{\rho\alpha_1^2\hat{\phi}_k(t)(\sum_{i=0}^{t-1}\hat{\phi}_k(i)\Delta v_k(i) + \epsilon_{k-1}(t))}{\lambda + \alpha_1^2\hat{\phi}_k^2(t)} \\
\qquad + \dfrac{\rho\alpha_1\hat{\phi}_k(t)\Big(\alpha_1 e_{k-1}(t+1) + \sum_{i=2}^{q}\alpha_i e_{k-i+1}(t+1)\Big)}{\lambda + \alpha_1^2\hat{\phi}_k^2(t)}, \\
\quad u_k(t) = \mathrm{sat}(v_k(t)),
\end{cases} \tag{12}
$$

where $\varepsilon > 0$ is sufficiently small, and $\hat{\boldsymbol{\phi}}_0(t)$ is the initial value of $\hat{\boldsymbol{\phi}}_k(t)$.

4. CONVERGENCE ANALYSIS

Before proceeding with the convergence analysis, the following technical lemmas are given.

Lemma 4. (Jury (1964)) Let

$$
A = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \\ 1 & 0 & \cdots & 0 \\ & \ddots & \ddots & \vdots \\ & & 1 & 0 \end{bmatrix}.
$$

If $\sum_{i=1}^{n}|a_i| < 1$, then $s(A) < 1$, where $s(A)$ is the spectral radius of A.

Lemma 5. (Huang (1984)) $A \in \mathbb{R}^{n\times n}$, and $s(A)$ is the spectral radius of A. Then, for any $\delta > 0$, there always exists a proper matrix norm $\|\cdot\|$, such that

$$
\|A\| < s(A) + \delta.
$$

The following theorem gives the convergence analysis of the proposed PIO based HODDILC method.

Theorem 6. Consider the nonlinear system in (1) under Assumptions 1-3, where the PIO based HODDILC is applied. If the observer gains l_1 and l_2 are chosen to satisfy $\max\{|2-l_1+\sqrt{l_1^2-4l_2}|/2, |2-l_1-\sqrt{l_1^2-4l_2}|/2\} < 1$, and the parameters λ and ρ are chosen such that

$$
\lambda > \rho^2 L_u^2/4, \tag{13}
$$

it is guaranteed that (i) the estimation $\hat{\boldsymbol{\phi}}_k(t)$ is bounded for all t and k; (ii) the bounded convergence of observation error is achieved along the batch direction; (iii) the bounded convergence of tracking error $e_k(t)$ is also achieved along the batch direction.

Proof. Subtracting $\boldsymbol{\phi}_k(t)$ from both sides of (8) and denoting $\tilde{\boldsymbol{\phi}}_k(t) \triangleq \boldsymbol{\phi}_k(t) - \hat{\boldsymbol{\phi}}_k(t)$ lead to

$$
\begin{aligned}
\tilde{\boldsymbol{\phi}}_k(t) &= \boldsymbol{\phi}_k(t) - \hat{\boldsymbol{\phi}}_k(t) \\
&= \boldsymbol{\phi}_k(t) - \hat{\boldsymbol{\phi}}_{k-1}(t) - \frac{\eta\Lambda_{k-1}(t)\Delta\boldsymbol{u}_{k-1}(t)}{\mu + \|\Delta\boldsymbol{u}_{k-1}(t)\|^2} \\
&= \boldsymbol{\phi}_k(t) - \boldsymbol{\phi}_{k-1}(t) + \tilde{\boldsymbol{\phi}}_{k-1}(t) \\
&\quad - \frac{\eta(\Delta\boldsymbol{u}_{k-1}^\top(t)\tilde{\boldsymbol{\phi}}_{k-1}(t) + \xi_{k-1}(t))\Delta\boldsymbol{u}_{k-1}(t)}{\mu + \|\Delta\boldsymbol{u}_{k-1}(t)\|^2} \\
&= \left(I - \frac{\eta\Delta\boldsymbol{u}_{k-1}(t)\Delta\boldsymbol{u}_{k-1}^\top(t)}{\mu + \|\Delta\boldsymbol{u}_{k-1}(t)\|^2}\right)\tilde{\boldsymbol{\phi}}_{k-1}(t) \\
&\quad + \boldsymbol{\phi}_k(t) - \boldsymbol{\phi}_{k-1}(t) - \frac{\eta\xi_{k-1}(t)\Delta\boldsymbol{u}_{k-1}(t)}{\mu + \|\Delta\boldsymbol{u}_{k-1}(t)\|^2},
\end{aligned} \tag{14}
$$

where we have used the fact $\Delta\boldsymbol{u}_{k-1}^\top\tilde{\boldsymbol{\phi}}_{k-1}(t)\Delta\boldsymbol{u}_{k-1}(t) = \Delta\boldsymbol{u}_{k-1}(t)\Delta\boldsymbol{u}_{k-1}^\top\tilde{\boldsymbol{\phi}}_{k-1}(t)$. Due to $|\xi_{k-1}(t)| \leq \beta_\xi$ for all t and k, it follows that

$$
\left\|\frac{\eta\xi_{k-1}(t)\Delta\boldsymbol{u}_{k-1}(t)}{\mu + \|\Delta\boldsymbol{u}_{k-1}(t)\|^2}\right\| \leq \frac{\eta|\xi_{k-1}(t)|\|\Delta\boldsymbol{u}_{k-1}(t)\|}{2\sqrt{\mu}\|\Delta\boldsymbol{u}_{k-1}(t)\|} \leq \frac{\eta\beta_\xi}{2\sqrt{\mu}}.
$$

Taking the norm on both sides of (14) and considering $\|\boldsymbol{\phi}_k(t)\| \leq L_u$ for all t and k give

$$
\|\tilde{\boldsymbol{\phi}}_k(t)\| \leq \left\|I - \frac{\eta\Delta\boldsymbol{u}_{k-1}(t)\Delta\boldsymbol{u}_{k-1}^\top(t)}{\mu + \|\Delta\boldsymbol{u}_{k-1}(t)\|^2}\right\| \|\tilde{\boldsymbol{\phi}}_{k-1}(t)\| + \beta_1,
$$

where $\beta_1 \triangleq 2L_u + \frac{\eta\beta_\xi}{2\sqrt{\mu}}$. Since $\eta \in (0, 2]$ and $\mu > 0$, then there exists a scalar $\rho_1 \in (0, 1)$ such that

$$
\left\|I - \frac{\eta\Delta\boldsymbol{u}_{k-1}(t)\Delta\boldsymbol{u}_{k-1}^\top(t)}{\mu + \|\Delta\boldsymbol{u}_{k-1}(t)\|^2}\right\| \leq \rho_1 < 1,
$$

which implies that

$$
\|\tilde{\boldsymbol{\phi}}_k(t)\| \leq \rho_1\|\tilde{\boldsymbol{\phi}}_{k-1}(t)\| + \beta_1 \leq \rho_1^k\|\tilde{\boldsymbol{\phi}}_0(t)\| + \frac{\beta_1}{1-\rho_1}.
$$

Due to the boundedness of initial estimation error $\tilde{\boldsymbol{\phi}}_0(t)$ for any t, it is concluded that $\tilde{\boldsymbol{\phi}}_k(t)$ is bounded, which, together with the boundedness of $\boldsymbol{\phi}_k(t)$, indicates that $\hat{\boldsymbol{\phi}}_k(t)$ is bounded for any t and k, and satisfies $|\hat{\boldsymbol{\phi}}_k(t)| \leq \beta_{\hat{\boldsymbol{\phi}}}$. Thus, the condition (i) holds.

By defining $\tilde{\xi}_k(t) \triangleq \xi_k(t) - \epsilon_k(t)$, then it follows from (5) and (7) that

$$
\chi_k(t+1) = (\bar{A} - L\bar{C})\chi_{k-1}(t+1) + \kappa_k(t), \tag{15}
$$

where $\chi_k(t+1) = [\tilde{y}_k(t+1), \tilde{\xi}_k(t)]^\top$, $L = [l_1, l_2]^\top$ and

$$
\bar{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \bar{C} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^\top, \kappa_k(t) = \begin{bmatrix} \Delta\boldsymbol{u}_k^\top(t)\tilde{\boldsymbol{\phi}}_k(t) + \Delta\xi_k(t) \\ \Delta\xi_k(t) \end{bmatrix}.
$$

It is easy to verify that $\kappa_k(t)$ is bounded for any t and k owing to the boundedness of $\xi_k(t)$, $\tilde{\boldsymbol{\phi}}_k(t)$ and $\boldsymbol{u}_k(t)$. By properly choosing the observer gains l_1 and l_2 satisfying $\max\{|2 - l_1 + \sqrt{l_1^2 - 4l_2}|/2, |2 - l_1 - \sqrt{l_1^2 - 4l_2}|/2\} < 1$, then $\bar{A} - L\bar{C}$ is Schur stable, indicating that the bounded convergence of $\chi_k(t)$ is achieved along the batch direction. The condition (ii) is satisfied. Note also that $\epsilon_k(t)$ is bounded and satisfies $|\epsilon_k(t)| \leq \beta_\epsilon$ for any t and k based on the boundedness of $\xi_k(t)$.

Based on Assumption 3 and the proof of Theorem 2 in Chi et al. (2020), there exists a diagonal matrix $\boldsymbol{\theta}_k(t)$ such that

$$
\Delta\boldsymbol{u}_k(t) = \boldsymbol{\theta}_k(t)\Delta\boldsymbol{v}_k(t),
$$

where $\boldsymbol{\theta}_k(t) = \mathrm{diag}\{\theta_k(0), \theta_k(1), \ldots, \theta_k(t)\}$, $\Delta\boldsymbol{v}_k(t) = [\Delta v_k(0), \Delta v_k(1), \ldots, \Delta v_k(t)]^\top$ and

$$
\theta_k(i) = \begin{cases} [0,1), & v_k(i) > u_+, \\ 1, & u_- \leq v_k(i) \leq u_+, \; i \in \mathbb{Z}_t. \\ [0,1), & v_k(i) < u_-, \end{cases}
$$

It follows from (5) and the definition of $e_k(t)$ that

$$
\begin{aligned}
e_k(t+1) &= [1 - \zeta_{1,k}(t)]e_{k-1}(t+1) + \tau_{1,k}(t) \\
&\quad - \frac{\rho\alpha_1\hat{\phi}_k(t)\phi_k(t)\theta_k(t)}{\lambda + \alpha_1^2\hat{\phi}_k^2(t)} \sum_{i=3}^{q} \alpha_i e_{k-i+1}(t+1),
\end{aligned} \tag{16}
$$

where $\zeta_{1,k}(t) = \rho\alpha_1(\alpha_1 + \alpha_2)\hat{\phi}_k(t)\phi_k(t)\theta_k(t)/[\lambda + \alpha_1^2\hat{\phi}_k^2(t)]$,

$$
\begin{aligned}
\tau_{1,k}(t) &= \frac{\rho\alpha_1^2\phi_k(t)\theta_k(t)}{\lambda + \alpha_1^2\hat{\phi}_k^2(t)} \left[\sum_{i=0}^{t-1} \hat{\phi}_k(i)\Delta v_k(i) + \epsilon_{k-1}(t)\right] \\
&\quad - \sum_{i=0}^{t-1} \phi_k(i)\Delta u_k(i) - \xi_k(t).
\end{aligned}
$$

Based on (16), we have

$$
\boldsymbol{e}_k(t+1) = \mathscr{A}_k(t)\boldsymbol{e}_{k-1}(t+1) + \tilde{\tau}_{1,k}(t), \tag{17}
$$

where $\boldsymbol{e}_k(t+1) = [e_k(t+1), \ldots, e_{k-q+2}(t+1)]^\top$, $\tilde{\tau}_{1,k}(t) = [\tau_{1,k}(t), 0, \ldots, 0]^\top$, $\zeta_{i,k}(t) = -\rho\alpha_1\alpha_{i+1}\hat{\phi}_k(t)\phi_k(t)\theta_k(t)/[\lambda + \alpha_1^2\hat{\phi}_k^2(t)]$, $i = 2, 3, \ldots, q-1$ and

$$
\mathscr{A}_k(t) = \begin{bmatrix} 1 - \zeta_{1,k}(t) & \zeta_{2,k}(t) & \cdots & \zeta_{q-2,k}(t) & \zeta_{q-1,k}(t) \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}.
$$

Note that

$$
\begin{aligned}
&|1 - \zeta_{k,1}(t)| + \sum_{i=2}^{q-1} |\zeta_{k,i}(t)| \\
&\quad = 1 - \frac{\rho\alpha_1(\alpha_1 + \alpha_2 - \sum_{i=3}^{q}\alpha_i)\phi_k(t)\hat{\phi}_k(t)\theta_k(t)}{\lambda + \alpha_1^2\hat{\phi}_k^2(t)}.
\end{aligned}
$$

In terms of $\alpha_1 + \alpha_2 - \sum_{i=3}^{q}\alpha_i \triangleq \bar{\alpha} > 0$ and the condition (13), it is easy to derive

$$
\begin{aligned}
0 < M_1 &\leq \frac{\rho\alpha_1(\alpha_1 + \alpha_2 - \sum_{i=3}^{q}\alpha_i)\phi_k(t)\hat{\phi}_k(t)\theta_k(t)}{\lambda + \alpha_1^2\hat{\phi}_k^2(t)} \\
&\leq \frac{\rho\alpha_1\bar{\alpha}|\phi_k(t)||\hat{\phi}_k(t)|}{2\sqrt{\lambda}\alpha_1|\hat{\phi}_k(t)|} \leq \frac{\rho L_u}{2\sqrt{\lambda}} < 1.
\end{aligned}
$$

Therefore, we have

$$
|1 - \zeta_{k,1}(t)| + \sum_{i=2}^{q-1} |\zeta_{k,i}(t)| < 1 - M_1 < 1,
$$

which, by Lemma 4, implies that $s(\mathscr{A}_k(t)) < 1$ for any t and k. Then by Lemma 5, there exists a sufficiently small constant δ such that $\|\mathscr{A}_k(t)\| \leq s(\mathscr{A}_k(t)) + \delta < \rho_2 < 1$.

Besides, we rewrite the unconstrained input $v_k(t)$ as

$$
\begin{aligned}
v_k(t) &= v_{k-1}(t) - \frac{\rho\alpha_1^2\hat{\phi}_k(t)}{\lambda + \alpha_1^2\hat{\phi}_k^2(t)} \left[\sum_{i=0}^{t-1} \hat{\phi}_k(i)\Delta v_k(i) + \epsilon_{k-1}(t)\right] \\
&\quad + \frac{\rho\alpha_1\hat{\phi}_k(t)}{\lambda + \alpha_1^2\hat{\phi}_k^2(t)} \sum_{i=3}^{q} \alpha_1 e_{k-i+1}(t+1) \\
&\quad + \frac{\rho\alpha_1(\alpha_1 + \alpha_2)\hat{\phi}_k(t)}{\lambda + \alpha_1^2\hat{\phi}_k^2(t)} [y_d(t+1) - y_{k-1}(t+1)]
\end{aligned} \tag{18}
$$

According to (4), it follows that

$$
\begin{aligned}
y_{k-1}(t+1) = y_0(t+1) &+ \sum_{i=0}^{t} \phi_{k-1,0}^{t}(i)[u_{k-1}(i) - u_0(i)] \\
&+ \sum_{i=0}^{t} \psi_{k-1,0}^{t}(i)[d_{k-1}(i) - d_0(i)]
\end{aligned}
$$

Applying the above equality to (18) gives

$$
v_k(t) = [1 - \vartheta_k(t)]v_{k-1}(t) + \tau_{2,k}(t), \tag{19}
$$

where

$$
\vartheta_k(t) = \rho\alpha_1(\alpha_1+\alpha_2)\hat{\phi}_k(t)\phi_{k-1,0}^{t}(t)\theta_{k-1}(t)/[\lambda + \alpha_1^2\hat{\phi}_k^2(t)],
$$

$$
\begin{aligned}
\tau_{2,k}(t) = &-\frac{\rho\alpha_1^2\hat{\phi}_k(t)}{\lambda+\alpha_1^2\hat{\phi}_k^2(t)}\left[\sum_{i=0}^{t-1}\hat{\phi}_k(i)\Delta v_k(i) + \epsilon_{k-1}(t)\right] \\
&+ \frac{\rho\alpha_1\hat{\phi}_k(t)}{\lambda+\alpha_1^2\hat{\phi}_k^2(t)}\left[\sum_{i=3}^{q}\alpha_i e_{k-i+1}(t+1) + (\alpha_1+\alpha_2)e_0(t+1)\right] \\
&+ \frac{\rho\alpha_1(\alpha_1+\alpha_2)\hat{\phi}_k(t)}{\lambda+\alpha_1^2\hat{\phi}_k^2(t)}\left\{-\sum_{i=0}^{t-1}\phi_{k-1,0}^{t}(i)\theta_{k-1}(i)v_{k-1}(i) + \right. \\
&\left.\sum_{i=0}^{t}\phi_{k-1,0}^{t}(i)\theta_0(i)v_0(i) - \sum_{i=0}^{t}\psi_{k-1,0}^{t}(i)[d_{k-1}(i) - d_0(i)]\right\}
\end{aligned}
$$

It follows from (13) that

$$
\begin{aligned}
0 < M_2 &\le \frac{\rho\alpha_1(\alpha_1+\alpha_2)\hat{\phi}_k(t)\phi_{k-1,0}^{t}(t)\theta_{k-1}(t)}{\lambda+\alpha_1^2\hat{\phi}_k^2(t)} \\
&\le \frac{\rho\alpha_1(\alpha_1+\alpha_2)|\phi_{k-1,0}^{t}(t)||\hat{\phi}_k(t)|}{2\sqrt{\lambda}\alpha_1|\hat{\phi}_k(t)|} \le \frac{\rho L_u}{2\sqrt{\lambda}} < 1,
\end{aligned}
$$

which indicates that $|1-\vartheta_k(t)| < 1 - M_2 < \rho_3 < 1$ for any t and k.

Next, the bounded convergence of tracking error $e_k(t)$ will be proved based on the double-dynamic analysis in Meng and Moore (2017). Step (I) Let $t = 0$. It follows that

$$
\tau_{1,k}(0) = \frac{\rho\alpha_1^2\phi_k(0)\theta_k(0)}{\lambda+\alpha_1^2\hat{\phi}_k^2(t)}\epsilon_{k-1}(0) - \xi_k(0), \tag{20}
$$

which is bounded due to the boundedness of $\phi_k(0)$, $\theta_k(0)$, $\hat{\phi}_k(0)$, $\epsilon_{k-1}(0)$ and $\xi_k(0)$. Therefore $\tilde{\tau}_{1,k}(0)$ is also bounded and satisfies $|\tilde{\tau}_{1,k}(0)| \le \beta_{\tilde{\tau}_{1,k}(0)} < \infty$. From (17), we have

$$
\begin{aligned}
\|\boldsymbol{e}_k(1)\| &\le \|\mathscr{A}_k(t)\|\|\boldsymbol{e}_{k-1}(1)\| + \beta_{\tilde{\tau}_{1,k}(0)} \\
&\le \rho_2^k\|\boldsymbol{e}_0(1)\| + \beta_{\tilde{\tau}_{1,k}(0)}/(1-\rho_2).
\end{aligned} \tag{21}
$$

Due to the boundedness of $\boldsymbol{e}_0(1)$, the bounded convergence of $\boldsymbol{e}_k(1)$ is satisfied, immediately indicating that $e_k(1)$ is boundedly convergent for any k and satisfies $\sup_k |e_k(1)| \le \beta_e(0) < \infty$. Then, it follows from (19) that

$$
\begin{aligned}
\tau_{2,k}(0) = &-\frac{\rho\alpha_1^2\hat{\phi}_k(0)}{\lambda+\alpha_1^2\hat{\phi}_k^2(0)}\epsilon_{k-1}(0) \\
&+ \frac{\rho\alpha_1\hat{\phi}_k(0)}{\lambda+\alpha_1^2\hat{\phi}_k^2(0)}\left[\sum_{i=3}^{q}\alpha_i e_{k-i+1}(1) + (\alpha_1+\alpha_2)e_0(1)\right] \\
&+ \frac{\rho\alpha_1(\alpha_1+\alpha_2)\hat{\phi}_k(0)}{\lambda+\alpha_1^2\hat{\phi}_k^2(0)}\left\{\phi_{k-1,0}^{0}(0)\theta_0(0)v_0(0)\right. \\
&\left.-\psi_{k-1,0}^{0}(0)[d_{k-1}(0) - d_0(0)]\right\}
\end{aligned} \tag{22}
$$

It is easy to verify that $\tau_{2,k}(0)$ is also bounded and assumed to satisfy $|\tau_{2,k}(0)| \le \beta_{\tau_{2,k}}(0)$. From (19), we have

$$
\begin{aligned}
|v_k(0)| &\le |1-\vartheta_k(0)||v_{k-1}(0)| + |\tau_{2,k}(0)| \\
&\le \rho_3^k\|v_0(0)\| + \beta_{\tilde{\tau}_{2,k}(0)}/(1-\rho_3).
\end{aligned} \tag{23}
$$

which implies that $v_k(0)$ is boundedly convergent for all k and satisfies $|v_k(0)| \le \beta_v(0)$.

Step (II) Suppose that for any $t = 0, 1, \ldots, T-1, T \in \mathbb{Z}_{N-1}$, both $e_k(t+1)$ and $v_k(t)$ are boundedly convergent and satisfy $|e_k(t+1)| \le \beta_e(t) < \infty$ and $|v_k(t)| \le \beta_v(t) < \infty$ for any k.

For $t = T$, one can easily check that

$$
|\tau_{1,k}(T)| \le \rho L_u(2T\beta_{\hat{\boldsymbol{\phi}}}\beta_{v,\max} + \beta_\varepsilon)/\lambda + 2TL_u u_{\max} + \beta_\xi \tag{24}
$$

with $\beta_{v,\max} = \max_{t\in\mathbb{Z}_{T-1}}\{\beta_v(t)\}$, $u_{\max} = \max\{|u_+|, |u_-|\}$. This implies that $\tilde{\tau}_{1,k}(T)$ is also bounded and satisfies $\tilde{\tau}_{1,k}(T) \le \beta_{\tilde{\tau}_{1,k}}(T)$. Following the same way as that in the case of $t = 0$, $\boldsymbol{e}_k(T+1)$ is boundedly convergent, therefore $e_k(T+1)$ is bounded and satisfies $|e_k(T+1)| \le \beta_e(T) < \infty$. Then, we have

$$
\begin{aligned}
|\tau_{2,k}(T)| \le \frac{\rho}{2\sqrt{\lambda}}\Big[&(2\beta_{\hat{\boldsymbol{\phi}}} + L_u)T\beta_{v,max} + \beta_\epsilon + \beta_e(T) \\
&+ 2(T+1)L_d\beta_d\Big] < \infty.
\end{aligned} \tag{25}
$$

Therefore, the bounded convergence of $v_k(T)$ can be ensured from (19). By mathematical induction, it is concluded that the bounded convergence of tracking error $e_k(t)$ and $v_k(t)$ are achieved. Then, the condition (iii) holds. The proof is complete. ■

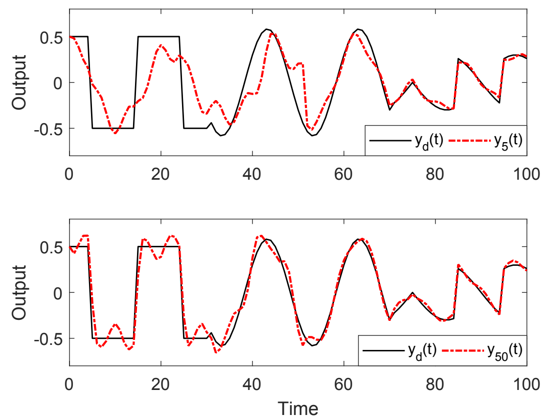

Fig. 1. Output tracking performance

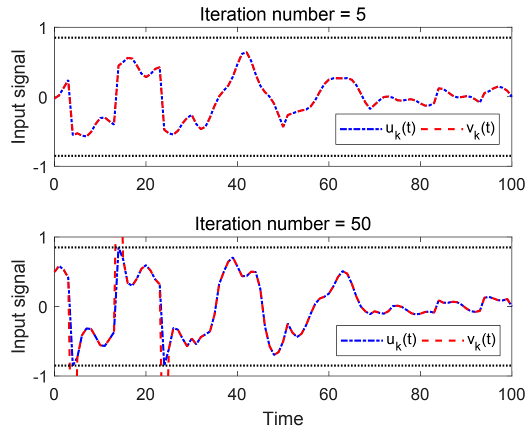

Fig. 2. Control signal with and without constraints

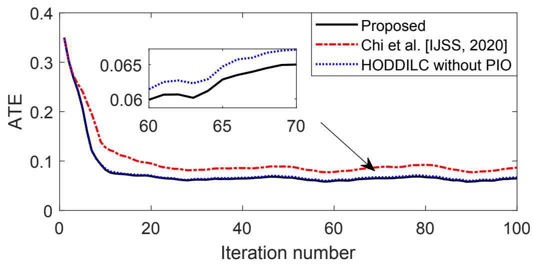

Fig. 3. ATE performance index

5. AN ILLUSTRATIVE EXAMPLE

Consider a nonlinear systems studied in Chi et al. (2020)

$$y_k(t+1) = \begin{cases} \dfrac{y_k(t)}{1+y_k^2(t)} + u_k^3(t) + d_k(t), \quad t \in [0,50], \\ \dfrac{y_k(t)y_k(t-1)y_k(t-2)(y_k(t-2)-1)+\alpha(t)u_k(t)}{1+y_k^2(t-1)+y_k^2(t-2)} \\ +d_k(t), \quad t \in (50,100], \end{cases}$$

where $\alpha(t) = 1+\text{round}(t/50), t \in \{0,1,\ldots,99\}$. The input constraints are given as $u_+ = 0.85$, $u_- = -0.85$. The external disturbance is assumed as $d_k(t) = 0.1\sin(t+0.2k)$ which is obviously nonrepetitive. The desired reference is taken as

$$y_d(t+1) = \begin{cases} 0.5 \times (-1)^{\text{round}(t/10)}, & t \in [0,30], \\ 0.5\sin(t\pi/10) + 0.3\cos(t\pi/10), & t \in (30,70], \\ 0.3\sin(t\pi/15) \times (-1)^{\text{round}(t/10)}, & t \in (70,100]. \end{cases}$$

For the simulation purpose, the initial parameters are chosen as $u_0(t) = 0$, $\epsilon_0(t) = 0$, $\hat{\phi}_0(t) = 0.2$ for $t \in \{0,1,\ldots,99\}$, $\varepsilon = 0.001$, $\eta = 1$, $\lambda = 0.7$, $\mu = 1$, $\rho = 0.8$, $l_1 = 0.9$ and $l_2 = 0.05$. The tracking results and control signal are shown in Figs.1 and 2, while the averaged tracking error defined by $\text{ATE}(k) = \sum_{t=1}^{N} |e_k(t)|/N$ is plotted in Fig.3. It is seen that the tracking performance is gradually improved as the iteration number increases. Also, the proposed method outperforms the HODDILC without PIO and the newly proposed observer-based DDILC in Chi et al. (2020). Note that the control signal by the proposed method obviously satisfies the input constraints.

6. CONCLUSION

In this paper, a novel PIO based HODDILC method has been proposed for nonlinear batch processes with nonrepetitive disturbances subject to input constraints. Compared with the recently developed observer-based DDILC method Chi et al. (2020), high-order tracking error information and control inputs in the previous time instants in the current batch are incorporated into ILC law design, such that the tracking performance could be further improved. Differing from most of the existing DDILC methods (Hou and Jin (2013); Chi et al. (2018a,b)) where all the uncertainties are lumped into the pseudo partial derivative or pseudo gradient to facilitate algorithm design and convergence analysis, the proposed method estimates the gradient itself and the uncertainty term by parameter estimation algorithm and PIO, respectively. Based on the contraction mapping principle, rigorous analysis has been carried out to clarify the convergence of tracking error. An illustrative example has well demonstrated the effectiveness and advantage of the proposed design.

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