Abstract: This paper presents the application of the time and frequency domain data-driven methods to the iterative decentralized PID tuning of MIMO process. Each single loop is sequentially excited to generate data required for retuning. The initial PID tuning gains are adjusted in order to match the defined decentralized MIMO reference model. The proposed strategy is applied to process simulations and to an experimental coupled tanks pilot plant.

Keywords: PID control, Data-driven tuning, Iterative Tuning, Process Control

1. INTRODUCTION

Most industrial processes are multi-input/multi-output (MIMO). For such systems, the interactions between inputs and outputs make the feedback controller design more difficult comparing to single-input/single-output (SISO) process. By adjusting the controller parameters of one loop can affect other loops performance due to the coupling between them.

Decentralized PID MIMO control is still widely used at the lower-levels for regulatory control (Shen et al., 2010). This approach aims to tune the controllers based on SISO methods aiming at compensating loop interactions (Acioli Júnior and Barros, 2012). According to the coupling level in the process, many decentralized techniques can be applied. For modest interactions, detuning methods, sequential loop closing methods or equivalent transfer function methods can be used.

In the sequential loop closing methods (Mayne, 1979), the idea is to treat the decentralized controller design problem for process with $n$ inputs $n$ outputs as a sequence of $n$ SISO projects. Thus, the loops are tuned and closed sequentially, one after another. One approach to tune SISO PID controllers is applying data-driven techniques. They determine parameters directly by using operational data or generated from an experiment (Gao et al., 2017). In most of the cases, it is used a reference model to describe the control system objective and an optimization is performed.

Data-driven MIMO PID tuning techniques have been proposed by extensions of SISO tuning versions. Those methods can be iterative as in Jansson and Hjalmarsson (2004), or based on a single experiment as in Campestrini et al. (2016). Most of those techniques are for discrete time domain controller, making more difficult to implement in PLCs (programmable logic controllers) or DCSs (distributed control systems). Moreover, most of these techniques use open-loop data for parameters computations. However, in some cases it is common to keep the loop closed due, for instance, to production constraints, safety reasons or unstable plants (Padilla et al., 2017).

In Gao et al. (2017), it is proposed a data-driven optimal tuning using the closed loop step response time domain data directly based on a reference model. In Moreira et al. (2018b), a frequency constraint is added to improve system robustness and stability. In Moreira et al. (2018a), it is proposed a technique to shape a desired closed-loop frequency response. These techniques only consider SISO systems. The iterative version of these techniques is presented in Moreira et al. (2018c).

In this article, the techniques presented in Gao et al. (2017), Moreira et al. (2018c) and Moreira et al. (2018a) are used to the PI/PID decentralized of MIMO plants readjust iteratively using sequential loop closing methods in closed-loop approach. The time and frequency data are generated by the sequentially application of a reference signal proposed in Barroso et al. (2015) in each loop. Optimal gains are computed for each loop by iteration. Performance is evaluated using frequency and time indexes in order to evaluate the results for each resulting adjustment.

This paper is organized as: The problem statement is presented in Section 2. The experiment design is developed in Section 3. The data-driven tuning techniques chosen are explained in Section 4. The proposed procedure is summarized in Section 5. The reference model selection is explained in Section 6. Simulations results are presented in Section 7. Experimental results are presented in Section 8. The conclusions are discussed in Section 9.
2. PROBLEM STATEMENT

Consider a MIMO closed loop $T$ as shown in Fig. 1, $r$ is the reference n-vector, $e$ is the error n-vector, $C$ the initial decentralized PID controller matrix, $u$ is controller action n-vector, $G$ is the n-input x n-output process matrix and $y$ is the output n-vector.

Each SISO PID controller in $C$ has the following form:

$$C_i(s) = K_p + \frac{K_i}{s} + K_ds$$  \hspace{1cm} (1)

where $K_p$, $K_i$ and $K_d$ are the respective Proportional, Integral and Derivative gains.

Assume initial controllers are known. The problem discussed in this paper can be stated as: Given a decentralized stable MIMO $n \times n$ closed loop $T$ with known stabilizing PID controllers $C_i(s)$. Without a parametric identification of the model $G$, find appropriate tuning gain values to shape and match a defined reference model $T_r$ as close as possible.

3. EXPERIMENT DESIGN

The proposed strategy consists of an iterative procedure. An excitation signal is sequentially applied to the MIMO closed loop system. This excitation signal is applied to each single loop sequentially to generate the required time and frequency domain data to retune the current PID controller. The procedure is repeated sequentially for all single loops to adjust the controller gains. In the next iteration, each control loops is excited again until there is a convergence in PID parameters computation. The excitation signal data can also be used to compute performance assessment indexes.

The chosen reference signal is explained in the next subsection. It is designed to generate frequency and time domain data to retune the controllers. Moreover, it is possible to estimate frequency domain performance indexes as gain and phase margins.

3.1 Closed Loop Excitation Signal

The reference signal is composed as the sequence of three different signals: a step, a standard relay test (Åström and Hägglund, 1984) and a phase margin experiment (de Arruda and Barros, 2003). An example of this excitation signal applied to each loop is shown in Fig. 2.

Delay Estimation  The cross-correlation method is applied in this paper in output $y(t)$ and reference $r(t)$ signals in the time interval where the step signal was applied ($t = 0$ and $t = T_1$), by the following formula:

$$\tilde{\tau}_d \approx \frac{1}{N} \arg \max_{\tau_d} \sum_k y(k)r(k - \tau_d), \hspace{1cm} (2)$$

where $N$ is the number of collected samples.

Gain and Phase Margins Estimation  From the experiment data, the crossover and critical frequencies, $\tilde{\omega}_c$ and $\tilde{\omega}_c$ respectively, can be estimated by measuring a stable limit cycle period during the time intervals $[T_1; T_2]$ and $[T_2; T_3]$. The process frequency response for those frequencies, ($G(j\tilde{\omega}_c)$) and ($G(j\tilde{\omega}_c)$), can be estimated using the Discrete Fourier Transform (DFT) data from the chosen stable limit cycles.

As the controller transfer function $C_i(s)$ is known, it is possible to compute the frequency responses $C_i(j\tilde{\omega}_c)$ and $C_i(j\tilde{\omega}_c)$. Therefore, the gain $A_m$ and phase margins $\phi_m$ are estimated by:

$$\hat{A}_m = \frac{1}{|G_i(j\tilde{\omega}_c)C_i(j\tilde{\omega}_c)|} \hspace{1cm} (3)$$

$$\hat{\phi}_m = \pi + \angle G_i(j\tilde{\omega}_c)C_i(j\tilde{\omega}_c) \hspace{1cm} (4)$$

Moreover, the estimated margins $\hat{A}_m$ and $\hat{\phi}_m$ can be used to evaluate the control system performance by comparing with the reference margins $A_{ref}$ and $\phi_{ref}$ stated by the reference model $T_r(s)$.

4. SISO DATA-DRIVEN TUNING TECHNIQUES

The retuned controller $\tilde{C}_i(s)$ is described by the following equations:

$$\tilde{C}_i(s) = (K_p + K_p^\Delta) + \frac{K_i + K_i^\Delta}{s} + (K_d + K_d^\Delta)s \hspace{1cm} (5)$$

where $K_p^\Delta$, $K_i^\Delta$ and $K_d^\Delta$ are the Proportional, Integral and Derivative gains increments respectively.

4.1 Time shaping

The time shaping follows from Gao et al. (2017). The technique is stated as follows.

Using time domain data collected from a closed-loop step change experiment or generated in operational procedures, the optimal increments vector $\theta_0 = [K_p^\Delta \ K_i^\Delta \ K_d^\Delta]^T$ that solves the following optimization problem:

$$\min_{\theta_0} J_1 = ||\Omega - \Phi \theta_0||_2^2 \hspace{1cm} (6)$$

where $\Omega = \left[H_r(T_1) - H_r(T_1) \ldots H_r((NT_s) - H_T(NT_s))\right]^T$, $\Phi = \left[\begin{array}{c} H_\Delta^2(T_1) \ H_\Delta^2(T_1) \ H_\Delta^2(T_1) \\ \vdots \ \vdots \ \vdots \\ H_\Delta^2((NT_s)) \ H_\Delta^2((NT_s)) \ H_\Delta^2((NT_s)) \end{array}\right]$.
\( H_\Delta \) are the step response of the terms \( T(s) \Delta_i(s) \) and
\[
\Delta_1(s) = \frac{1}{K_p + \frac{K_r}{s}} + K_d s, \quad \Delta_2(s) = \frac{1}{s} \frac{1}{K_p + \frac{K_r}{s}}
\]
\[
\Delta_3(s) = \frac{1}{K_p + \frac{K_r}{s}} + K_d s.
\]
As optimization problem (6) is convex and the matrices are constant, the solution vector \( \theta_0 \) can be obtained by:
\[
\theta_0 = (\phi^T \phi)^{-1} \phi^T \Omega \quad (7)
\]

4.2 Time shaping with frequency constraint

The optimization problem described in section 4.1 can be extended to guarantee some frequency domain conditions. These are important to improve the system robustness and stability characteristics as gain or phase margins. Hence, it can be required in a control system project that the frequency response in a certain points must be equal or closer to the reference model.

Thus, a constraint in the frequency domain is inserted in the following steps:

- Initialization: From the initial decentralized MIMO system \( T \). Obtain the initial controllers \( C \), define the desired references models \( T_r \) and a convergence tolerance \( \epsilon \).
- Retuning: Repeat steps (a) and (b) until all controllers have been tuned
  a) Experiment: Apply the proposed reference signal to excite the selected single control loop to collect the response data.
  b) Tuning: Use a data-driven SISO PID tuning technique to adjust the controller.
- Evaluation: Compute the optimal gains increments
  a) Return to the step (2): case the optimal gains increments were bigger than the specified tolerance \( \epsilon \).
  b) End: case the optimal gains increments were smaller than the specified tolerance \( \epsilon \).

The procedure block diagram is shown in Fig. 3.

5. ITERATIVE MIMO TUNING PROCEDURE

The developed procedure can be summarized in the following steps:

1. Initialization: From the initial decentralized MIMO system \( T \). Obtain the initial controllers \( C \), define the desired references models \( T_r \) and a convergence tolerance \( \epsilon \).
2. Retuning: Repeat steps (a) and (b) until all controllers have been tuned
   a) Experiment: Apply the proposed reference signal to excite the selected single control loop to collect the response data.
   b) Tuning: Use a data-driven SISO PID tuning technique to adjust the controller.
3. Evaluation: Compute the optimal gains increments
   a) Return to the step (2): case the optimal gains increments were bigger than the specified tolerance \( \epsilon \).
   b) End: case the optimal gains increments were smaller than the specified tolerance \( \epsilon \).

The procedure block diagram is shown in Fig. 3.
Table 1. Controllers gains for Time Shaping

<table>
<thead>
<tr>
<th>Iteraction</th>
<th>Controller 1</th>
<th>Controller 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_p$</td>
<td>$K_i$</td>
</tr>
<tr>
<td>1</td>
<td>1.585</td>
<td>0.250</td>
</tr>
<tr>
<td>2</td>
<td>1.612</td>
<td>0.251</td>
</tr>
<tr>
<td>3</td>
<td>1.612</td>
<td>0.253</td>
</tr>
</tbody>
</table>

where $\tau_c$ is the closed loop tuning parameter and $\tau_d$ is the process delay.

For this control system design, it is possible to define the $T_r(s)$ by selecting the desired gain $A_m$ or phase margins $\phi_m$. This is done by the development in Acitoi Júnior and Barros (2011) and Ho et al. (2001) that obtained the following equations:

$$\tau_c = \beta \tau_d$$  \hspace{1cm} (13)

$$\beta = \frac{2A_m}{\pi} - 1$$  \hspace{1cm} (14)

$$\phi_m = \frac{\pi}{2} \left(1 - \frac{1}{A_m}\right)$$  \hspace{1cm} (15)

According to the equations above, by measuring the time delay $\tau_d$ and defining $\beta$, $A_m$, or $\phi_m$ it is possible to obtain the reference model as equation (12). In this paper, the gain margin is defined.

7. SIMULATION RESULTS

To evaluate the proposed MIMO decentralized PID strategy, consider the reflux and vapor flow and the temperatures of plates 4 and 17 of the destillation column from Luyben and Vinante (1972). The model is:

$$\begin{bmatrix} R \\ V \end{bmatrix} = \begin{bmatrix} \frac{2.2}{7s+1} & e^{-s} \frac{1.3}{7s+1} & e^{-0.3s} \\ \frac{2.8}{9s+1} & e^{-1.8s} \frac{4.3}{9s+1} & e^{-0.35s} \end{bmatrix} \begin{bmatrix} T_4 \\ T_{17} \end{bmatrix}$$  \hspace{1cm} (16)

The initial decentralized PID controllers are obtained from example 1 of Vázquez and Morilla (2002):

$$C_1(s) = 0.88 + \frac{0.4835}{s} + 0.0308s$$  \hspace{1cm} (17)

$$C_2(s) = 2.70 + \frac{1.5084}{s} + 0.1971s$$  \hspace{1cm} (18)

According to the IMC PI design equations (14 and 15), the reference closed-loop transfer functions ($T_r(s)$) are different for various delays estimations. A Gaussian noise with zero mean and variance of 0.0001 is added in system outputs signals in all simulations.

The estimated delays are 1.06 and 0.31 seconds for each single loop. For the reference margins $A_{ref} = 3$ and $\phi_{ref1} = 60^\circ$ the reference model is defined as:

$$T_r = \begin{bmatrix} 1 \\ 0.9645s + 1 \end{bmatrix} e^{-1.06s} \begin{bmatrix} 0 \\ 0.2821s + 1 \end{bmatrix} e^{-0.31s}$$  \hspace{1cm} (19)

The iterative procedure is applied to the system for all data-driven techniques discussed with a tolerance $\epsilon = 0.1$. The respective controllers gains for each method are listed in Tables 1, 2 and 3. For all techniques the procedure converged in a few iterations.
The valve that connects the tanks is kept constant during the experiments. The initial reference for the controllers are 52% and 50% for loop 1 and 2 respectively. For the reference margins are $A_{ref} = 5$ and $\phi_{ref} = 72^\circ$ for both loops.

The iterative procedure is applied to the system for all data-driven techniques discussed with a tolerance $\epsilon = 0.01$. The respective controllers gains for each method are listed in Tables 6, 7 and 8. Moreover, the frequency constraint used in the time shape with frequency constraint is the critical frequency.

For all techniques, the procedure have converged almost within the same number of iterations for different solutions. The difference in the final controllers gains can be analyzed in the performance indexes listed in Tables 9 and 10 for frequency and time domain respectively.

For the time shaping technique and time shaping with frequency constraint, it is noticed a better fit for the frequency domain, specially in phase margin, and a small deceasing in the NRMSE criteria. The frequency shaping tuning technique improves the $NRMSE_1$ and the margins however it decreases the $NRMSE_2$. Those small difference in the time domain performance is justified by the initial controllers that have a good fit. It also can be noticed a high improvement in the coupling effect in the loop 2 by the IAE index while in the loop 1 it remains in similar values. The respective outputs for a step reference signal in the time domain are shown in Fig. 7 and 8.

9. CONCLUSIONS

In this paper, it was discussed time and frequency domain data-driven methods to a iterative decentralized PID tuning to MIMO systems. Using the procedure that excites...

REFERENCES