Energy and Carbon-Constrained Production Planning with Parametric Uncertainties

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Abstract: The intent of this paper is to develop a mathematical model considering uncertainties in aggregate production planning problems. It is being a great advance to consider uncertainty in production planning problems compared to models that do not account for uncertainty which results in poor planning decisions. The paper deals with the planning problems with multiple process routes to satisfy single product demand. Robust optimization is used to target parametric uncertainty to immune the model against it. The objective of the problem is to minimize the overall production cost, capping the total carbon emission and energy consumption. A detailed methodology is presented to develop a robust optimization counterpart handling parametric uncertainties. The model has the ability to control the degree of conservatism for every constraint using budget parameter and guarantees feasibility for the robust optimization problem. The applicability of the model is explained using a case study. The impact of different budget parameters on the objective value is studied, it will assist the planner to prepare the production plan with the known uncertainty level.

***Keywords**:* Robust optimization, Carbon emission, Energy consumption, Convex optimization, Production planning

1. INTRODUCTION

Aggregate production planning (APP) models are one of the most salient models in the planning of industries. Developing a production planning model while focusing on constrained carbon emission and energy saving can provide an efficient approach for sustainable development [Sinha and Chaturvedi, 2019]. However, some key input data of production and energy planning models, such as cost, energy consumption and demand specifications can be uncertain [Mirakyan and De-Guio, 2015]. In such a scenario traditional plan may result in inferior solutions which would significantly affect planning performance [Sagawa and Nagano, 2021].

To address the problem of uncertainty, literature formulations can be classified into different categories; scenario-based stochastic programming, data-driven machine learning methodologies and robust counterpart optimization. Out of them, the first two methods require historical knowledge of data in order to handle uncertainties, which may not be available or difficult to acquire. As an alternative approach, robust optimization avoids such shortcomings and target uncertainties with a predefined bounded uncertainty set.

Mula et al. [2006] presented a review of different models for production planning under uncertainty. It comprises conceptual, analytical, intelligence and simulation-based mathematical models used for APP. Further, robust optimization methodologies have been developed and widely applied in APP for multisite, supply chain networks etc. Modarres and Izadpanahi [2016] proposed a robust formulation for APP focusing on energy saving. However, very few works of literature proposed an APP using different process routes. A deterministic dual objective model to minimize carbon emission and energy consumption from multiple process routes is graphically solved by Sinha and Chaturvedi [2018]. To handle parametric uncertainties associated with each process route, a robust counterpart formulation is proposed in this paper.

The aim of the robust formulation is to make the mathematical model immune against the parametric uncertainties in the inputs. Soyster [1973] advocated uncertainties using a linear optimization approach to find a solution that fits with all data in a given uncertainty range. The result of this model happens to be over-conservative. Afterwards, in convex programming with ellipsoidal uncertainty sets, Ben-Tal and Nemirovski [1999] developed a rigorous optimization approach to solve parameter uncertainty. They developed robust equivalents for linear optimization and quadratically constrained programming. Later, Bertsimas and Sim [2004] claimed the worst-case value for all of the unknown parameters at the same time is improbable. To handle parametric uncertainties, they introduced a linear formulation having a mechanism to control the conservative level and optimality.

In this paper, a robust formulation is presented for an APP problem having multiple process routes, each process route has its respective emission factor (EF), specific energy consumption (SEC) and production cost (per ton). These parameters happen to be uncertain in a bounded interval. To immune the model against these uncertainties a robust counterpart is proposed. This model results to be linear with energy and emission limits along with an objective to minimize the production cost. The applicability of the proposed approach is illustrated by applying it to a case study based on the steel industry having multiple process routes.

2. PROBLEM DEFINITION AND MATHEMATICAL FORMULATION

Given Z_i process routes to produce a finished product from raw material. With the increasing demand of the future, P_i i.e. the variable amount of production level from the i^{th} process route needs to be enhanced.

The APP aims to plan production with a restriction on total carbon emission and total energy consumption. Here each process route has an emission factor (EF_i) which is uncertain and can vary in the region $EF_i \in [\overline{EF_i} - \widehat{EF_i},\ \overline{EF_i} + \widehat{EF_i}]$. Similarly, each process route has a specific energy consumption (SEC_i) which is also uncertain and can vary in the region $SEC_i \in [\overline{SEC_i} - \widehat{SEC_i},\ \overline{SEC_i} + \widehat{SEC_i}]$. The maximum level of production from i^{th} process route is specified fo be P_i^{max}. Also production cost per unit production (C_i) from each process route is also specified to be within the region process route $C_i \in [\overline{C_i} - \widehat{C_i},\ \overline{C_i} + \widehat{C_i}]$.

The objective is to minimize the production cost satisfying the limits of carbon emission and energy consumption. A schematic of this problem is shown in Fig. 1.

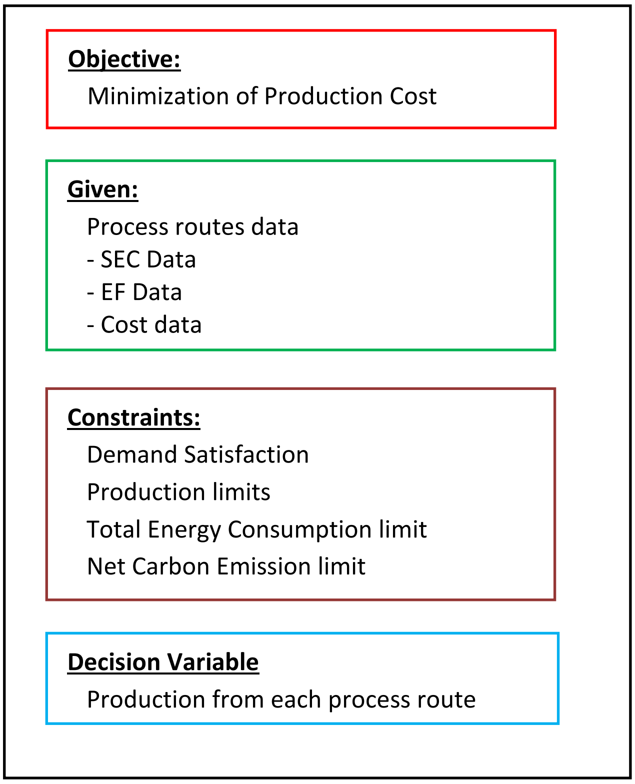

Fig 1. Schematic representation of the defined problem

2.1 Deterministic Mathematical Formulation

This section gives the mathematical formulation taking the nominal values. The mathematical model comprises the following sets, variables, parameters, and constraints.

Sets

$I = \{i | i = process\ routes\}$

Parameters

EF_i — *emission factor of process route 'i'*

SEC_i — *specific energy consumption of process route 'i'*

C_i — *production cost per unit product of process route 'i'*

P_i^{max} — *maximum production level of process route 'i'*

P_d — *total production demand*

CEL — *carbon emission limit*

ECL — *energy consumption limit*

Variables

$P_i =$ Amount of production from process route 'i'

Constraints

The most important constraint is to meet the production demand; the constraint is expressed in (1). Each process route P_i has the maximum production limit defined by (2).

$$\sum_{i=1}^{Z_S} P_i = P_d \tag{1}$$

$$P_i \leq P_i^{max} \tag{2}$$

The restriction on total carbon emission (CEL) and total energy consumption (ECL) is given by (2) and (3).

$$\sum_{i=1}^{Z_S} P_i \times EF_i \leq CE_l \tag{3}$$

$$\sum_{i=1}^{Z_S} P_i \times SEC_i \leq EC_l \tag{4}$$

The objective is to minimize the total production cost i.e. given in (4):

$$minimize \sum_{i=1}^{Z_S} P_i \times C_i \tag{5}$$

2.2 Robust Mathematical Formulation including uncertainty

This section presents the robust mathematical formulation incorporating the given uncertainties. The constraints of the deterministic mathematical formulation are modified using the concept proposed by Bertsimas and Sim [2004]. The mathematical model comprises the following additional variables and parameters followed by the modified constraints.

Additional Parameters

$\overline{EF_i}$ *nominal value of emission factor of process route 'i'*

$\widehat{EF_i}$ *uncertainity in emission factor of process route 'i'*

$\overline{SEC_i}$ *nominal value of specific energy consumption of process route 'i'*

$\widehat{SEC_i}$ *uncertainity in specific energy consumption of process route i*

$\overline{C_i}$ *nominal value of production cost per unit product from route P_i*

$\widehat{C_i}$ *uncertainity in production cost per unit production from route P_i*

Γ^E *uncertainity level in emission*

Γ^{SEC} *uncertainity level in energy consumption*

Γ^C *uncertainity level in production cost*

Additional variables

$z^E,, q_i^E, z^{SEC}, q_i^{SEC}, z^C, q_i^C$ *positive auxilarrly varibles introduced due to duality theorem*

Modified constraints

Equation (3) that restricts the carbon emission limit is modified as follows (6 and 7).

$$\sum_{i=1}^{z_s} P_i\overline{EF_i} + \Gamma^E z^E + \sum_i^{z_s} q_i^E \leq CEL, \quad \forall\, i \tag{6}$$

$$z^E + q_i^E \geq P_i \times \widehat{EF_i}, \qquad \forall\, i \tag{7}$$

Similarly, Equation (4) that limits energy consumption is modified as (8) and (9).

$$\sum_{i=1}^{z_s} P_i\overline{SEC_i} + \Gamma^{SEC} z^{SEC} + \sum_i^{z_s} q_i^{SEC} \leq ECL, \quad \forall\, i \tag{8}$$

$$z^{SEC} + q_i^{SEC} \geq P_i \times \widehat{SEC_i}, \quad \forall\, i \tag{9}$$

Further, the objective which aims to minimize (refer to (5)) the production cost (PC) is modified as (10) and (11).

$$\sum_{i=1}^{z_s} P_i C_i + \Gamma^C z^C + \sum_i^{z_s} q_i^C \leq PC \quad \forall\, i \tag{10}$$

$$z^C + q_i^C \geq P_i \times \widehat{C_i}, \quad \forall\, i \tag{11}$$

The overall formulation can be solved as a linear programming formulation. Probability violation methodology with respect to uncertainty realization can be adapted from Bertsimas and Sim (2004).

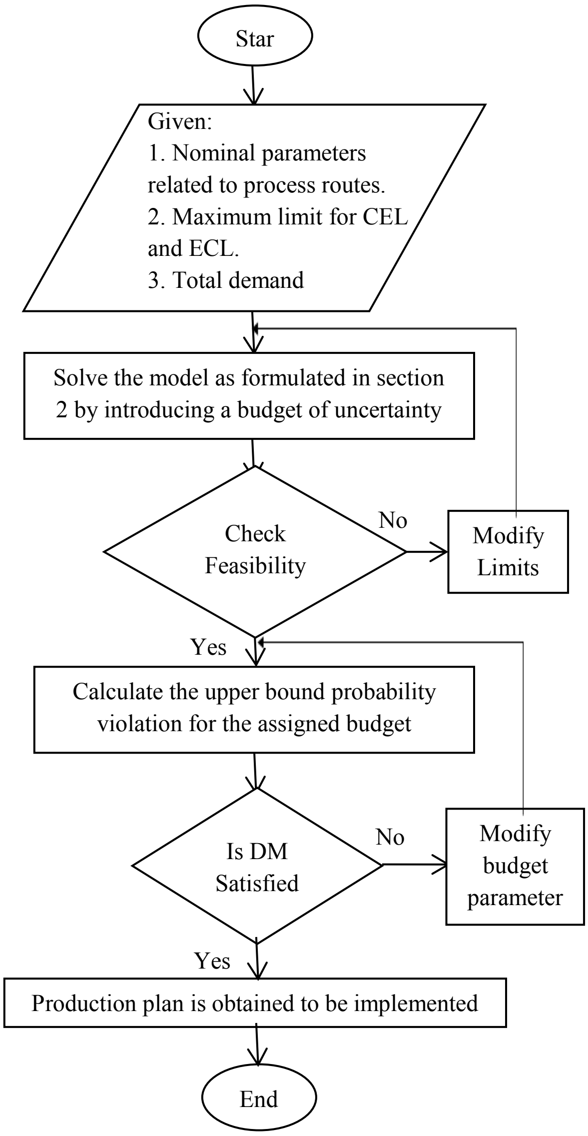

Fig. 2. Flowchart for the methodology of APP with parametric uncertainty

The flow chart in Fig. 2 shows an APP methodology with uncertain parameters for optimal reliability of production cost. First, the proposed mathematical model is solved using given data by assigning the budget parameter for each constraint; production cost (Γ^C), carbon emission(Γ^E), and energy consumption (Γ^{SEC}). A feasible model will result in attaining minimum cost with a production plan for the assigned level of uncertainty. If the true values of the parameter violate the capped limit of constraints, then the model results to be infeasible; however, this could be resolved with different cases, discussed later in this section. Next, calculate the upper bound probability violation for the assigned budget parameter. If the values are satisfactory for the decision-maker (DM) then the procedure may end by obtaining a production plan, which can guarantee the desired reliability constraints. If the DM is not satisfied, then the whole procedure should be repeated with the new value of the budget parameter until the DM can make an appropriate trade-off between the surplus production cost and calculated probability violation value.

With an objective to minimize production cost, there could be a possibility of the optimization model results being infeasible for a specific uncertainty level. The reason may be that the constrained limit for total carbon emission and energy consumption might be violating and not be satisfied with the production limit of the process routes. The issue can be resolved with the following cases.

1. By modifying the capped limits (CEL) of total carbon emission and total energy consumption (ECL), keeping the production limit constant (P_i^{max}).
2. By modifying the limit of the production amount P_i^{max} from the process routes and keeping the capped limits constant.

For the first case, the capped limits could be extended until the model outputs feasible for its respective known budget parameter. For the second case, the production limit could be modified as per the possibility and choice of DM. It could be modified by increasing the limit of the route having the lowest cost. Other methodologies can also be implemented like the graphical approach presented by Al-Mayyahi et al (2013) for target resource planning.

3. ILLUSTRATIVE EXAMPLE

The proposed methodology is applied to a case study based on the Indian steel industry. As per the report by the Centre of Science and Technology [2009], India is the third-largest steel producer in the world, it is also estimated that steel production in 2030-31 will be around 302 million MT (mMT). India's steel production is carried out mainly through four process routes. These are Scrap-Electric Arc Furnace (Scrap-EAF), Gas – based Direct Reduced Iron-Electric Arc Furnace (Gas-based DRI-EAF), Blast Furnace/Basic Oxygen Furnace (BF/BOF) and Coal-based DRI-EAF.

Table 1: Description of process routes for the example (Indian steel industry 2030-31)

Process routes	EF (MT CO_2/t product)	SEC (GJ/t product)	Production Cost ($k/t product)	Maximum projected production in 2030-31 (mMT)
Scrap-EAF	1.1±0.11	20±4	50±3	32
Gas-based DRI-EAF	1.5±0.35	21.9±2	60±3	19
BF/BOF	2.1±0.4	29±4	30±2	95
Coal-based DRI-EAF	3.8±0.5	23.4±3	25±2	180

Each process routes have its SEC, EF adapted from Sinha and Chaturvedi [2019]. An arbitrary cost is also assumed required to produce per ton of product from each process route. With the unpredictable scenarios and behaviour of process routes, these parameters can vary from their nominal value in a bounded interval. The amount of product (steel) required from each process route is presented in Table 1 with their respective uncertain process parameters. The aggregate production required from all the process routes is 302 mMT as total demand.

The objective of the problem is to optimize production cost and to estimate it considering parametric uncertainties while satisfying the demand. Here the total emission and energy consumption is capped with 1000 MT CO_2 and 8300 GJ respectively. First, the deterministic problem is solved and the objective value is calculated as $8700000. However, the parameters i.e. SEC, EF and C (cost per ton product) may change but are considered to vary in their respective bounded region. With the objective to minimize the production cost under uncertainty, the worst-case scenario corresponds to the maximum value of all the parameters. It should also be noted that aggregate forecasts are more accurate than individual ones suggest that the "true values" taken by Γ will belong to a much narrower range. For the illustrated example four process routes are available, therefore the possible range of each budget parameter will be [0,4]. Here, the DM should choose the budget parameter considering the perturbations in process route parameters.

The model was solved using the GAMS 24.8.2, XPRESS solver on the computer (Intel(R) Core(TM) i5 (3 GHz), 8 GB RAM) and the results were obtained in a fraction of seconds.

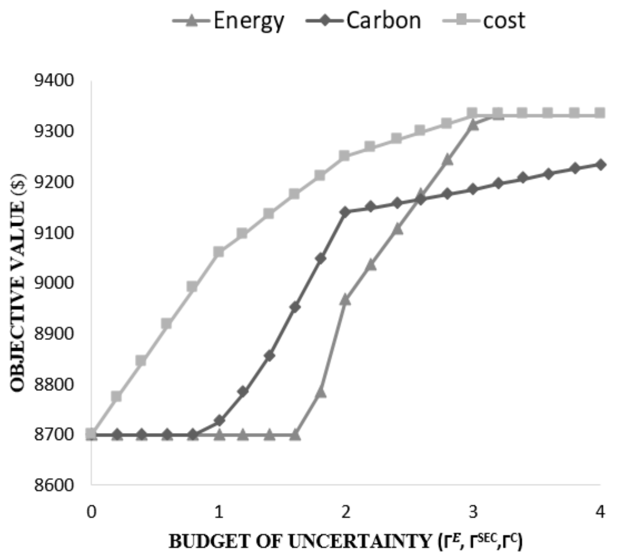

Fig. 3. Impact on the objective value with variation in the budget parameter of cost (Γ^C), carbon emission(Γ^E), and energy consumption (Γ^{SEC}).

Uncertainty in EF: In this case, bounded and symmetric uncertainty in the emission factor of each process route is assumed. The nominal EF along with the variability scale is presented in Table 1. In the previous section, it was stated that EF_i (emission factor of the i^{th} process route) can vary in the region $EF_i \in [\overline{EF_i} - \widehat{EF_i},\ \overline{EF_i} + \widehat{EF_i}]$. The parameters are considered to be varying to the right-hand side spread (leading to worst-case) from the nominal value of the bounded interval.

The level of uncertainty will be controlled by Γ^E and its corresponding effect to the objective value (production cost) can be observed in Fig. 3. It is also observed that the production cost is not affected until $\Gamma^E = 0.8$, showing that this level of uncertainty will not affect the production plan. Beyond this level, the model looks for other possible production plans to minimize objective value satisfying the emission and energy constraints. A steep rise is observed in production cost between the level of uncertainty (0.8, 2), which then steadily increases. These drastic and unpredictable changes in objective value are evident because the model looks for suitable production plans to carry out production within defined limits on the expense of production cost. Table 2 presents the effect of the upper bound of probability violation with varying budget parameter. It is observed that with an increase in budget parameter, the objective value increases. The surplus amount is also known as the price of robustness. This provides a choice to the decision-maker for a trade-off between the price of robustness and the upper bound of the probability violation.

Uncertainty in SEC: Similarly**,** in this case, true values of SEC appears in the bounded and symmetric region $[\overline{SEC_i} - \widehat{SEC_i},\ \overline{SEC_i} + \widehat{SEC_i}]$. Table 1 presents the nominal SEC of each process route along with the scale of variability. The degree of uncertainty is regulated by the Γ^{SEC} and can be seen in Fig. 3 to signify its resulting influence on the objective value. Similar to the previous case, till $\Gamma^{SEC} = 1.6$, no impact on the production plan and objective value been observed. It is also noticed that the feasible production plan can only be achieved up to $\Gamma^{SEC} = 3.6$, with an objective of \$ 9375750. Further than this degree, the limit of overall capping of energy consumption could be exceeded, leading to infeasibility. The consequence of the upper bound of probability violation with variance is presented in Table 2.

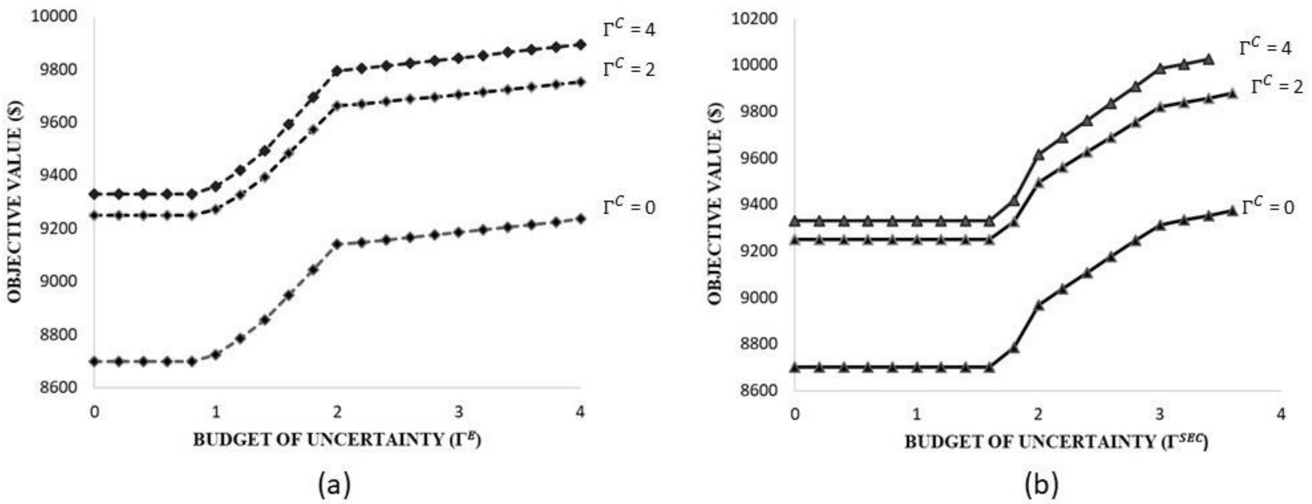

Fig. 4. Impact of multiple uncertainties on objective value (a) production cost/ton and EF, (b) production cost/ton and SEC

Uncertainty in production cost: In the same manner, the cost (C_i) varies in the interval $[\overline{C_i} - \widehat{C_i},\ \overline{C_i} + \widehat{C_i}]$. The realization of objective value concerning the budget parameter is observed in Fig. 3. A gradual increase is been observed in the production cost with an increasing level of uncertainty. It will result in feasible solutions for any budget parameter lies between [0,4]. For the worst-case i.e. $\Gamma^C = 4$, the resultant objective value will be \$10246710, with a probability of constraint violation to be 0.0625. However, the objective value results to be constant beyond $\Gamma^C = 3$, showing that only 75% of uncertain parameters (cost) influence the objective value.

A detailed study considering two uncertainties at a time is explained using Fig. 4. With an increase in the budget parameter (Γ^C), the curve shifts upwards considering uncertainties in production cost. Fig. 4(a) demonstrates the effect on objective value due to uncertainty in production cost/ton and EF, whereas, Fig. 4(b) accounts for uncertainty in production cost/ton and SEC. In both figures, the curve shift with a huge margin when Γ^C is modified from 0 to 2, on the other hand, a comparatively smaller shift is noticed from Γ^C = 2 to 4. This is due to the uncertainty level in cost and the latter shift has a lower impact on objective value because it results to be constant beyond $\Gamma^C = 3$.

Table 2. Solution data of example

Emission Factor			
Budget of Uncertainty	0	2	4
Objective Value (\$k)	8700	9150	9235.28
Probability of constraint violation	0.71	0.295	0.0625
Specific Energy Consumption			
Budget of Uncertainty	0	2	4
Objective Value (\$k)	8700	8967.56	Infeasible
Probability Constraint Violation	0.71	0.295	

Production Cost			
Budget of Uncertainty	0	2	4
Objective Value ($k)	8700	9250	10246.71
Probability Constraint Violation	0.71	0.295	0.0625

A production plan considering all uncertainties is presented in Table 3. With their respective value of $\Gamma^{C}, \Gamma^{E}, \Gamma^{SEC}$ the presented plan should be followed to satisfy the demand with the capped limits of energy consumption and carbon emission. For the mentioned level of uncertainty, and to perform production within the capped limits, a surplus approximate amount of $1076080 is required to immune the model against uncertainty.

Table 3. Production plan with uncertainty

$\Gamma^{C}, \Gamma^{E}, \Gamma^{SEC}$	2.5, 2, 2
Production Cost	$ 9776080
Production Plan (mMT/process route)	
P1	32
Scrap-EAF	13.3
Gas-based DRI-EAF	89.7
BF/BOF	167
Total Demand	302 mMT

The outcomes of the proposed work will assist DM in the following ways:

- Obtaining a production schedule that should be followed adhering to restricted standards.
- Estimating the production cost in uncertain scenarios.
- Provides flexibility to the model and the ability to modify it based on uncertainty level.

4. CONCLUSION

With an increase in concern over carbon emission, energy consumption and production cost; constrained planning with uncertainty is important. In this study, a robust counterpart programming model is developed to integrate uncertainties in parameters related to the APP with multiple process routes. Total carbon emission and total energy consumption required to satisfy the demand is capped and the optimization problem is solved to minimize production cost. This formulation do not increase the problem size significantly and maintains linearity, It also provides control over the feasibility and optimality for the robust optimization problem. The proposed methodology is explained with a case study of steel production in India. In the example for a set of prescribed uncertainty levels, results depict that approximately an additional 12.3% capital would be required to satisfy demand under the constrained scenario. A production plan is also presented with an upper bound of probability violation for each constraint. The planning problem can also be implemented by constraining cost objective and optimizing the model for carbon emission and energy consumption. In future work, similar planning problems would be linked with scheduling for sustainable production.

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