Dynamic Modeling of a Batch Sludge Settling Column by Partial Differential Non Linear Equations with a Moving Interface

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Abstract: The further goal of the model presented in this paper is to propose a digital tool to predict the behaviour of a main equipment of the water resource recovery facilities, the secondary settling tank. Most of the time, models of this liquid/solid two-phase system are based on a dynamic solid phase mass balance and a settling velocity constitutive expression stemming from the Kynch'assumption. The model presented here is based on both dynamic mass and dynamic momentum balance equations and on an explicit representation of the sludge blanket depth. Therefore, it is able to be used in a wider range of operating conditions than existing models in the literature. It includes effective solid stress appearing when the solid particles concentration is above a given threshold and a moving interface above which there is no solid particles: the sludge blanket. A study of the main physical phenomena into a batch sludge sedimentation column (hydrodynamics of two-phase suspensions) is usually carried out before a faithful representation of continuous sedimentation within a continuous settling tank is deduced. The batch settler is divided into three zones (clarification, thickening and compression). A one dimensional implicit hybrid partial differential non linear equations (Hybrid NL-PDE's) state space representation is proposed based on the different dynamics in each of the three zones as well as the constitutive equations and the boundary conditions. Some simulations are given and compared with sludge blanket height measurements in a batch settling column.

Keywords: Dynamic Model; Partial Differential Equations (PDE's); Non linear system; Mobile interface; Constitutive equations; Boundary Conditions; Batch sludge settling column; simulations

1. INTRODUCTION

Continuous sedimentation/consolidation is an important solid/liquid separation process widely used in wastewater treatment plants, mining, pulp and paper, chemical, food, and many other process industries as well as in estuarine or coastal zones. These processes have many features in common, particularly the relative flow of solid particles and fluid as the underlying basic principle, Burger (2000), Garrido et al. (2003), Li et al. (2014).
Inside the mixture, the solid particles settle and form a porous bed, when their concentration exceeds a given threshold named percolation value. The liquid phase flows through the porous network that leads to an increase of the interparticle stress which reduces the settling velocity of the solid particles, Toorman (1996).

1-D dynamic models of settling tanks represent yet the best compromise between complexity and the significance of the considered phenomena. Three types of models are described in the literature; first, the models that are based on a dynamic solid particles mass balance coupled to a constitutive equation, the so-called batch settling veloc-

ity, Vesilind (1968), Takacs et al. (1991), Diehl (2000), Queinnec (2001), David et al. (2009), Burger et al. (2013), secondly the models that are based on a dynamic solid particles mass balance and on a static momentum balance, Burger (2000), Garrido et al. (2003) and thirdly the models that are based on both a dynamic solid particles mass balance and a dynamic momentum balance, Chauchat et al. (2013), Cadet et al. (2015), Valentin et al. (2020). In the second and third cases, constitutive equations have to be used to express the forces acting on the particles: the pressure force, the effective solid stress, the gravity and the drag force.

In the first case, Takacs et al. (1991) extended the flux approach to low concentrations and proposed a double exponential settling velocity model based on Vesilind expression, Vesilind (1968). It has been used to model batch and continuous settling tanks in normal operating conditions. But Queinnec (2001) showed the limitation of this first approach where the constitutive equation for the solid particles velocity depends only on the solids concentration, Kynch (1952). David et al. (2009) concluded that the Takacs method often works satisfactorily in normal

operating conditions but, during extreme events such as storms, the solid particles concentration may be decreasing with depth and the Takacs method fails.

All these observations highlight the need to deepen methods that effectively represent all operating conditions in sedimentation/consolidation operation.

This paper presents an implicit nonlinear partial derivative equations (PDE) model of the batch sludge settling column with one moving interface, the sludge blanket, and the thickening/compression concentration threshold. This model is valid in a wide range of operating conditions. It considers neither the semi-empirical Vesilind hindered settling velocity, Vesilind (1968) nor the Takacs semi-empirical expression, Takacs et al. (1991), and is based on dynamic mass and dynamic momentum balance equations. Through this batch sedimentation study, our goal is to develop a model that can be used for a further design of a control of the water quality at the top outlet of the continuous settling tanks. To reach this goal, the model must take into account the various behaviours of the mixture inside the settling tank and not only the nominal operation.

Continuous and Boolean variables as well as the partial differential and algebraic equations to describe the main physical phenomena taking place in the batch sludge settling column (hydrodynamics of two-phase suspensions) have been identified and are presented in a structured way. Two discontinuous phenomena have been detected and integrated into the dynamic model which provides a hybrid character to this nonlinear model, Valentin et al. (2007).

2. A 1D DYNAMIC PHYSICAL MODEL

At a starting time, t_0, the batch settling column is completely filled with organic sludge at an initial concentration. Time elapses to allow the sludge to thicken and to compress with the weight as body force. Fig. 1 gives a schematic view of the evolution of the sludge thickening and settling over times t_i such that $t_3 > t_2 > t_1 > t_0$.

Thus, the batch column is modelled by considering three zones, clarification, thickening and compression. Three different behaviours appear separated by one moving interface, the sludge blanket, and the thickening/compression concentration threshold :

- the moving interface, the sludge blanket, is located at depth $z = z_v(t)$. It separates the clarification zone which does no longer contain any solid particles and the thickening zone which contains solid particles,
- the solid particles concentration threshold is located at depth $z = z_c(t)$ where a change of behaviour takes place because the solid particles volume fraction exceeds a threshold above which an interparticle stress between the solid particles appears (when they are closer to each other).

This is another approach than Valentin et al. (2020) which assumed that the water was not necessarily clear at the top of the tank (zero solid particles concentration), then a two zones schematic view was presented.
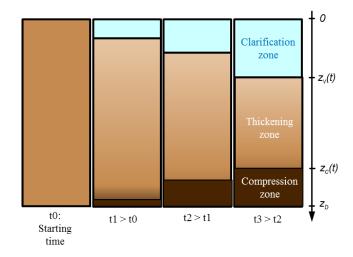

Figure 1. One-dimensional schematic view of organic sludge settling evolution in a batch settling column.

2.1 1D Mass and Momentum balances in the liquid/solid two-phase zones

The dynamic model describing the behaviour of the sludge in the two-phase zones of the batch settling column (thickening and compression zones) comes from the dynamic mass and momentum balances under the following commonly used simplifying assumptions, Burger (2000), Garrido et al. (2003), Chauchat et al. (2013), Li et al. (2014):

1. The liquid and solid phases completely fill the settler, then its volume is constant.
2. There is no biological activity in the settling tank, Burger (2000).
3. The solid particles have the same size and shape, Garrido et al. (2003), David et al. (2009), Diehl (2000).
4. There is a uniform particle concentration at a given depth, David et al. (2009), Diehl (2000).
5. The vessel wall friction is negligible.
6. Suspensions are flocculated completely before sedimentation.
7. The solid particles are small with respect to the containing vessel and have the same density, Garrido et al. (2003).
8. Solid particles and fluid are incompressible (No creation of flocks or filaments), Garrido et al. (2003), David et al. (2009). Then, solid phase density, ρ_s (kg/m^3), and liquid phase density, ρ_l (kg/m^3) are constant, Diehl (2000), Chauchat et al. (2013).
9. There is no mass transfer between the solid particles and the fluid, Garrido et al. (2003).
10. The batch settling column has a constant cross-sectional area, S.

Let consider both the thickening and the compression zones, the volume of which is noted $V(t)$. $V(t)$ decreases with time. They contain the liquid/solid two-phase sludge mixture. Then, dynamic mass and dynamic momentum balances can be written for the two phases, liquid and solid. Let $\varepsilon_s(z, t)$ denote the solid particles volume fraction with t the time and z the depth from the top of the batch settling column. $\varepsilon_l(z, t)$ denotes the liquid volume fraction. The solid particles mass concentration is then

$C_s(z,t) = \rho_s \varepsilon_s(z,t)$. Let $v_s(z,t)$ (m/s) denote the solid phase Eulerian average velocity and $v_l(z,t)$ (m/s) the liquid phase Eulerian average velocity.

Remark 1. To improve the readability, an abuse of notations is done by omitting (z,t) in the following equations.

The dynamic mass balances can be written as the next two partial differential equations (PDE's) for the liquid phase and the solid phase, in the most general way:

Liquid phase mass Balance:
$$\partial_t(\rho_l \varepsilon_l) = -\partial_z(\rho_l \varepsilon_l v_l) \tag{1}$$

Solid phase mass Balance:
$$\partial_t(\rho_s \varepsilon_s) = -\partial_z(\rho_s \varepsilon_s v_s) \tag{2}$$

As well, the dynamic momentum balance equations can be written as two PDE's for the liquid phase and the solid phase, Chauchat et al. (2013), Drew (1982), Martin et al. (1994):

Liquid phase momentum balance:
$$\partial_t(\rho_l \varepsilon_l v_l) = -\partial_z(\rho_l \varepsilon_l v_l^2) + \varepsilon_l \rho_l g - \varepsilon_l \partial_z P$$
$$-r(\varepsilon_s)(v_l - v_s) \tag{3}$$
with:

$\varepsilon_l \rho_l g$	volumetric gravitational force (body force)
$\partial_z P$	gradient of the pore pressure (hydrodynamic pressure)
$r(\varepsilon_s)(v_l - v_s)$	Stokes like drag force i.e. liquid-solid dynamic interaction force standing for viscous friction between the two phases. $r(\varepsilon_s)$ is the resistance coefficient. Chauchat et al. (2013).

Solid phase momentum Balance:
$$\partial_t(\rho_s \varepsilon_s v_s) = -\partial_z(\rho_s \varepsilon_s v_s^2) + \varepsilon_s \rho_s g - \varepsilon_s \partial_z P - \partial_z \sigma_e$$
$$+r(\varepsilon_s)(v_l - v_s) \tag{4}$$
with:

$\partial_z \sigma_e$	gradient of the interparticle stress between the solid particles. Burger (2000)

2.2 Specific physical algebraic equations in the two-phase zone

As the sludge is a two-phase (liquid, solid) suspension, the following algebraic equation is valid:
$$\varepsilon_l(z,t) + \varepsilon_s(z,t) = 1 \tag{5}$$

Moreover, the sum of the two mass balances (1) and (2) knowing (5) gives:
$$\partial_z(\varepsilon_l v_l + \varepsilon_s v_s) = 0 \tag{6}$$
As the operation in the settling column is batch, the liquid and solid velocities at the bottom are:
$$v_l(z_b,t) = 0 \text{ and } v_s(z_b,t) = 0 \tag{7}$$
Then, the following algebraic equation stands:
$$\varepsilon_l(z,t)v_l(z,t) + \varepsilon_s(z,t)v_s(z,t) = 0 \tag{8}$$

Therefore, $\varepsilon_l(z,t)$ can be calculated from (2) and (5) and $v_l(z,t)$ can be deduced from (4), (5) and (8):
$$v_l(z,t) = \frac{-\varepsilon_s(z,t)v_s(z,t)}{(1 - \varepsilon_s(z,t))} \tag{9}$$

2.3 The three deduced EDP modeling the two-phase zone

Thus, the four dynamic balances can be expressed only in terms of the solid particles volume fraction, $\varepsilon_s(z,t)$, the solid particles volumic flux, $f_s(z,t) = \varepsilon_s(z,t)v_s(z,t)$ and the pore pressure, P. The gradient of the pore pressure, $\partial_z P$, is calculated from both the liquid phase and the solid particles momentum balances, (3), (4), and according to (8), $\partial_t(f_l + f_s) = 0$:
$$\partial_t \varepsilon_s = -\partial_z f_s \tag{10}$$

$$\varepsilon_s^2 \partial_t f_s = f_s^2 \partial_z \varepsilon_s - 2\varepsilon_s f_s \partial_z f_s - \frac{\varepsilon_s^3}{\rho_s}\partial_z P - \frac{\varepsilon_s^2 \partial_{\varepsilon_s} \sigma_e}{\rho_s}\partial_z \varepsilon_s$$
$$+\varepsilon_s^3 g - \frac{r(\varepsilon_s)\varepsilon_s f_s}{\rho_s(1 - \varepsilon_s)} \tag{11}$$

$$0 = \frac{(1 - 2\varepsilon_s)f_s^2}{(1 - \varepsilon_s)^2}\partial_z \varepsilon_s - \frac{2\varepsilon_s f_s}{(1 - \varepsilon_s)}\partial_z f_s + \varepsilon_s^3(\frac{1}{\rho_l} - \frac{1}{\rho_s})\partial_z P$$
$$-\frac{\varepsilon_s^2}{\rho_l}\partial_z P - \frac{\varepsilon_s^2 \partial_{\varepsilon_s} \sigma_e}{\rho_s}\partial_z \varepsilon_s + \varepsilon_s^2 g$$
$$-\frac{r(\varepsilon_s)\varepsilon_s f_s}{(1 - \varepsilon_s)}(\frac{1}{\rho_l} - \frac{1}{\rho_s}) \tag{12}$$

Remark 2. Using the liquid phase and the solid particles momentum balances to calculate the pore pressure, as above, avoids an additional constitutive equation to calculate it, as it was proposed in Valentin et al. (2020).

2.4 Physical parameters' constitutive equations

Constitutive (closure) expressions come from experimental data. Different equations have been proposed by various authors in several contexts (wastewater from cities, mines, estuarine or coastal zones ...). Li et al. (2014) presented a very interesting critical review with most of the approaches (non-exhaustively). $\sigma_e(\varepsilon_s)$ and $r(\varepsilon_s)$ can be chosen among them.

For example, the constitutive equations presented by Garrido et al. (2003) for $\sigma_e(\varepsilon_s)$ and by Chauchat et al. (2013), based on Toorman (1996), for $r(\varepsilon_s)$ can be used:

$$\sigma_e(\varepsilon_s) = \alpha(\varepsilon_s)\sigma_0 \frac{\varepsilon_s^{n_s} - \varepsilon_c^{n_s}}{\varepsilon_c^{n_s}} \tag{13}$$
$$r(\varepsilon_s) = \rho_l g/K(\varepsilon_s) \text{ with } K(\varepsilon_s) = A_k \varepsilon_s^{-2/(3-n_r)} \tag{14}$$
with σ_0, n_s, A_k and n_r different constant parameters characterizing the sludge (permeability K, ...) and $\alpha(\varepsilon_s)$, a Boolean parameter which depends on the settler zone and defined as follows:

$$\alpha(\varepsilon_s) = \begin{cases} 0 & \text{for } \varepsilon_s \leq \varepsilon_c \\ 1 & \text{for } \varepsilon_s > \varepsilon_c \end{cases} \tag{15}$$

with ε_c, the solid volume fraction compression threshold. $\alpha(\varepsilon_s)$ is equal to zero in the thickening zone where the

particles are quite far from each other due to a low concentration. Thus, the constitutive equation of $\sigma_e(\varepsilon_s)$ depends on the zones of the batch settling column. The Boolean parameter $\alpha(\varepsilon_s)$ defines a change of settling behaviour at depth $z = z_c(t)$. The formulation ensures that $\sigma_e(\varepsilon_s)$ is a continuous function at $\varepsilon_s = \varepsilon_c$. Then the mixture inside the batch settling column has two different behaviours in the thickening zone and in the compression zone. The need for continuous and Boolean variables in the settler model makes it hybrid, Valentin et al. (2007).

2.5 Motion equation of the moving interface between the one-phase clarification zone and the two-phase zones, $z_v(t)$

Inside the mixture, the solid particles settle in the thickening zone and form a porous bed in the compression zone. These two zones contain solid particles and liquid while the upper clarification zone only contains liquid. Let calculate the solid particles mass variation in the two lowest zones which contain solid particles, with volume $V(t)$:

$$\frac{dM_s}{dt} = \frac{d}{dt}\left[\rho_s S \int_{z_v^+(t)}^{z_b} \varepsilon_s(z,t)dz\right] \quad (16)$$

According to the Leibniz rule, $\frac{dM_s}{dt}$ can be written as follows:

$$\frac{dM_s}{dt} = \rho_s S \left[\int_{z_v^+(t)}^{z_b} \partial_t \varepsilon_s(z,t)dz - \varepsilon_s(z_v^+,t)\frac{dz_v(t)}{dt}\right] \quad (17)$$

Taking into account the continuity balance (2), the batch settling operation and further calculations, $\frac{dM_s}{dt}$ can be written:

$$\frac{dM_s}{dt} = \rho_s S \left[f_s(z_v^+,t) - \varepsilon_s(z_v^+,t)\frac{dz_v(t)}{dt}\right] \quad (18)$$

On the other side, the system being closed, $\frac{dM_s}{dt} = 0$. Then:

$$\varepsilon_s(z_v^+,t)\frac{dz_v(t)}{dt} = f_s(z_v^+,t) \quad (19)$$

2.6 Boundary conditions

The boundary conditions are defined at the two boundaries of the liquid/solid two-phase zone, thickening and compression zones, i.e. at the sludge blanket depth, $z_v(t)$, which is the moving interface, and at the bottom of the batch settling column, z_b. Variables ε_s, f_s and P are continuous at the thickening/compression threshold, when $\varepsilon_s(z,t) = \varepsilon_c$. Three boundary conditions are necessary for the three first order EDPs.

The velocity of the solid particles is zero at the bottom of the batch column, $v_s(z_b,t) = 0$. From this condition can be deduced two boundary conditions concerning convected flow variables (Duidam (2009)), standing in EDPs (2) and (4):

- particles mass flux:
$$\rho_s f_s(z_b,t) = 0, \quad (20)$$
- particles momentum flux:
$$\rho_s f_s(z_b,t)v_s(z_b,t) = 0. \quad (21)$$

Since equations (10) and (11) are finally used, the same equivalent boundary condition is used twice:

$$f_s(z_b,t) = 0, \quad (22)$$

It is clear from equations (20) and (21) that this single condition makes sense simultaneously for the two equations (10) and (11).

Let calculate the total momentum variation in the batch settling column in order to express a boundary condition at the moving interface, $z_v(t)$:

$$\frac{d}{dt}\frac{\mathcal{P}_t}{S} = \frac{d}{dt}\left[\int_0^{z_b} \rho_l f_l(z,t)dz + \int_{z_v^+(t)}^{z_b} \rho_s f_s(z,t)dz\right] (23)$$

According to the Leibniz rule, the sludge blanket motion and the batch settling operation, $\frac{d}{dt}\frac{\mathcal{P}_t}{S}$ can be written as follows:

$$\frac{d}{dt}\frac{\mathcal{P}_t}{S} = \int_0^{z_v^-(t)} \rho_l \partial_t f_l(z,t)dz + \int_{z_v^+(t)}^{z_b} \rho_l \partial_t f_l(z,t)dz$$
$$-\rho_l f_l(z_v^+,t)\frac{dz_v(t)}{dt} + \int_{z_v^+(t)}^{z_b} \rho_s \partial_t f_s(z,t)dz$$
$$-\rho_s f_s(z_v^+,t)\frac{dz_v(t)}{dt} \quad (24)$$

Taking into account the liquid and solid momentum balances (3) and (4), the batch settling operation, the zero liquid velocity in the clarification zone and further calculations, $\frac{d}{dt}\frac{\mathcal{P}_t}{S}$ can be written:

$$\frac{d}{dt}\frac{\mathcal{P}_t}{S} = -\int_0^{z_v^-(t)} \partial_z P(z,t)dz$$
$$-\int_{z_v^+(t)}^{z_b} \rho_l \partial_z \left(\frac{f_l^2(z,t)}{\varepsilon_l(z,t)}\right)dz - \int_{z_v^+(t)}^{z_b} \partial_z P(z,t)dz$$
$$-\int_{z_v^+(t)}^{z_b} \rho_s \partial_z \left(\frac{f_s^2(z,t)}{\varepsilon_s(z,t)}\right)dz - \int_{z_v^+(t)}^{z_b} \partial_z \sigma_e dz$$
$$-[\rho_l f_l(z_v^+,t) + \rho_s f_s(z_v^+,t)]\frac{dz_v(t)}{dt} + \frac{W_t}{S} \quad (25)$$

with W_t the total weight of the sludge mixture in the batch column.

On the other side, the system being closed, the total momentum variation in the batch settling column, $\frac{d}{dt}\frac{\mathcal{P}_t}{S}$, is only equal to the pressure forces plus the total weight of the sludge mixture:

$$\frac{d}{dt}\frac{\mathcal{P}_t}{S} = P_{atm} - P(z_b,t) - \sigma_e(z_b,t) + \frac{W_t}{S} \quad (26)$$

Finally:

$$\varepsilon_s(z_v^+,t)P(z_v^+,t) = \frac{\rho_l f_s^2(z_v^+,t)}{(1 - \varepsilon_s(z_v^+,t))}$$
$$+\varepsilon_s(z_v^+,t)[P(z_v^-,t) + \sigma_e(z_v^+,t)] \quad (27)$$

with:

$$P(z_v^-,t) = P_{atm} + \rho_l z_v g \quad (28)$$

Now let us write the state-space representation of the sludge settling in batch operation in a column with the three different dynamics in each of the three zones, clarification, thickening and compression given by EDPs (10), (11) and (12), $\sigma_e(\varepsilon_s,\alpha)$, $r(\varepsilon_s)$ and $\alpha(\varepsilon_s)$ constitutive equations, (13)-(14)-(15) taken as examples, the boundary conditions (22) and (27) and the dynamic of the moving interface, the sludge blanket (19).

3. THE IMPLICIT NON LINEAR STATE SPACE HYBRID MODEL

The state, X, is of dimension 3 with the solid particles volume fraction, $\varepsilon_s(z,t)$, the solid particles volumic flux, $f_s(z,t)$ and the pore pressure, P:

$$X = \begin{pmatrix} \varepsilon_s(z,t) \\ f_s(z,t) \\ P(z,t) \end{pmatrix}$$

After some manipulations of (10), (11) and (12), the implicit hybrid non linear PDE state space representation of the batch settling column is:

$$\begin{cases} E(X)\partial_t X = \mathcal{A}(X, \partial_z X, r, \alpha) \\ Y = \mathcal{C}(X) \end{cases} \quad (29)$$

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \rho_s \varepsilon_s^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (30)$$

$$\mathcal{A} = \mathcal{A}_1(X, \sigma_e(X, \alpha))\partial_z X + \mathcal{A}_2(X, r), \quad (31)$$

$$\mathcal{A}_1 = \begin{pmatrix} 0 & -1 & 0 \\ \rho_s f_s^2 - \varepsilon_s^2 \partial_{\varepsilon_s} \sigma_e & -2\rho_s \varepsilon_s f_s & -\varepsilon_s^3 \\ A_1^* & -\frac{2\varepsilon_s f_s}{(1-\varepsilon_s)} & A_1^{**} \end{pmatrix}, \quad (32)$$

$$A_1^* = \frac{(1-2\varepsilon_s)f_s^2}{(1-\varepsilon_s)^2} - \frac{\varepsilon_s^2 \partial_{\varepsilon_s} \sigma_e}{\rho_s},$$

$$A_1^{**} = -\varepsilon_s^2 \left(\frac{(1-\varepsilon_s)}{\rho_l} + \frac{\varepsilon_s}{\rho_s} \right),$$

$$\mathcal{A}_2 = \begin{pmatrix} 0 \\ \rho_s \varepsilon_s^3 g - \frac{r\varepsilon_s f_s}{(1-\varepsilon_s)} \\ \varepsilon_s^2 g - \frac{r\varepsilon_s f_s}{(1-\varepsilon_s)}\left(\frac{1}{\rho_l} - \frac{1}{\rho_s}\right) \end{pmatrix} \quad (33)$$

and with $\sigma_e(\varepsilon_s, \alpha)$, $r(\varepsilon_s)$ and $\alpha(\varepsilon_s)$ constitutive equations, (13)-(14)-(15), the boundary conditions (22) and (27) and the dynamic of the moving interface, the sludge blanket (19). The matrix $\mathcal{C}(X)$ depends on the output of interest, Y.

4. DYNAMIC SIMULATIONS

In this section, the organic sludge settling in a batch column of height $z_b = 1.8m$ is simulated. The results are compared to the sludge blanket height measurements. The organic sludge characteristics parameters are $\rho_s = 1050 kg/m^3$ and $\rho_l = 1000 kg/m^3$. The permeability parameter, A_k, and the solid volume fraction compression threshold, ε_c, are estimated with a least square method that minimizes the quadratic difference between the $N_m = 22$ measurements and the N_m values simulated with the model presented in the above section. $A_k = 7.53 \times 10^{-5} m/s$ with 95% confidence interval $\pm 5.4 \times 10^{-20} m/s$ and $\varepsilon_c = 5.3 \times 10^{-3}$ with 95% confidence interval $\pm 2.6 \times 10^{-8}$. The values of the other parameters come from Garrido et al. (2003) and Chauchat et al. (2013) paper with the necessary adaptations to represent organic and not mineral sludge. A comparison between simulated sludge blanket height and the N_m measurements is given in Fig. 2. And a comparison between the simulated average solid particles concentration under sludge blanket and values calculated from the

sludge blanket height measurements, assuming that the concentration is uniform under the sludge blanket, is given in Fig. 3. In both cases, the simulated values fit with the experimental data. The average of the absolute value of the relative errors between each measured sludge blanket height value and the corresponding simulated value is 3.5%. The average of the absolute value of the relative errors between the average solid particles concentration under sludge blanket and the corresponding simulated value is 4.8%. The sludge settles well and the solid particles accumulate at the bottom of the batch column where their average concentration, C_s, increases. The height of clarified water zone increases with time and separation by settling occurs as expected.
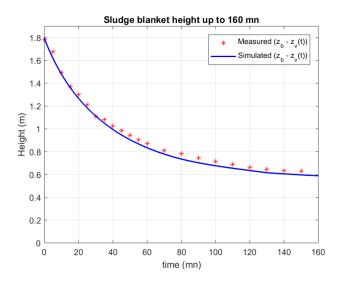
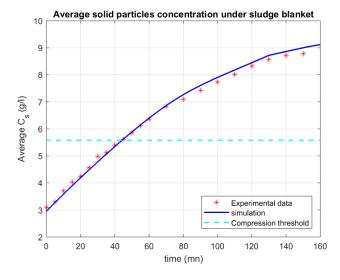Figure 2. Sludge blanket height in a batch settling column.

Figure 3. Average solid particles concentration under sludge blanket in a batch settling column.

5. CONCLUSIONS AND PERSPECTIVES

In this paper, an implicit hybrid partial differential non linear equations (Hybrid NL-PDE's) representation of a batch settling column has been presented. It takes into

account one moving interface, the sludge blanket, and the thickening/compression concentration threshold and is able to be used in a wider range of operating conditions than existing models in the literature. Next step is to extend this structured Hybrid NL-PDE's representation of a batch settling column to the continuous secondary settler involved in wastewater treatment plants. Indeed in practice, settling tanks are still largely used but they may undergo dysfunction due to gravity settling problems or to the quantity or the quality of sludge. Besides new technologies like membrane filtration are often proposed as alternatives to settling tanks but are not yet up to expectations. Therefore it is still interesting to model and optimize the behaviour of existing wastewater treatment unitary equipment's and to propose control strategies for a more efficient and compact installation and for a wide range of operating conditions. It will be part of a new digital tool to predict the behaviour of a main equipment of the Water Resource Recovery Facilities, the secondary settling tank.

6. ACKNOWLEDGMENTS

7. NOTATIONS

Index i stands for liquid phase or solid phase (particles).

$\varepsilon_i(z,t)$	solid (liquid) phase volume fraction
ε_c	solid volume fraction thickening/compression threshold
$\rho_i \ (kg/m^3)$	solid (liquid) phase density
$C_i(z,t)$ (kg/m^3)	solid (liquid) phase mass concentration $C_i(z,t) = \rho_i \varepsilon_i(z,t)$
$f_i(z,t) \ (m/s)$	solid (liquid) phase average volumetric flux
$P(z,t) \ (Pa)$	excess pore pressure
$P_{atm} \ (Pa)$	atmospheric pressure
r $(kg.m^{-3}.s^{-1})$	resistance coefficient of the drag force proposed by Darcy and Gersevanov in a two-phase model
S	cross sectional area of the column
$\sigma_e(\varepsilon)$	effective solid stress function (Pa)
$v_i(z,t) \ (m/s)$	solid (liquid) phase average velocity
$v_m(t) \ (m/s)$	mixture (bulk) average velocity
$V(t)$	volume of the two lowest zones which contain solid particles, thickening and compression zones
$z_v(t) \ (m)$	sludge blanket depth (moving interface)
$z_c(t) \ (m)$	the thickening/compression interface
$z_b \ (m)$	cylindric batch settling column depth

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