A closed-loop frequency domain PI retuning technique for multivariable systems

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Abstract: A Proportional-Integral (PI) controller frequency domain retuning technique for square multivariable process is presented in this paper. Assuming knowledge of the current PI controller, the controller gains increments are computed in such that the new closed-loop matches with a desired reference model. This is performed using only the closed-loop response at certain frequencies, without knowledge of the process model. Effectiveness of the proposed method is shown in the simulation results. The maximum singular value is used to show the robustness. Finally, an experimental result is presented.

Keywords: PI control, frequency domain, multivariable, retuning, process control

1. INTRODUCTION

Multivariable process control design is usually complex due to the interaction between input and output variables. Despite the developments in the advanced control techniques, Proportional-Integral (PI)/Proportional-Integral-Derivative (PID) controllers are still the most used in the industry (Nisi et al., 2019), because they provide the easiest and most effective solutions in most applications.

The multivariable processes control schemes based on PI/PID can be divided as: centralized control (Besta and Chidambaram, 2017), decentralized control (Vu and Lee, 2010) and decoupling control (Sun et al., 2016; Garrido et al., 2018). When interactions are not modest, centralized or decoupling control should be used. Decoupling control design can be approached in two ways (Garrido et al., 2016): a decoupler combined with a diagonal decentralized controller or a purely centralized controller.

The control design methods are classified into: model-based or data-driven. In model-based methods the first step is to identify a reduced process model, then the controller is tuned using this model (e.g. Kumar et al. (2012), Ram and Chidambaram (2015)). Identifying these models may not be an easy task. Moreover, the desired performance may not be achieved due to inevitable modeling errors.

In the past twenty years, many data-driven or model-free tuning techniques have been proposed (Formentin et al., 2019). The data-driven methods tune the controller parameters using operational or generated data from an experiment instead of an explicit parametric model. For example, in Hjalmarsson (1999), the Iterative Feedback Tuning (IFT) method is extended to multiple-input multiple-output (MIMO) processes. In Huff et al. (2019), the optimal controller identification (OCI) is based on the Prediction Error identification problem. These methods may have difficulty in practical implementation, due to the number of experiments needed (IFT) or the high computational cost.

Despite numerous existing control tuning techniques, more than 60% of industrial controllers suffer from certain types of malfunctions (Gao et al., 2017). Such malfunctions can occur for two reasons: 1) the controller has been poorly tuned or 2) the plant structure has changed.

A data-driven method for simultaneous performance assessment and retuning of industrial PID controllers is presented in Gao et al. (2017). In this procedure the optimum controller is tuned using closed-loop step response data directly based on a reference model. A frequency restriction was introduced by da Silva Moreira et al. (2018b) in order to improve robustness and ensure stability. In da Silva Moreira et al. (2018a), a closed-loop data-driven PID tuning technique using only frequency domain data is presented. However, all of these methods are presented only to single-input-single-output systems.

In this paper, the technique presented in da Silva Moreira et al. (2018a) is extended to square MIMO processes. The increments of the initial centralized/decentralized controllers gains of a MIMO process are computed using only frequency domain data points. The explicit process model is not required in this method. The idea is to adjust the closed-loop behavior to be close to a diagonal reference model. The final controller is centralized decoupling controller.

⋆ This work was supported by the CNPQ.

The paper is organized as follows. In section 2, the problem statement is defined. The frequency domain retuning method for MIMO process is proposed in section 3. The simulation and experimental results are presented in sections 4 and 5, respectively. Finally, the conclusion is presented in section 6.

2. PROBLEM STATEMENT

Consider a Linear Time-Invariant (LTI) MIMO process $\mathbf{G}(s) \in \mathbb{C}^{m \times m}$ with m inputs and m outputs and a controller $\mathbf{C}(s) \in \mathbb{C}^{m \times m}$. The closed-loop is given by:

$$\mathbf{T}(s) = (\mathbf{I} + \mathbf{G}(s)\mathbf{C}(s))^{-1}\mathbf{G}(s)\mathbf{C}(s), \qquad (1)$$

where $\mathbf{T}(s) \in \mathbb{C}^{m \times m}$, $\mathbf{I} \in \mathcal{R}^{m \times m}$ is an identity matrix and $\mathbf{C}(s)$ is a centralized PI controller given by:

$$\mathbf{C}(s) = \begin{bmatrix} C_{11}(s) & C_{12}(s) & \cdots & C_{1m}(s) \\ C_{21}(s) & C_{22}(s) & \cdots & C_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ C_{m1}(s) & C_{m2}(s) & \cdots & C_{mm}(s) \end{bmatrix} \qquad (2)$$

or a decentralized PI controller:

$$\mathbf{C}(s) = \begin{bmatrix} C_{11}(s) & 0 & \cdots & 0 \\ 0 & C_{22}(s) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_{mm}(s) \end{bmatrix} \qquad (3)$$

with $C_{ij}(s) = Kp_{ij} + \frac{Ki_{ij}}{s}$, Kp_{ij} and Ki_{ij} are the Proportional and Integrative tuning gains, respectively and $i, j = 1, 2, ..., m$.

Assume an arbitrary closed-loop $\mathbf{T}(s)$ with a known initial PI controller $\mathbf{C}(s)$. A closed-loop reference model $\mathbf{T}_r(s)$ is defined. The problem statement is to obtain controller gains increments to yield closed-loop matching the $\mathbf{T}_r(s)$ using only closed-loop frequency response data and without the knowledge of the process model $\mathbf{G}(s)$.

3. FREQUENCY DOMAIN RETUNING

Consider an initial closed-loop ($\mathbf{T}(s)$) with a known initial PI controller (2) or (3). The retuning controller is given by:

$$\bar{\mathbf{C}}(s) = \mathbf{C}(s) + \mathbf{C}^{\Delta}(s), \qquad (4)$$

where $\mathbf{C}^{\Delta}(s)$ represent the controller gains increments matrix.

Lemma 1. Given the closed-loop desired reference model:

$$\mathbf{T}_r(s) = \begin{bmatrix} Tr_{11}(s) & 0 & \cdots & 0 \\ 0 & Tr_{22}(s) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Tr_{mm}(s) \end{bmatrix}, \qquad (5)$$

the $\mathbf{C}^{\Delta}(j\omega)$ value is given by:

$$\mathbf{C}^{\Delta}(j\omega) = \mathbf{C}(j\omega)\mathbf{T}(j\omega)^{-1}(\mathbf{T}_r(j\omega) - \mathbf{T}(j\omega))\mathbf{S}_r^{-1}(j\omega), \qquad (6)$$

where $\mathbf{S}_r(s) = \mathbf{I} - \mathbf{T}_r(s)$ is the reference model Sensitivity Function.

Proof. By definition

$$\mathbf{S}(s) = (\mathbf{I} - \mathbf{T}(s)) = (\mathbf{I} + \mathbf{G}(s)\mathbf{C}(s))^{-1}. \qquad (7)$$

Multiplying (7) by $\bar{S}^{-1}(s) = (\mathbf{I} + \mathbf{G}(s)\bar{\mathbf{C}}(s))$:

$$\mathbf{S}(s)\bar{\mathbf{S}}^{-1}(s) = (\mathbf{I} + \mathbf{G}(s)\mathbf{C}(s))^{-1}(\mathbf{I} + \mathbf{G}(s)\bar{\mathbf{C}}(s)), \qquad (8)$$

and $\mathbf{S}_r(s)$:

$$\mathbf{S}(s)\bar{\mathbf{S}}^{-1}(s)\mathbf{S}_r(s) = (\mathbf{I} + \mathbf{G}(s)\mathbf{C}(s))^{-1} (\mathbf{I} + \mathbf{G}(s)\bar{\mathbf{C}}(s))\mathbf{S}_r(s). \qquad (9)$$

The idea is to make the new closed-loop close to the reference model. Consider $\bar{\mathbf{S}}(s) = \mathbf{S}_r(s)$:

$$\mathbf{S}(s) = (\mathbf{I} + \mathbf{G}(s)\mathbf{C}(s))^{-1}(\mathbf{I} + \mathbf{G}(s)\bar{\mathbf{C}}(s))\mathbf{S}_r(s). \qquad (10)$$

Substituting (4) into (10):

$$\mathbf{S}(s) = (\mathbf{I} + \mathbf{G}(s)\mathbf{C}(s))^{-1} (\mathbf{I} + \mathbf{G}(s)(\mathbf{C}(s) + \mathbf{C}^{\Delta}(s)))\mathbf{S}_r(s) \qquad (11)$$

$$\mathbf{S}(s) = [(\mathbf{I} + \mathbf{G}(s)\mathbf{C}(s))^{-1}(\mathbf{I} + \mathbf{G}(s)\mathbf{C}(s)) + (\mathbf{I} + \mathbf{G}(s)\mathbf{C}(s))^{-1}(\mathbf{G}(s)\mathbf{C}(s)\Delta(s))]\mathbf{S}_r(s), \qquad (12)$$

with $\Delta(s) = \mathbf{C}^{-1}(s)\mathbf{C}^{\Delta}(s)$,

$$\mathbf{S}(s) = [\mathbf{I} + \mathbf{T}(s)\Delta(s)]\mathbf{S}_r(s), \qquad (13)$$

$$\mathbf{S}(s) - \mathbf{S}_r(s) = \mathbf{T}(s)\Delta(s)\mathbf{S}_r(s), \qquad (14)$$

$$\mathbf{T}_r(s) - \mathbf{T}(s) = \mathbf{T}(s)\Delta(s)\mathbf{S}_r(s), \qquad (15)$$

$$\Delta(s) = \mathbf{T}^{-1}(s)(\mathbf{T}_r(s) - \mathbf{T}(s))\mathbf{S}_r^{-1}(s). \qquad (16)$$

Substituting $\Delta(s) = \mathbf{C}^{-1}(s)\mathbf{C}^{\Delta}$:

$$\mathbf{C}^{\Delta}(s) = \mathbf{C}(s)\mathbf{T}(s)^{-1}(\mathbf{T}_r(s) - \mathbf{T}(s))\mathbf{S}_r^{-1}(s). \qquad (17)$$

Considering $s = j\omega$ in (17) obtain (6), where $\mathbf{C}^{\Delta}(j\omega) \in \mathbb{C}^{m \times m}$.

Considering the PI controller, the elements of the $\mathbf{C}^{\Delta}(j\omega)$ are of the form:

$$C_{ij}^{\Delta}(s) = Kp_{ij}^{\Delta} + \frac{Ki_{ij}^{\Delta}}{s}. \qquad (18)$$

Thus, Kp_{ij}^{Δ} and Ki_{ij}^{Δ} are computed as shown in lemma 2.

Lemma 2. The parameters Kp_{ij}^{Δ} and Ki_{ij}^{Δ} of the $\mathbf{C}^{\Delta}(j\omega)$ are computed by:

$$Kp_{ij}^{\Delta} = (\Phi_{r_{ij}}^T \Phi_{r_{ij}})^{-1}\Phi_{r_{ij}}^T \Omega_{r_{ij}} \qquad (19)$$

$$Ki_{ij}^{\Delta} = (\Phi_{i_{ij}}^T \Phi_{i_{ij}})^{-1}\Phi_{i_{ij}}^T \Omega_{i_{ij}}, \qquad (20)$$

where

$$\Phi_{r_{ij}} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \qquad (21)$$

$$\Omega_{r_{ij}} = \begin{bmatrix} \Re(C_{ij}^{\Delta}(j\omega_1)) \\ \Re(C_{ij}^{\Delta}(j\omega_2)) \\ \vdots \\ \Re(C_{ij}^{\Delta}(j\omega_N)) \end{bmatrix}, \qquad (22)$$

$$\Phi_{i_{ij}} = \begin{bmatrix} -1/\omega_1 \\ -1/\omega_2 \\ \vdots \\ -1/\omega_N \end{bmatrix}, \qquad (23)$$

$$\Omega_{i_{ij}} = \begin{bmatrix} \Im(C_{ij}^{\Delta}(j\omega_1)) \\ \Im(C_{ij}^{\Delta}(j\omega_2)) \\ \vdots \\ \Im(C_{ij}^{\Delta}(j\omega_N)) \end{bmatrix}, \quad (24)$$

$C_{ij}^{\Delta}(j\omega)$ is given by (6), $\Re()$ and $\Im()$ are the real and imaginary parts, respectively, $\omega_1 > 0$ and ω_N is the frequency where the phase of the reference model is $-90°$.

Proof. We know that $\mathbf{C}^{\Delta}(j\omega) \in \mathbb{C}^{m\times m}$. Then, the frequency response of the each element of the $\mathbf{C}^{\Delta}(j\omega)$ is the form $\mathbf{C}_{ij}^{\Delta}(j\omega) = a + bj$, where $a = \Re(\mathbf{C}_{ij}^{\Delta}(j\omega))$ and $b = \Im(\mathbf{C}_{ij}^{\Delta}(j\omega))$.

Substituting $s \to j\omega$ into (18), for a frequency we have:

$$Kp_{ij}^{\Delta} + \frac{Ki_{ij}^{\Delta}}{j\omega} = \Re(\mathbf{C}_{ij}^{\Delta}(j\omega)) + j\Im(\mathbf{C}_{ij}^{\Delta}(j\omega)). \quad (25)$$

Separating the real and imaginary terms:

$$Kp_{ij}^{\Delta} = \Re(\mathbf{C}_{ij}^{\Delta}(j\omega)) \quad (26)$$

$$-\frac{Ki_{ij}^{\Delta}}{\omega} = \Im(\mathbf{C}_{ij}^{\Delta}(j\omega)). \quad (27)$$

Considering N frequencies points:

$$\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} Kp_{ij}^{\Delta} = \begin{bmatrix} \Re(\mathbf{C}_{ij}^{\Delta}(j\omega_1)) \\ \Re(\mathbf{C}_{ij}^{\Delta}(j\omega_2)) \\ \vdots \\ \Re(\mathbf{C}_{ij}^{\Delta}(j\omega_N)) \end{bmatrix}, \quad (28)$$

$$\begin{bmatrix} -1/\omega_1 \\ -1/\omega_2 \\ \vdots \\ -1/\omega_N \end{bmatrix} Ki_{ij}^{\Delta} = \begin{bmatrix} \Im(\mathbf{C}_{ij}^{\Delta}(j\omega_1)) \\ \Im(\mathbf{C}_{ij}^{\Delta}(j\omega_2)) \\ \vdots \\ \Im(\mathbf{C}_{ij}^{\Delta}(j\omega_N)) \end{bmatrix}. \quad (29)$$

The frequency responses of the closed loop system can be evaluated by applying the fast Fourier transform (FFT) to the measured reference (r) and output (y) signals.

4. SIMULATION RESULTS

In this section, the presented retuning method is applied to two processes.

The centralized controller (Ram and Chidambaram, 2015) and decentralized (Vu and Lee, 2010) retuning of the TITO process are shown in example 1. In the example 2, a 3×3 process is considered.

The frequency range used was $[\omega_1, \omega_N]$, with $\omega_1 > 0$ and ω_N is the frequency where the phase of the reference model $\mathbf{T}_r(s)$ is $-90°$. The sampling period was ($T_s = 1$ s).

To MIMO closed-loop processes is more robust if the closed-loop maximum singular value is smaller within an interested frequency range. Thus, the maximum singular values curve of the initial ($\mathbf{T}(s)$) and retuned ($\bar{\mathbf{T}}(s)$) closed-loop is used to compare the two closed-loops.

4.1 Example 1

Consider the binary distillation column Wood-Berry process (Wood and Berry, 1973):

$$\mathbf{G}(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}. \quad (30)$$

The reference model considered is:

$$\mathbf{T}_r = \begin{bmatrix} \dfrac{1}{6.549s+1}e^{-1s} & 0 \\ 0 & \dfrac{1}{15.2817s+1}e^{-3s} \end{bmatrix}. \quad (31)$$

Initial controller is centralized - The initial controller (Ram and Chidambaram, 2015) is:

$$\mathbf{C}(s) = \begin{bmatrix} 0.3140 + \dfrac{0.0471}{s} & -0.3058 - \dfrac{0.0458}{s} \\ 0.1068 + \dfrac{0.0160}{s} & -0.2072 - \dfrac{0.0310}{s} \end{bmatrix}. \quad (32)$$

The retuned controller is:

$$\bar{\mathbf{C}}(s) = \begin{bmatrix} 0.2335 + \dfrac{0.0347}{s} & 0.1085 - \dfrac{0.0067}{s} \\ 0.0539 + \dfrac{0.0134}{s} & 0.03226 - \dfrac{0.0051}{s} \end{bmatrix}. \quad (33)$$

The closed-loop step response is shown in Fig. 1. As expected, the responses of the proposed closed-loop followed the desired responses ($\mathbf{T}_r$).

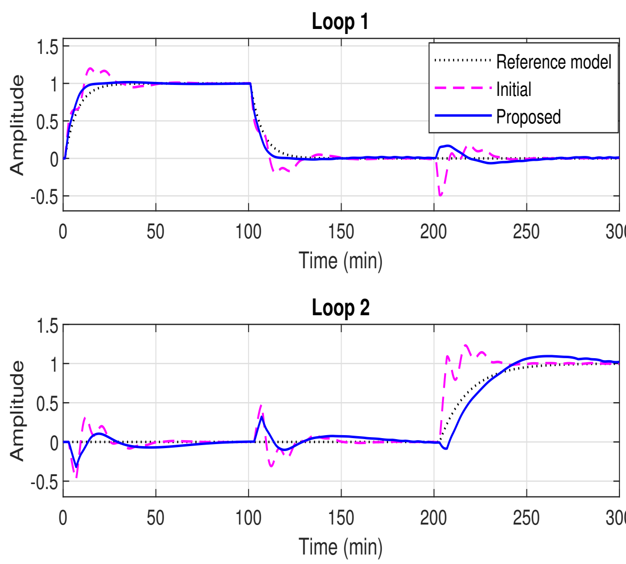

Fig. 1. Closed-loop responses of system (30) with controllers (32) and (33) - initial centralized controller - example 1

The maximum singular value curve of closed-loop is shown in Fig. 2. Note that the curve corresponding to the proposed controller is below the curve of the initial controller. This indicates most favorable robustness of the proposed controller.

Initial controller is decentralized - Now consider the initial controller (Vu and Lee, 2010) is:

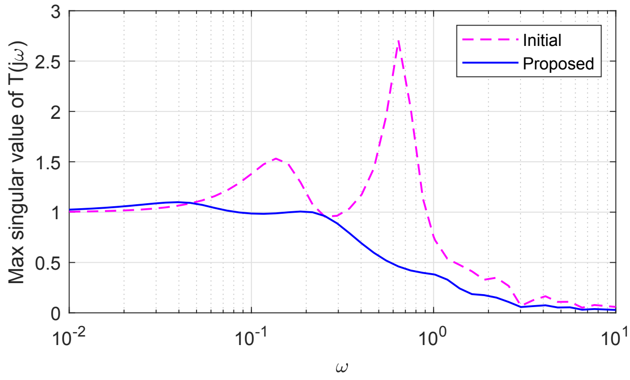

Fig. 2. Evaluation of system robustness - initial centralized controller - example 1

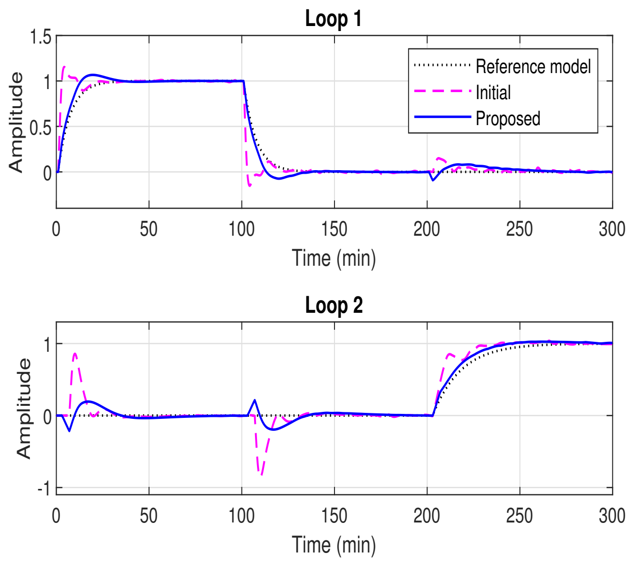

Fig. 3. Closed-loop responses of system (30) with controllers (34) and (35) - initial decentralized controller - example 1

$$\mathbf{C}(s) = \begin{bmatrix} 0.7319 + \dfrac{0.0744}{s} & 0 \\ 0 & -0.0772 - \dfrac{0.0102}{s} \end{bmatrix}. \quad (34)$$

The retuned controller is:

$$\bar{\mathbf{C}}(s) = \begin{bmatrix} 0.1979 + \dfrac{0.0312}{s} & -0.0582 - \dfrac{0.0058}{s} \\ 0.0339 + \dfrac{0.0097}{s} & -0.0590 - \dfrac{0.0060}{s} \end{bmatrix}. \quad (35)$$

The closed-loop step response is shown in Fig. 3, where it is possible obverse that the proposed closed-loop matches with the defined reference model.

The maximum singular value curve of closed-loop is show in Fig. 4. The curve corresponding to the proposed controller is below the curve of the initial controller.

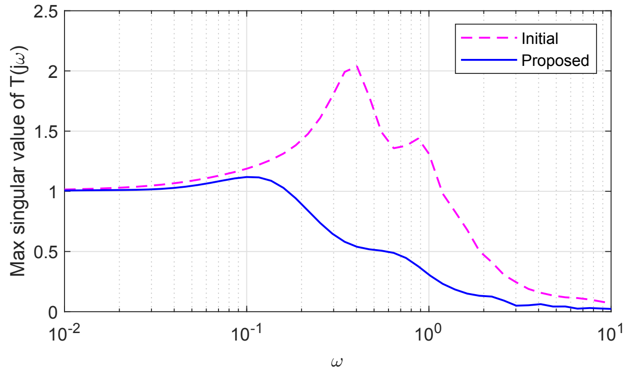

Fig. 4. Evaluation of system robustness - initial decentralized controller - example 1

4.2 Example 2

Consider the binary ethanol-water system of a pilot plant distillation column (Ogunnaike et al., 1983):

$$\mathbf{G}(s) = \begin{bmatrix} \dfrac{0.66e^{-2.6s}}{6.7s+1} & \dfrac{-0.61e^{-3.5s}}{8.64s+1} & \dfrac{-0.0049e^{-s}}{9.06s+1} \\ \dfrac{1.11e^{-6.5s}}{3.25s+1} & \dfrac{-2.36e^{-3s}}{5s+1} & \dfrac{-0.01e^{-1.2s}}{7.09s+1} \\ \dfrac{-34.68e^{-9.2s}}{8.15s+1} & \dfrac{46.2e^{-9.4s}}{10.9s+1} & \dfrac{0.87(11.61s+1)e^{-s}}{(3.89s+1)(18.8s+1)} \end{bmatrix}. \quad (36)$$

The reference model is:

$$\mathbf{T}_r = \begin{bmatrix} \dfrac{1}{7.64s+1}e^{-2.6s} & 0 & 0 \\ 0 & \dfrac{1}{14.19s+1}e^{-3s} & 0 \\ 0 & 0 & \dfrac{1}{20.52s+1}e^{-s} \end{bmatrix}, \quad (37)$$

and the initial controller (Ram and Chidambaram, 2015) is:

$$\mathbf{C}(s) = \begin{bmatrix} 1.5215 + \dfrac{0.3804}{s} & -0.291 - \dfrac{0.0727}{s} & 0.0052 + \dfrac{0.0013}{s} \\ 0.5918 + \dfrac{0.1479}{s} & -0.3865 - \dfrac{0.0966}{s} & -0.0011 - \dfrac{0.0003}{s} \\ 29.2240 + \dfrac{7.3060}{s} & 8.9282 + \dfrac{2.2320}{s} & 0.8419 + \dfrac{0.2105}{s} \end{bmatrix}. \quad (38)$$

The retuned controller is:

$$\bar{\mathbf{C}}(s) = \begin{bmatrix} 0.7993 + \dfrac{0.2990}{s} & -0.0409 - \dfrac{0.0337}{s} & -0.0015 - \dfrac{0.0005}{s} \\ 0.1557 + \dfrac{0.1170}{s} & -0.0847 - \dfrac{0.0447}{s} & -0.0011 - \dfrac{0.0001}{s} \\ -23.07 + \dfrac{6.163}{s} & -7.9350 + \dfrac{0.9727}{s} & 0.3425 + \dfrac{0.0820}{s} \end{bmatrix}. \quad (39)$$

The closed-loop step response is shown in Fig. 5.

The maximum singular value plot of the closed-loop is shown in Fig. 6. Again, the curve produced by the proposed controller is below the curves provided by the initial controller.

5. EXPERIMENTAL RESULT

In this section, the retuning method is applied to didactic temperature module. The step response and the mean squared error are used to compare the initial and proposed closed-loop.

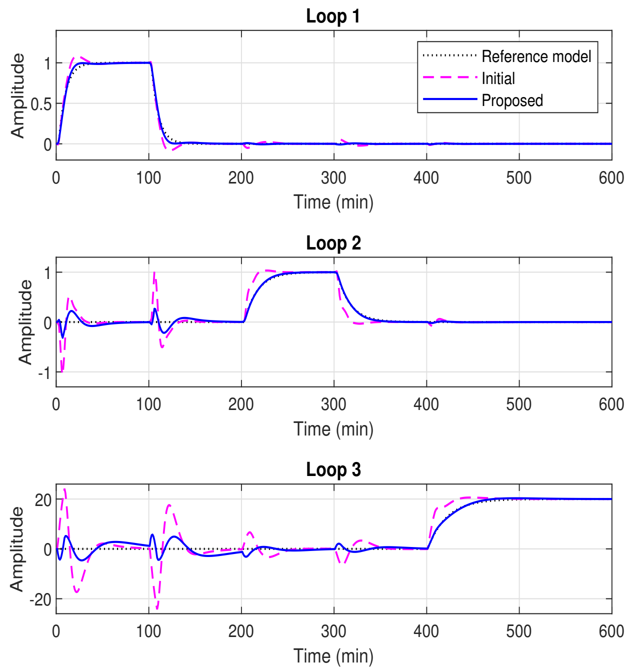

Fig. 5. Closed-loop responses of system (36) with controllers (38) and (39) - example 2

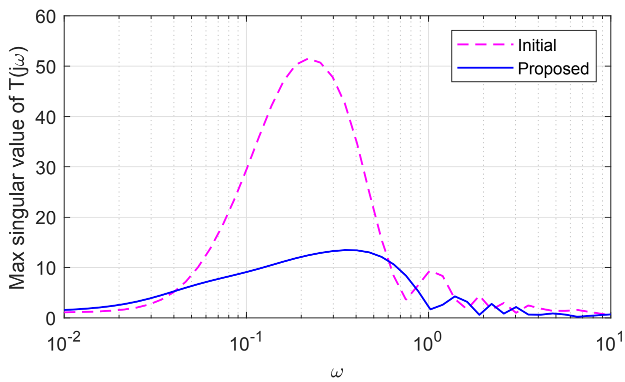

Fig. 6. Evaluation of system robustness - example 2

The temperature module uses the principle of heat dissipation caused by field effect transistors and the spread of that heat on the printed circuit board (see Fig. 7). The module was designed with an Arduino shield connection layout. The USB cable is used to connected with the computer (more details see Lima et al. (2018)).

The initial decentralized controller is:

$$\mathbf{C}(s) = \begin{bmatrix} 0.0257 + \dfrac{0.0001}{s} & 0 \\ 0 & 0.0252 + \dfrac{0.0001}{s} \end{bmatrix} \tag{40}$$

and the reference model is:

$$\mathbf{T}_r = \begin{bmatrix} \dfrac{1}{47s+1}e^{-8.5s} & 0 \\ 0 & \dfrac{1}{19.56s+1}e^{-13s} \end{bmatrix}. \tag{41}$$

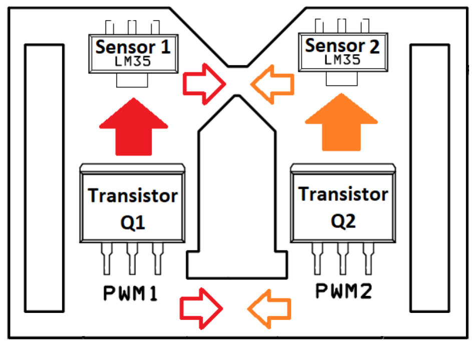

Fig. 7. Schematics of the didactic module

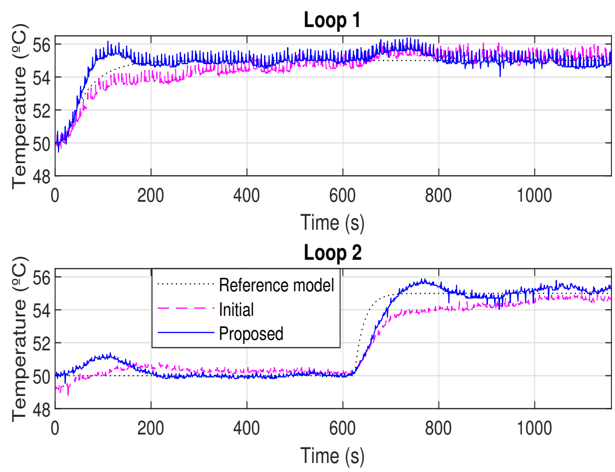

Fig. 8. Closed-loop responses with controllers (40) and (42) - experiment

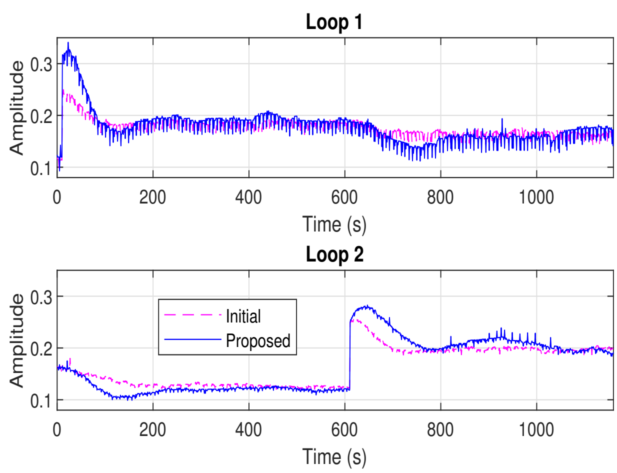

Fig. 9. Initial and proposed control signal - experiment

The retuned controller is the centralized controller given by:

$$\bar{\mathbf{C}}(s) = \begin{bmatrix} 0.0395 + \dfrac{0.00036}{s} & -0.0164 - \dfrac{0.00014}{s} \\ 0.0070 - \dfrac{0.00025}{s} & 0.0268 + \dfrac{0.00038}{s} \end{bmatrix}. \tag{42}$$

The closed-loop step response and the signal control are shown in the Figs. 8 and 9. It can be observe the loop

Table 1. Mean square error

	Loop 1	Loop 2
Initial	1.6964	1.2669
Proposed	0.9391	0.9886

2 steady state error with the initial controller, which was reduced with the proposed controller. The mean square error is shown in Table 1, where it is possible to observe a reduction of 44% and 21% of the loop 1 and 2 error, respectively.

6. CONCLUSION

In this paper a retuning centralized/decentralized PI controller method was present. Using frequency domain data the controller parameters increments was computed. To prove the effectiveness of the method we presented simulation and experimental results.

In the examples was observe that with retuning it is possible to improve the performance of centralized or decentralized control and decrease the coupling. When the initial controller is a decentralized, the final controller is a centralized decoupling controller, as shown in the section 4.1. The reduction of the maximum singular value of the proposed closed-loop when compared to the initial loop showed the technique's robustness.

In the experimental result, we used a didactic temperature module. As presented in section 4.1, the initial decentralized controller was retuned and we obtained a centralized controller. In this case, we observed a reduction in mean square error.

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