2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

A Coordinated Optimization Approach to Volt/VAr Control for Large Power Distribution Networks

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Abstract— **Electric distribution networks are operated under a number of constraints in order to deliver power at a certain quality and reliability level. A distributed management system (DMS) is a supervisory control layer in the distribution system used by the utilities for managing distribution assets in a coordinated fashion. For large distribution systems (those consisting of thousands of nodes and multiple tens of capacitor banks and voltage regulators), an integrated Volt/VAr Control (IVVC), which maximizes asset lifetime, is non-trivial due to the size of the search space for determining the optimal settings of these devices. This paper presents coordinated optimization approach to IVVC for large power distribution networks that will enable a more optimal operation of the distribution network while maximizing distribution control asset lifetime through the minimization of unnecessary device switching.**

I. INTRODUCTION

The basic structure of an electric system consists of various hardware such as generators, power lines, transformers, feeders, and SCADA equipment for generation, transmission, and distribution of electricity to the end consumers. The power industry is operated under complex rules to provide reliable electricity to the end user. Power produced by generators is first transmitted over high voltage lines (66 KV to 765 KV) and then distributed at low voltage levels (below 66 KV). The control of bulk power over high-voltage transmission lines is markedly different than that of distributing power in the low-voltage distribution circuit. The delivery of electricity typically utilizes a supervisory control and data acquisition system (SCADA) that provides monitoring and control from generation through the step-down substation to detect the need for an increase/reduction in generating resources, and to respond to system instabilities. Today's electric system is inefficient with losses occurring throughout the transmission and distribution system due to increased load growth.

In general, electric distribution networks are operated under a number of constraints in order to deliver power at a certain quality and reliability level. Distribution control assets, such as capacitor banks, voltage regulators, whose operation is governed by local controllers based on locally sensed variables such as voltage or reactive power, are employed by the utilities to facilitate and to maintain an acceptable variation of voltage magnitude and an acceptable level of power factor within the distribution system. Traditionally, these devices are operated under fixed schedules, based on voltage, time of day or some other local parameters, and their operations are disjointed from one another. Any changes to the system configuration or in power demands, or due to the intermittent behavior of renewable generation can result in higher, lower, or oscillatory voltages in the system, resulting in a decreased effectiveness of operation. A computationally efficient optimal control policy for integrated Volt/VAr control (IVVC) is a critical component of a DMS [1] for improved power factor, device life, and load adjustment via conservation voltage reduction [2].

Most prior work on IVVC is based on determining the optimal solution (on/off, tap settings) only for the current time instead of an optimal schedule for the entire day [3], [4], [5], [6], [7], [8], [9]. IVVC is further complicated by the introduction of intermittent renewable power sources in the distribution grid, which can cause unacceptably large voltage fluctuations. Performing IVVC for only the current time point (or in a feedback control approach) can compensate for large voltage fluctuations at the expense of frequent switching of the cap bank and voltage regulator control devices. Since the lifetime of a capacitor or voltage regulator is dependent on the number of switching operations performed, it is desired to minimize the number of device operations during the day while at the same time achieving the desire level of voltage and reactive power control. For example, a capacitor bank is typically design so that it can withstand 40,000-50,000 on/off switching cycles for an average of 6-7 switches per day, but if the controls for the cap bank allow this number of switching operations to increase then the lifetime will be correspondingly reduced. Thus, any local or global control algorithms resulting in frequent switching of device setting are likely to be discarded by the Utility in an attempt to prevent premature device failure. In this sense, an optimal IVVC should consider minimizing the number of switches as well as improving the power factor or minimizing losses, which are commonly used as objectives.

There are two critical needs in daily scheduling of reactive power sources and adjusting voltage regulator tap ratios. The first need is an accurate load (active, reactive) forecast. In order to minimize the number of switches of capacitors, we apply model of the distribution network to estimate the behavior of the network as though it were driven by the forecasted load. The models estimate the impact of nonlinearities, and are used to predict the behavior of dependent variables (i.e. outputs: node voltages, losses) of the modeled dynamic distribution network with respect to changes in the independent variables (i.e. control device settings). Second need is computational speed so that the determination of the

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978-1-4577-0079-8/11/$26.00 ©2011 AACC

control device settings can be accomplished within a small time period. The algorithm for scheduling should run fast enough to provide updates in 5-15 minutes for large radial distribution networks. That is why more recent methods such as genetic algorithms may be undesirable even though they are capable of finding global minima.

In this paper, we present a a coordinated optimized IVVC algorithm that is computationally efficient at the expense of optimality (an approximate solution to the minimum cost of the objective function) for large radial distribution networks (e.g., those which contains thousands of nodes, tens of cap banks, and tens of load tap changers/ voltage regulators. In [10], it is discussed that the search space for the optimal device settings that minimize the cost of the objective function is $(2^{CB} \times 33^{VR})^{TP}$ where the number 2 stands for the two switching states (On/Off) of each of "CB" capacitor banks, "33" stands for the thirty three tap positions (from -16 to 16) of each of VR/LTC or voltage regulators, and "TP" stands for the number of time points in one twenty-four hour optimization time window. Clearly, the penalty for attempting to minimize the number of switching operations is a search space that is exponentially large. For example, if CB = 20, VR =10, and TP = 288 (the number of five minute time windows in twenty-four hours) then the number of combinations of control device settings is "Infinite". Even if, the number of devices were reduced to CB = 10, VR = 4, "33" tap setting changed to 4 (through expert power systems knowledge, and TP = 7, the resulting search space is greater than 1.6x1063! Therefore, we felt we needed to foresake search for the"true" global optimum, and coordinated several algorithms to reduce the search space to a reasonable value.

In this paper, we focus on real-time scheduling of device settings, only, and expand upon the approach described in [10]. We present a coordinated optimized IVVC algorithm that is computationally efficient at the expense of optimality (an approximate solution to the minimum cost of the objective function) for large radial distribution networks (e.g., those which contains thousands of nodes, tens of cap banks, and tens of load tap changers/ voltage regulators. There are several factors that contribute to the increased efficiency of our new approach compared to dynamic programming of [10] or genetic algorithms. First, we consider optimization of (discrete) capacitors and voltage regulators as separate but dependent problems. Modularity of the algorithms for different types of control devices provides flexibility to increase optimality, allowing integration with other more optimal but less efficient algorithms as needed. Our new algorithm provides: (1) an optimal daily schedule for capacitor banks based on minimizing the total VARs at the head of the distribution substation, (2) these cap banks settings are used to determine an tap settings based on leveling the average voltage and setting the average voltage to achieve some level of objective function cost minimization within appropriate sections of the distribution network, and (3) finalizing the VR tap settings using the dynamic programming algorithm discussed in [10]. Step 2 of our new approach codifies the "expert power systems knowledge" that was used to reduce the search space in [10].

Second, we partition the problem to supportive, subordinate distribution control systems, such as a microgrid control system (MCS) for microgrids or a substation-based, Distribution Automation (DA) SCADA which assumes the responsibility of CB and VR control for those nodes of the distribution grid assigned to its area of responsibility (AOR) in coordination with the overall DMS. Each of the subordinate distribution control systems coordinates its objective function for optimization with the DMS. This partitioning enables a large distribution network to be reduced to a manageable size. Each AOR is treated as a spot load by the top-level DMS. Each control system whether it be a MCS or a DA SCADA has the responsibility to coordinate the IVVC objective function with the DMS, provide DMS with its load forecast, This partitioning distributes IVVC operations so that no one control system is forced to bear the burden of the entire network IVVC control.

II. PROBLEM STATEMENT

Distribution Management System (DMS) is a supervisory control layer in the distribution system for managing distributed energy generation, microgrids (MGs), loads such as homes and buildings, energy storage systems, switchable VAR sources, and voltage regulators (VRs) or load tap changers (LTCs). A Microgrid Control System (MCS) is the most complex, subordinate asset of the DMS. The MCS manages the internal resources of the microgrids in support of the DMS for optimal distributed generation dispatch as well as Volt/VAR control in the distribution system. The DMS and MGC communicate objective functions and aggregated information on their assets to achieve at least three objectives independently or as simultaneous objectives:

- loss minimization
- load reduction
- power factor correction

for global coordinated control of the distribution operation. Incorporating MSCs into the DMS is advantageous in reducing the complexity of the IVVC because the loads and assets within local control of MCSs are aggregated as a single spot load for load forecasting and power flow computations.

The IVVC is composed of two separate but dependent steps as shown in Fig. 1. First step is the optimal commitment of CBs. The objective in this step is to increase (assuming need for reactive power) the power factor to greater than or equal to a desired value. Our challenge is in two-fold. First, CBs have discrete values. Second, the number of switches of CBs should not increase beyond a certain limit since the lifetime of a CB is correlated with the number of times it is switched on or off. Also the topology of the CB distribution within the AOR is known to the CB commitment algorithm. Finally, the CB commitment algorithm requires forecasted total reactive load for the AOR over 24 hours scheduling cycle. Second step is the adjustment of VR and transformer tap ratios. The second algorithm has the objective to flatten the average load voltage curve over the course of the day and maintain the average load voltage to a desired level over the 24 hour scheduling cycle. The desired level over the 24-hour scheduling cycle. The desired level for the average voltage is set to achieve a desired objective using the AOR's conservation voltage reduction (CVR) factor [2]. CVR factor is the percent change in load consumption resulting from a percent change in voltage [11]. CVR factors in the range of 0.7 have been found to be typical in Northwest Energy Efficiency Alliance's (NEEA) Distribution Efficiency Initiative (DEI) and EPRI's Green Circuits. Adjustment of feeder voltage at the substation and along the distribution feeders has long been used to maintain service voltage within the limits set forth in industry standards, such as ANSI C84.1 in North America [11]. The third algorithm is the Dynamic Programming algorithm of [10]. The second algorithm codifies the "expert" knowledge identified in [10] used to speed the rate of convergence of the algorithm by re-dicing the number of tap setting for the baseline determined by leveling the average voltage to a desired value.

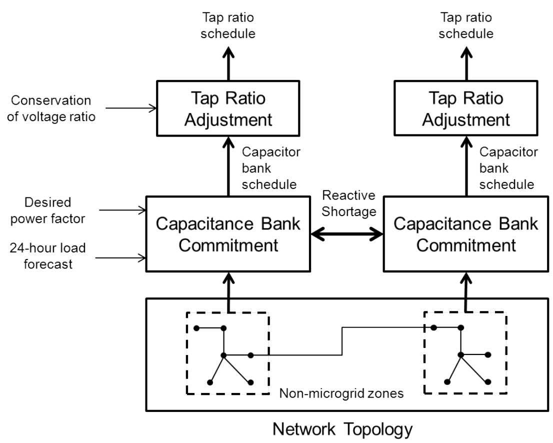

Fig. 1. The IVVC algorithm is composed of CB commitment and tap-ratio adjustment algorithms run in series.

III. OPTIMAL CAPACITANCE BANK COMMITMENT

Our optimal CB commitment algorithm is composed of three stages described as follows

- Divide the network in the AOR into zones
- Distribute VAr shortage to zones with inadequate CBs to achieve desired power factor
- Solve the modified Knapsack problem with the branch-and-bound algorithm.

Currently, Stage I is the only stage that has not been automated. Thus, a system expert is needed to perform this stage, but this is typically done once prior to final design and commissioning of the system. There are various factors that can be considered when dividing the network into zones. It may be the case that some zones have VAr resources that can supply reactive power to achieve the desired power factor while some zones do not have adequate resources. This is when the second stage comes into picture. When a zone does not have adequate resources to achieve the desired power factor then other zones with excess CBs should help out the zone in need of VAr resources. There are two possible methods to have zones help each other to achieve a common goal: 1) Have the VARs shared among zones or 2) have the shortage shared among the zones. The second method is easier to implement algorithmically because of the way Stage III is designed to switch CBs on/off at a minimum rate for unbalanced 3-phase circuits.

We now describe the algorithm for Stage - II. Input to the algorithm from previous stage is the zone map. The zone map is a graph G=(V,E) where V is the set or vertices and E is the set of edges. Each vertex corresponds to a zone. There is an edge between two zones if and only if there is a bus in one zone that is connected to bus in the other zone. In the following we will describe how zones in need of help to achieve desired power factor can get help from their neighboring zones with adequate or excess VAr resources. The other inputs to Stage - II are total reactive power shortage curves over time based on the desired power factor and total reactive power supplied by CBs for each zone for each phase. The output of Stage - II is the updated reactive power shortage curve for each zone for each phase based on how much of the neighboring zones shortages' need to be taken care for. It may be the case that a zone that needs help has more than one neighbor. In order to account for this case, we incorporated a distribution mechanism that allows a zone to distribute its reactive shortage over to its neighbors based on neighboring zones' total reactive resources.

The final stage of the CB commitment algorithm is to solve for the optimal on/off times of each CB over the 24-hour scheduling period The input to this stage from Stage - II is the updated reactive power shortage curve such that the desired power factor will be achieved in each zone at each phase within some error margins based on reactive power resources. Any mismatch in the resource distribution is taken care of in Stage - II. The remaining inputs to the algorithm are:

- Desired power factor
- CB VARs per zone per phase
- Branching size
- Maximum overshoot factor
- Percent overshoot off-time span
- Switch-on and inter-switch on sensitivities

Stage III focuses only on finding the switch on/off times for the CBs. The next two inputs are desired power factor and reactive power input from each CB for each zone and phase. The rest of the inputs are parameters for the algorithm. These parameters to the algorithm may influence the optimality of the branch-and-bound algorithm. For example, a higher the branching size results in a more optimal the CB commitment result.

Our approach in this final stage of the overall CB commitment algorithm is to describe the problem as a knapsack problem (KP). A KP is described as follows: given a knapsack of size W and items with weights and profits, maximize the total profits of items in the knapsack while keeping the total weight of the knapsack below W. The KP-based definition of the CB commitment considers CBs as items. The profit associated with each CB is the total reactive power supplied by the CB during its commitment period. Since the predicted reactive power curve is a time-varying function and is, in general, not a constant function, the profit for a CB depends on the switching order of the CBs. The weight associated with each CB is simply the fixed reactive power supplied by each CB upon switching on.

A branch-and-bound (BB) algorithm is used to solve KP. Other algorithms examined include dynamic programming and greedy. The construction of the BB algorithm for the KP-based CB commitment problem requires calculation of the area under the reactive power shortage curve when a CB is switched on. This area calculation is carried in two steps. First, the time spans during which the reactive power of the CB is less than or equal to the reactive power shortage are extracted. Second, the total duration of these time spans is multiplied by the reactive power supplied by the CB.

The KP-based description of the CB commitment naturally overcomes the challenges stated in the problem definition. First, the problem description considers the CBs as items with discrete weights. Thus it does not require finding a continuous reactive power solution and discretization of the solution. Second, the problem is formulated in a way that results in switching a CB on and off only once. Finally, the BB solution approach aligns with visual commitment of the CBs as would be done by a power systems expert. For example, if the reactive power shortage curve is shaped like a pyramid the an intuitive commitment of the CBs is to have the CBs with higher reactive power on longer than the CBs with low reactive power. In other words, a power systems expert would want to maximize the coverage of the area under the reactive load curve.

We now describe the mathematical formulation and algorithmic solution to the CB commitment problem. Suppose that a hitch-hiker has to fill up his knapsack by selecting among a finite number of objects. Each object has weight or size and value. The hitch-hiker wants to maximize the overall value of the objects in the knapsack while keeping the overall weight (size) below a certain level.

Let $Q(k)$ be the reactive power generated at the substation for $k = 1, \ldots, K$ with sampling period T such that $k \times T = 24$ is equivalent to 24 hours. Let ϕ and ϕ_d be the current and desired power factors. Then, the desired reactive power at the substation is

$$Q_d(k) = \frac{\sqrt{1-\phi_d^2}}{\phi_d} P(k) \tag{1}$$

where $P(k)$ is the real power at the substation and is a function of $Q(k)$ and ϕ as follows

$$P(k) = \frac{\phi}{\sqrt{1-\phi^2}} \tag{2}$$

The shortage reactive power at the substation Q_s is the difference between the reactive power generated at the substation and desired reactive power

$$Q_s(k) = Q(k) - Q_d(k) \tag{3}$$

Our goal is to utilize CBs to provide the shortage reactive power at the substation. There are two ways to formulate the CB commitment algorithm: 1) Under-coverage and 2) Over-coverage. The under-coverage algorithm we try to the cover the area under the shortage curve. That is the CBs provide reactive power below the shortage curve while minimizing the uncovered area between shortage and total reactive power from CBs over their switched on periods. In the over-coverage algorithm CBs provide reactive power above the shortage curve while minimizing the area between the total CBs and shortage.

In the following, we first describe the under-coverage formulation and then the over-coverage one. Let $x(k)$ be an ordered sequence of CBs such that $x_i(k) = 1$ if CB_i is switched on at time k and $x_i(k) = 0$ otherwise. Let w_i be the reactive power supplied by CB_i where $i = 1 \cdots N$ and N is the number of CBs. The profit associated with a CB varies as a function of the shortage reactive power and previous CBs that has been switched on. First, we illustrate the profit calculation in an example and then present the formal description.

Formally, the profit is calculated as follows. Let W_i be the total reactive power of the CBs in the ordered sequence $x_i(k)$ that has been switched on prior to i^{th} CB. Then, the profit associated with CB_i is equal to

$$p_i = l_i w_i \tag{4}$$

We can now state the optimal CB commitment problem as a variation on KP optimization problem as follows

$$\text{Maximize} \qquad \sum_i^N p_i x_i(k) \tag{5}$$

$$\text{Subject to} \qquad \sum_i^N w_i x_i(k) \leq Q_s(k) \tag{6}$$

where w_i and p_i are as described above. Since we implicitly incorporate the information on under-coverage of the shortage curve and weight through profit and we can simplify the problem for algorithm development as follows

$$\text{Minimize} \qquad A - \sum_i^N p_i(w_i, Q_s, k) x_i(k) \tag{7}$$

where $A = \sum_{k=1}^K Q_s(k)$ is the area under the shortage curve and profit is a function of weight and shortage curve over time as described above.

The over-coverage algorithm is the same as under-coverage algorithm except the calculation of profits associated with each CB. The profit is still the multiplication of the length and weight. But the length is calculated differently. The optimization problem is formulated to minimize the difference between the area covered by CBs and the shortage curve instead of the difference between the shortage curve and CBs.

Both under- and over-coverage algorithms require calculating the duration during which CBs are switched on. For example, the profit of switching a CB on depends on how long it stays on. However, CB lifetime depends on the number of switches. Having CBs switched 6-7 times a day will result in average lifetime of 30 years. Thus if we double the number of switches we halve the lifetime of the CB. That is why while calculating the duration we need to make sure that number of switches is minimized. In order to calculate the duration l_i given relative shortage function Q over 24 hours (sampled as low as every 5 minutes) and weight T we first use a threshold function F to identify periods where Q is greater than T. Mathematically,

$$F(Q,T) = \begin{cases} 1, & Q \geq T \\ 0, & Q < T \end{cases} \tag{8}$$

After we calculate F, we calculate the indices (or times) for positive and negative slopes of F. Each positive (negative) slope corresponds to a switch on (off) point. Each positive slope should be followed by a negative slope. If the index of the first positive slope is greater than the index of the first negative slope then there is a missing positive slope at time t=0. If the index of the last negative slope is smaller than the index of the last positive slope ten there is a missing negative slope at the last time point. Once the positive and negative slopes are matched, we need to analyze the duration of switch-ons and inter-switch-ons. If the switch-on duration is too small then we need to eliminate the first switch-on to minimize number of switches. If the inter-switch-on is too small then we need to combine the two switches.

In developing an algorithm to solve this problem, we wish to provide an intuitive and computationally efficient solution. There are several ways of solving KPs. KPs are combinatorial algorithms and solutions to combinatorial algorithms include but not limited to greedy algorithm, dynamic programming, and branch-and-bound [12], [13]. Dynamic programming performs an enumeration of all the feasible points but it differs from the branch-and-bound (BB) algorithm in the sense that it works backwards from the last decisions to the earlier ones. Suppose that we made n decisions to solve a combinatorial optimization problem. According to dynamic programming approach the last k¡ndecisions must be optimal as well. That is the completion of an optimal sequence of the decisions must be optimal [12].

IV. OPTIMAL TAP RATIO ADJUSTMENT

The voltage profiles can greatly influence the system losses. Our goal in this section is to describe an algorithm that changes the tap settings of Load Tap Changers (LTCs) and/or Voltage Regulators (VRs) to improve the voltage profile. There are two steps in improving the voltage profile. First is to flatten the average voltage profile. The average voltage profile for a given zone and phase is the arithmetic mean of the voltages from load buses for the given zone and phase.

The average voltage profile is quantized to enable computation of the relative tap changes to flatten the voltage profile regardless of the voltage mean of the average voltage profile. In this stage we use the algorithm to calculate the duration of CB switch on time to calculate the tap ratio changes staring and end points over time as well. The switch-on and inter-switch-on sensitivities may result in increased number of tap changes yet will result in more flat voltage profile over time. Thus there is a tradeoff between the sensitivity levels and number of tap changes required to flatten the voltage profile.

The second important stage is the part where the mean of the average voltage is calculated to bias the relative tap ratios to calculate their absolute values such that when with the new tap ratios result in the desired mean voltage level. The desired mean voltage level is calculated with respect to desired CVR factor. Note that the tap ratio adjustment algorithm runs for each zone and for each phase. Finally, the output of the tap ratio adjustment algorithm is the tap ratios for each LTC or VR and for each phase over the course of the 24 hour scheduling period.

V. CASE STUDY: IEEE123 RADIAL FEEDER

In this section, we describe the Matlab/OpenDSS simulation built to test the performance of various Volt/VAr optimization algorithms. The flow chart of the Matlab/OpenDSS simulation is shown in Fig. 2. OpenDSS is a publicly available power flow software. OpenDSS is designed to simulate utility distribution systems in arbitrary detail for most types of analysis related to distribution planning [14]. OpenDSS takes scripts as input and is easy to integrate into Matlab environment. Currently, we have scripts for modified IEEE 13 and 123 circuits [15] and are working on building an 8000 bus circuit.

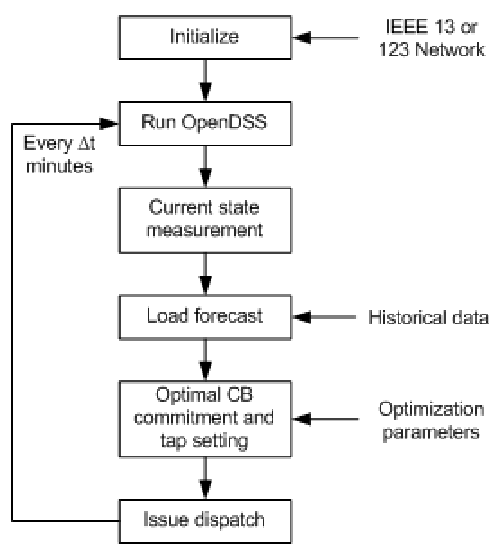

Fig. 2. The flow chart of the Matlab/OpenDSS simulation for testing various Volt/VAr optimization algorithms.

We obtain load forecast for a given time period (24 hours or longer) based on historical data using least squares estimation. Since the optimal CB commitment and tap settings are performed based on the load forecast, more accurate the forecast better the performance of these Volt/VAr optimization algorithms. Even though the simulation is every Δt minutes, it may be more adequate to run the load forecast algorithm every 3-4 hours. Similarly, for the CB commitment and VR/LTC tap setting algorithms, it may be more practical to run the algorithms every 15-30 minutes.

We run the Matlab/OpenDSS simulation on IEEE123 with additional CBs and phases for existing VRs and LTC to obtain average voltage profiles without the CBs and taps at zero setting and with the optimal CB commitment and tap settings. The average voltage profile drops below 0.95 p.u. while the maximum average voltage is 0.99 p.u. without any control. The end-of-line voltages varies between 0.93 p.u. and 1.01 p.u. As a result of the optimal CB commitment and VR/LTC tap setting algorithm, the average voltage is flattened and end-of-line voltages stay within 0.95 and 1 p.u. as shown with dashed lines in Fig. 3 The power factor at the substation is also improved. The power factor without the optimal CB commitment is around 0.95. The power factor with the optimal CB commitment is within 0.98 and 1 averaging around 0.99 over 24 hours.

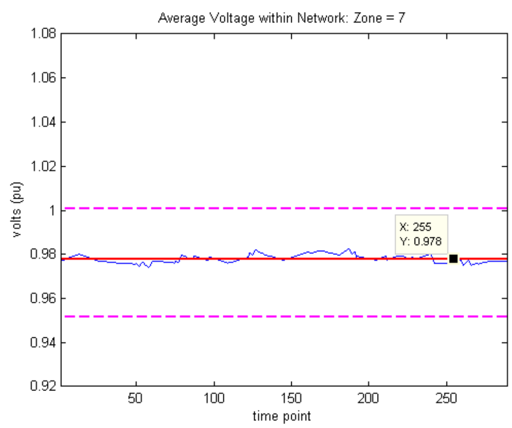

Fig. 3. The average voltage profile based on perfect knowledge of the network with optimal CB commitment and tap settings.

VI. CONCLUSION

We developed a new method to calculate optimal CB commitment and tap ratio adjustments over the course of a day. The algorithms are unique in the sense that they consider an optimal daily commitment instead of running an instance of the algorithm for every load sample time. This we believe results in computational savings, eventually, resulting in lower run time. The runtime of the optimal CB algorithm is dependent on the number of CBs, not the size of the network, and the number of zones. The optimal CB commitment and tap ratio adjustment algorithms can be run separately as long as the CBs are supplied to the tap ratio adjustment algorithm properly. The tap ratio algorithm can include an additional step to guarantee the tap ratio settings result in the desired voltage after the bias to the taps are calculated. This can be achieved by running a second iteration of the tap adjustment algorithm with a new desired voltage level updated based on the results from the first iteration. Overall, the developed algorithms achieve the objectives of improving the power factor and reducing system losses by switching VAr sources and adjusting tap ratios accordingly while keeping the number of switches and number of tap ratio changes well within their daily limits, thereby, maximizing the lifetime of these control devices. These algorithms support the increased penetration of renewable power sources since they can place the average voltage of the portion of the network containing the intermittent renewable at a level such that the voltage fluctuations do not exceed the ANSI 84.1 limits; thereby, mitigating the impacts to the cap bank and voltage regulator control devices.

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