2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

ACC 2011 Tutorial Session: An Introduction to Option Trading from a Control Perspective

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Abstract— **The purpose of this tutorial session is to explain how control-theoretic tools and associated mathematical concepts can be used in option trading. No previous knowledge of options will be assumed. After explaining the theory and mechanics of options and introducing the requisite mathematical models, the speakers will present a number of examples to demonstrate application of various trading algorithms, option hedging techniques and the use of both technical and fundamental analysis. The session will also include discussion of new and exciting research problems for the control field. One main theme of this tutorial session is that trading concepts can be explained in the context of a basic feedback loop with the control corresponding to modulation of the amount invested as a function of time.**

I. Intended Audience and Perspective

The target audience for this session is members of the control community that are seeking an easy-to-digest introduction to option trading from a systems-theoretic point of view. In this context, our goal is to bring the attendee "up to speed" and then include discussion of new research directions having both theoretical and applied components. Modelling of markets will be described under the assumption that the audience is uninitiated in option theory. Accordingly, considerable time will be dedicated to tutorial material and the differences between the feedback control approach and existing literature in the financial journals. The session will also include a review of basic background terminology associated with stock trading: margin, short selling, bid-ask spreads, liquidity, volatility, technical indicators, to name a few.

Integrated into the exposition will be the instructors' personal perspectives based on many years of trading and their initial work in this new line of research. Unlike classical approaches in finance, the approach espoused in this session does not rely on any type of stochastic model for the stock price $p(t)$; e.g., a geometric Brownian motion model point is the starting point in much of the financial literature. Instead, we view $p(t)$ as an uncertain external input against which we seek to achieve robust performance via modulation of the amount invested $I(t)$ in stock or options.

II. Trading via Feedback Control Methods

In recent years, there has been a surge of interest in the application of classical control methods to stock trading and finance; for example, see [1]-[19]. To differentiate the viewpoint in this tutorial with existing literature in the control field, for example, see [20] where predictive stochastic differential equation models are the central ingredient, the emphasis here is on the practical application of feedback from a systems perspective. In line with this, the control approach makes heavy use of familiar systems tools such as linear feedback, LQR methods, and convex optimization. A second salient feature of this research direction is the way the stock price $p(t)$ is handled. In much of their recent work, the session organizers have been evolving to the point of view that $p(t)$ is to be treated as an external input with no predictive model for its evolution. Thus, feedback induced robustness properties play the central role in determining investment strategies, not questionable predictive models.

Consistent with the instructors' perspective indicated above, a main theme of this tutorial session is that trading concepts, involving both stocks and options, can be explained in the context of a basic feedback loop with associated state equations. Figure 1 shows a block diagram of the basic information flow and feedback loop corresponding to either stock or options trading. In this context, information such as price $p(t)$ and volume $v(t)$ is transferred from the broker to the trader. In turn, the trader uses this information to determine an investment level $I(t)$, the control signal, which is then fed back to the broker who executes the corresponding transactions based on this time-varying investment level. In summary, the control system perspective in this tutorial emphasizes the feedback aspects of trading and the role it can play in providing robust performance.

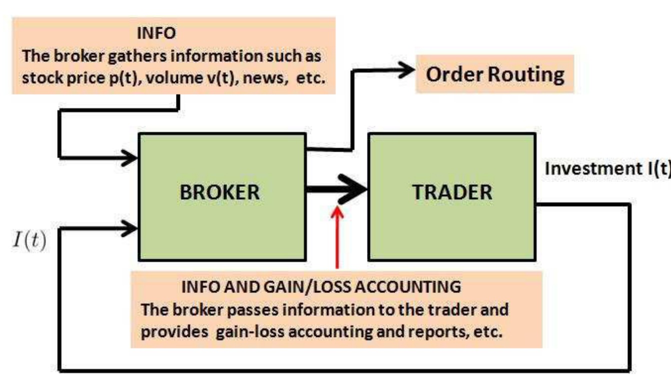

Fig. 1. Feedback Control Algorithm Strategy Resides in Trader Block

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III. Session Outline

The talks in this session will introduce the attendee to stock and option trading, explain classical option pricing theory in control-friendly terms, and introduce exciting new research directions for the control community. Those seeking further reading on classical option pricing theory may wish to refer to well-known textbooks by Luenberger [23] and Hull [24].

Talk 1: *Stock Market Basics Underlying Option Theory*

Key words include long and short positions, brokerage costs, bid-ask spreads, market orders, limit orders, stop orders and variants thereof, margin, performance metrics, benchmarks, capital preservation and draw-down and feedback control formulation and dynamics. The highlight of this section is the formulation of state equation dynamics which can be used to analyze the evolution of profits and losses from trading. For example, using the notation above, with $I_0 = I(0)$ being the initial investment, using a linear feedback $I = I_0 + Kg$, under idealized market conditions involving smooth price variations, as seen in [1] and [2], the state equations can be solved in closed form with the resulting trading gains given by

$$g(t) = \frac{I_0}{K}\left[\left(\frac{p(t)}{p(0)}\right)^K - 1\right].$$

Talk 2: *Option Basics*

Key words include put and call options, central ideas underlying option pricing, option lingo such as time value and intrinsic value and the mechanics of trading options. A highlight of this section is the presentation of examples to dispel the following erroneous belief which is held by many: *Option traders are much greater risk-takers than their stock trading counterparts*. The truth of the matter is that it is often the case that many option traders are actually more risk-averse than stock traders. For example, in a market with low volatility, the purchase of put options can be a cheap way to create portfolio insurance. A second possibility is that a trader wants to trade a high-flying volatile stock with limited downside risk. In many cases, it turns out this goal can be accomplished via the purchase of deeply-in-the-money call options.

Talk 3: *Black-Scholes Option Pricing*

Key words include option pricing argument, risk-neutral pricing, effect of model parameters, effects of interest rate and volatility and the Greeks. The highlight of this section is the celebrated Black-Scholes equation [21], [22] which determines the fair market value of an option $f(p, t)$ as a function of the stock price p, the elapsed time t and the risk-free rate of return r. This equation is given by

$$\frac{\partial f}{\partial t} + rp\frac{\partial f}{\partial p} + \frac{1}{2}\sigma^2 p^2 \frac{\partial^2 f}{\partial p^2} = rf$$

and has boundary conditions which depend on the nature of the option being priced. For example, for the case of a European call option with the strike price X, we enforce the condition $f(p, T) = \max\{p - X, 0\}$ where T is the terminal time. As explained in the tutorial, remarkably, this equation turns out to admit a closed-form solution.

Talk 4: *Options as Building Blocks*

Key words include combining options of different types, combining stock and options, classical trades such as spreads, covered calls, covered puts, collars and butterflies and robustness considerations. A highlight of this talk is the construction of the so-called profit-loss diagram. This diagram is a plot of profit or loss as a function of the underlying stock price at option expiration. For example, when Apple stock closed at \$335.06 on April 8, 2010, the May 345-strike and 360-strike calls were trading at \$9.35 and \$4.45 respectively. At that time, a trader purchasing one contract, representing 100 shares, of the 345-calls and selling one contract of the 360-strike calls, would be facing the profit-loss diagram shown in Figure 3.

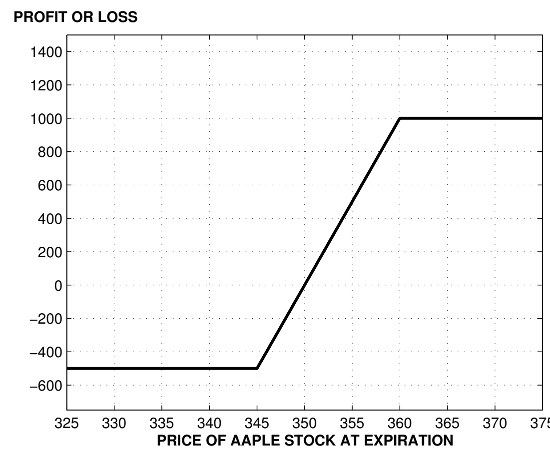

Fig. 2. Profit-Loss Diagram for May 2011 Bull Call Spread on Apple

Talk 5: *Case Study and Introduction to Hedging*

This talk will consist of a case study involving the so-called "miracle" obesity drug companies Vivus, Arena Pharmaceutical and Orexigen. During 2010-2011, there has been a "race" of sorts among these companies for FDA (Food and Drug Administration) approval. Accordingly, the stock prices of these companies were extremely volatile with associated option prices correspondingly high. One of the highlights of this talk is the demonstration of various option trades which were possible during that time period. We take the point of view of the trader seeking to play the FDA control process with limited downside risk while preserving the possibility of "large" returns. This relates to one of the themes in Talk 2. That is, option traders playing volatile high-flyers are not necessarily greater risk-takers than their stock trading counterparts.

The highlight of this section is the construction of a somewhat complex trade called an "iron condor." We consider the point of view of traders during June and July of 2010, the days preceding the FDA's deliberation on the Vivus drug called Qnexa. With the "theory of the trade" being that the FDA will delay making a definitive decision, perhaps requesting additional data, one possibility is to take a bet that during the month of July, the Vivus stock price will stay confined to a range which is within thirty percent of its July 6 price which is \$10.05. To indicate how an appropriate trade can be constructed while greatly limiting downside exposure, we imagine a trader willing to lose no more than \$10K should a worst-case scenario occur; e.g., a strongly negative FDA decision could result in a "crash" of the stock price. To this end, we apply the concepts presented in Talk 4 to obtain the profit-loss diagram for an iron condor involving four option trades each consisting of 128 contracts; see Figure 2.

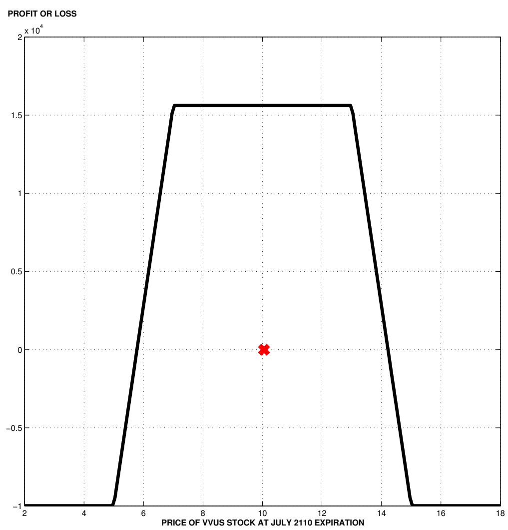

Fig. 3. Profit-Loss Diagram for July 2010 Vivus Iron Condor

Talk 6: *Connections to Control*

Key words include implied volatility and volatility trading, the delta hedge, hedging over time, control based hedging methods. The highlight of this talk is a case example in which a dynamic hedging feedback strategy is used to exploit a statistical arbitrage opportunity on DJX index options on the Dow Jones Industrial Average following the 2008 market crash. The arbitrage strategy involved selling over-priced out-of-the-money puts and dynamically delta-hedging in order to lock in a profit, regardless of the directional movement of the Dow.

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