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Improving Transient Stability of Multi–Machine Power Systems: Synchronization via Immersion of a Pendular System

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Abstract— **In this paper the problem of designing excitation controllers to improve the transient stability of multimachine power systems is addressed adopting two new perspectives. First, instead of the standard formulation of stabilization of an equilibrium point, we aim here at the more realistic objective of keeping the difference between the generators rotor angles bounded and their speeds equal—which is called synchronization in the power literature—and translates into a problem of stabilization of a set. Second, we adopt the classical viewpoint of power systems as a set of coupled nonlinear pendula, and express our control objective as ensuring that some suitable defined pendula dynamics are (asymptotically) immersed into the power system dynamics. Our main contribution is the *explicit computation* of a control law for the 2–machine system that achieves *global synchronization*. The same procedure is applicable to the n–machine case, for which the existence of a locally stabilizing solution is established.**

I. INTRODUCTION

Oscillations in power systems occur due to sudden faults and transients. Damping these oscillations, generically known as "transient stability improvement", is a critical issue that is witnessing increased interest in the new deregulated market [9]. Several control actions are available to accomplish this task, in this paper we consider the classical field excitation of the generators, see [10] where a switched series capacitor is used instead.

Transient stabilization has been traditionally formulated in terms of enlarging the domain of attraction of an operating equilibrium point [2], approach that has been widely adopted by the control community, see *e.g.*, [5], [11] and references therein. As early as 1974, [13], it was recognized that the requirement of convergence to an equilibrium point is too stringent for engineering applications—see also [8] and the recent discussion in [6]. In particular, it is argued in [13] that convergence of the rotor angles to a fixed point is not required. Indeed, due to the action of secondary control loops operating at a slower time scale, it suffices to keep their difference bounded during the transient period, in this case it is said that the power system *synchronizes* [1]. Adopting this viewpoint the control objective is now stabilization of a set, which is the problem addressed in this paper.

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Another novelty of this paper is that the controller is designed with the objective of making the power system asymptotically behave like some suitably defined nonlinear coupled pendula. This approach has several advantages.

1) It is consistent with a widely adopted viewpoint of the swing equations as a set of nonlinear coupled pendula, which should oscillate in a synchronized manner [1], [2]. See [12] for a recent interesting analysis of the instability mechanisms.
2) It provides a physical interpretation of the control action. In particular, physical intuition can be used to define the desired pendula dynamics, whose potential energy and dissipation functions are free to the designer.
3) The controller design task can be neatly formulated using the recently introduced immersion and invariance (I&I) methodology [3], [4], where (lower order) target dynamics that capture the desired behavior of the controlled system are first defined and then the control is designed to ensure that the target dynamics are (asymptotically) immersed into the system dynamics.

The main difficulty for the successful application of I&I is the need to solve a partial differential equation. In this paper this difficulty is obviated with a suitable selection of the target dynamics and assuming that all generators are actuated and have the same relative damping. The I&I strategy is applied to asymptotically stabilize the equilibrium of a single machine infinite bus system using a controllable series capacitor in [10] and generator excitation in [7].

This paper is organized as follows. A brief introduction to the I&I control synthesis is given in Section II. The model and the control problem are presented in Section III. Using the I&I strategy, we compute in Section IV a globally synchronizing control law for the 2–machine power system. The extension to the n–machine case, which yields a local result, is done in Section V. Section VI includes the application of the proposed technique to a classical example. Finally, we conclude with some remarks in Section VII.

II. IMMERSION AND INVARIANCE

The method of I&I for observer design, stabilization and adaptive control of nonlinear systems has been proposed in [3], and has been recently summarized in [4]. The following result from [3] will be instrumental for our developents.

Theorem 1 Consider the system

$$\dot{x} = f(x) + g(x)u, \tag{1}$$

with state $x \in \mathbb{R}^n$ and control $u \in \mathbb{R}^m$, and an assignable equilibrium point $x^\star \in \mathbb{R}^n$ to be stabilized. Let $s < n$, and assume we can find mappings

$$\alpha : \mathbb{R}^s \to \mathbb{R}^s, \quad \pi : \mathbb{R}^s \to \mathbb{R}^n, \quad c : \mathbb{R}^n \to \mathbb{R}^m,$$
$$\phi : \mathbb{R}^n \to \mathbb{R}^{n-s}, \quad \psi : \mathbb{R}^{n\times(n-s)} \to \mathbb{R}^m,$$

such that the following hold.

(H1) *(Target system)* The system

$$\dot{\xi} = \alpha(\xi), \tag{2}$$

with state $\xi \in \mathbb{R}^s$, has an asymptotically stable equilibrium at $\xi^\star \in \mathbb{R}^s$ and $x^\star = \pi(\xi^\star)$.

(H2) *(Immersion condition)* For all $\xi \in \mathbb{R}^s$

$$f(\pi(\xi)) + g(\pi(\xi))c(\pi(\xi)) = \frac{\partial \pi(\xi)}{\partial \xi}\alpha(\xi). \tag{3}$$

(H3) *(Implicit manifold)* The set identity

$$\begin{aligned} \mathcal{M} &:= \{x \in \mathbb{R}^n \mid x = \pi(\xi) \text{ for some } \xi \in \mathbb{R}^s\} \\ &= \{x \in \mathbb{R}^n \mid \phi(x) = 0\} \end{aligned} \tag{4}$$

holds.

(H4) *(Manifold attractivity and trajectory boundedness)* All trajectories of the system

$$\begin{aligned} \dot{z} &= \frac{\partial \phi(x)}{\partial x}\left[f(x) + g(x)\psi(x,z)\right] \\ \dot{x} &= f(x) + g(x)\psi(x,z), \end{aligned} \tag{5}$$

are bounded and satisfy

$$\lim_{t\to\infty} z(t) = 0. \tag{6}$$

Then, $x^\star$ is a globally asymptotically stable equilibrium of the closed loop system

$$\dot{x} = f(x) + g(x)\psi(x, \phi(x)).$$

Theorem 1 lends itself to the following interpretation. Given the system (1) and the target dynamical system (2) find, if possible, a manifold $\mathcal{M}$, which can be rendered invariant and attractive, and such that the restriction of the closed–loop system to $\mathcal{M}$ is described by $\dot{\xi} = \alpha(\xi)$. The control law $u = c(\pi(\xi))$ render the manifold invariant. A measure of the distance of the system trajectories to the manifold $\mathcal{M}$ is given by z, called off–the–manifold coordinate. Our aim is to design a control law $u = \psi(x, z)$ that drives to zero the coordinate z and keeps the system trajectories bounded.

III. MODEL AND PROBLEM FORMULATION

We consider a large–scale power system that consists of n generators interconnected through a *lossy* transmission network. The dynamics of the j–th machine using reduced network model with excitation are represented by the classical three–dimensional flux decay model[1]

$$\begin{aligned} \dot{\delta}_j &= \omega_j \\ M_j\dot{\omega}_j &= P_{m_j} - D_j\omega_j - G_{jj}E_j^2 - \\ &\quad -E_j \sum_{k=1,k\neq j}^{n} E_k Y_{jk} \sin(\delta_j - \delta_k + \alpha_{jk}) \\ \dot{E}_j &= -a_j E_j + b_j \sum_{k=1,k\neq j}^{n} E_k \cos(\delta_j - \delta_k + \alpha_{jk}) + \\ &\quad + \tfrac{1}{\tau_j}\left(E_{F_j}^\star + v_j\right), \end{aligned} \tag{7}$$

where we have defined

$$Y_{jk} := \sqrt{G_{jk}^2 + B_{jk}^2}, \;\; \alpha_{jk} := \arctan\frac{G_{jk}}{B_{jk}}$$
$$a_j := \frac{1}{\tau_j}[1 - B_{jj}(x_{dj} - x'_{dj})], \; b_j := \frac{Y_{jk}}{\tau_j}(x_{dj} - x'_{dj}).$$

We observe that if the network is lossless—that is if we neglect the transfer conductances G_{ij}—then $\alpha_{ij} = 0$. The field voltage E_{F_j} is split in two terms, $E_{F_j}^\star + v_j$. The first is constant and fixes the equilibrium value, while the second one is the *control action*.[2]

The standard formulation of transient stability presumes that the system (7) with $v = 0$ has a stable equilibrium $(\delta_j, \omega_j, E_j) = (\delta_j^\star, 0, E_j^\star)$, and the purpose of the control is to enlarge its domain of attraction. As explained in the introduction, we follow the formulation of [13] and [8], where a trajectory of the (uncontrolled or closed–loop) system is said to be transiently stable if the initial conditions belong to an open domain of attraction of the set

$$\mathcal{S}_n := \{\delta_j - \delta_k = c_{jk},\;\; \omega_j = \omega_k,\;\; E_j = E_j^\star,\; j \neq k\} \subset \mathbb{R}^{3n}, \tag{8}$$

where $E_j^\star > 0$ is some desired equilibrium value for E_j and c_{jk} are some non–negative constants that measure the admissible (steady–state) deviations of the rotor angles. Obviously, the assumption of existence of an open–loop stable equilibrium, which is consistent with the system operation, is retained. We recast then the transient stabilization problem as asymptotic stabilization of the set $\mathcal{S}_n$.

Definition 1 [1] A control v is said to (globally) synchronize the power system (7) if the set $\mathcal{S}_n$ is made (globally) attractive.

To solve this problem we apply the I&I strategy. We consider first, in the next subsection, the case of 2–machine power systems and then we generalize the study to the

[1]This corresponds to eq. (9) of [5], see also [11], to which we refer the reader for the definition of the symbols and additional details of the model.

[2]Throughout the paper the subindices j, k range in the set $\{1, \dots, n\}$, clarification that is omitted for brevity. The symbol without subindex denotes a column vector with all the elements piled up, *e.g.*, $v := \mathrm{col}(v_1, \dots, v_m)$

n–machine case.

Remark 1 In [13] it is shown that in the case of nonzero damping, the definition above is equivalent to the following. A trajectory is transiently stable if its initial conditions belong to an open domain of attraction of the set

$$\{\delta_j - \delta_k = c_{jk}, \quad \omega_j = 0, \;\; E_j = E_j^\star, \; j \neq k\} \subset \mathbb{R}^{3n}.$$

IV. I&I CONTROL FOR TWO–MACHINE SYSTEM

A. System dynamics

The dynamics of the two–machine system are obtained using equations (7), which yields the 6–th order model

$$\begin{array}{rcl}
\dot\delta_1 &=& \omega_1 \\
\dot\omega_1 &=& -D_1\omega_1 + P_1 - G_{11}E_1^2 - \\
&& -Y_{12}E_1E_2\sin(\delta_1 - \delta_2 + \alpha) \\
\dot E_1 &=& -a_1E_1 + b_1E_2\cos(\delta_1 - \delta_2 + \alpha) + \frac{1}{\tau_1}\left(E_{F_1}^\star + v_1\right) \\
\dot\delta_2 &=& \omega_2 \\
\dot\omega_2 &=& -D_2\omega_2 + P_2 - G_{22}E_2^2 + \\
&& Y_{21}E_1E_2\sin(\delta_1 - \delta_2 - \alpha) \\
\dot E_2 &=& -a_2E_2 + b_2E_1\cos(\delta_2 - \delta_1 + \alpha) + \frac{1}{\tau_2}\left(E_{F_2}^\star + v_2\right),
\end{array} \tag{9}$$

where, with an obvious abuse of notation, we assigned

$$D_i \leftarrow \frac{D_i}{M_i}, P_i \leftarrow \frac{P_{m_i}}{M_i}, G_{ii} \leftarrow \frac{G_{ii}}{M_i}, \; i = 1, 2$$

as well as $Y_{12} \leftarrow \frac{Y_{12}}{M_1}, Y_{21} \leftarrow \frac{Y_{21}}{M_2}$, and defined $\alpha := \alpha_{12} = \alpha_{21}$. We will assume in the sequel that $P_1 \neq P_2$.

Now, since we are interested in the angle and velocity differences, we define the state variables

$$x_1 = \delta_1 - \delta_2, x_2 = \omega_1 - \omega_2, x_3 = E_1, x_4 = E_2.$$

At this point we make the critical assumption of *uniform relative damping*

$$\frac{D_i}{M_i} =: D, \tag{10}$$

for $i = 1, 2$, to get

$$\begin{array}{rcl}
\dot x_1 &=& x_2 \\
\dot x_2 &=& -Dx_2 + P_1 - P_2 - G_{11}x_3^2 + G_{22}x_4^2 - \\
&& -Y_{12}x_3x_4\sin(x_1 + \alpha) - Y_{21}x_3x_4\sin(x_1 - \alpha) \\
\dot x_3 &=& -a_1x_3 + b_1x_4\cos(x_1 + \alpha) + \frac{1}{\tau_1}\left(E_{F_1}^\star + v_1\right) \\
\dot x_4 &=& -a_2x_4 + b_2x_3\cos(-x_1 + \alpha) + \frac{1}{\tau_2}\left(E_{F_2}^\star + v_2\right).
\end{array} \tag{11}$$

The control objective is then to asymptotically stabilize, with a well–defined domain of attraction, the equilibrium

$$x^\star = (x_1^\star, 0, x_3^\star, x_4^\star) := (c_{12}, 0, E_1^\star, E_2^\star). \tag{12}$$

Remark 2 Notice that, even though the dynamics of x_3 and x_4 can be arbitrarily assigned with v_1, v_2, these signals enter as products on the second state equation of (11). Consequently, standard techniques (*e.g.*, backstepping, control Lyapunov functions) cannot be applied to stabilize this system.

B. Target dynamics

To design a stabilizing controller for the 2–machine system (11) we verify the conditions of Theorem 1. Towards this end, we first select as target dynamics a simple damped pendulum system, that is

$$\begin{array}{rcl}
\dot\xi_1 &=& \xi_2, \\
\dot\xi_2 &=& -R(\xi)\xi_2 - V'(\xi_1).
\end{array} \tag{13}$$

The pendulum has an asymptotically stable equilibrium $\xi^\star = (\xi_1^\star, 0)$, which is ensured as follows.

(i) The potential energy function $V(\xi_1)$ satisfies

$$\left\{ \begin{array}{rcl} V'(\xi_1^\star) &=& 0, \\ V''(\xi_1) &>& 0. \end{array} \right. \tag{14}$$

(ii) The damping function verifies $R(\xi) > 0$.

For simplicity, we select

$$V(\xi_1) = -\beta\cos(\xi_1 - \xi_1^\star),$$

for some $\beta > 0$.

C. Immersion condition

Since the objective of I&I is to render the manifold $\mathcal{M}$, defined in (4), asymptotically attractive it is reasonable to choose the mapping $\pi : \mathbb{S}\times\mathbb{R} \to \mathbb{S}\times\mathbb{R}\times\mathbb{R}\times\mathbb{R}$ as follows

$$\pi(\xi_1, \xi_2) := \begin{bmatrix} \xi_1 \\ \xi_2 \\ \pi_3(\xi) \\ \pi_4(\xi) \end{bmatrix}, \tag{15}$$

where $\pi_3(\xi), \pi_4(\xi)$ are functions to be defined. Moreover, the condition $x^\star = \pi(\xi^\star)$ of Theorem 1, imposes the constraints

$$\xi_1^\star = x_1^\star, \; \pi_3(\xi^\star) = x_3^\star, \; \pi_4(\xi^\star) = x_4^\star. \tag{16}$$

Some simple calculations show that, selecting $R = D$, the immersion condition (3), reduces to the simple quadratic algebraic equation[3]

$$\begin{bmatrix} \pi_3(\xi) & \pi_4(\xi) \end{bmatrix} A(\xi_1) \begin{bmatrix} \pi_3(\xi) \\ \pi_4(\xi) \end{bmatrix} = s(\xi_1), \tag{17}$$

where

$$\begin{array}{rcl}
A(\xi_1) &:=& \begin{bmatrix} G_{11} & a_{12}(\xi_1) \\ a_{12}(\xi_1) & -G_{22} \end{bmatrix} \\
s(\xi_1) &:=& P_1 - P_2 + \beta\sin(\xi_1 - x_1^\star) \\
a_{12}(\xi_1) &:=& \dfrac{Y_{12}}{2}\sin(\xi_1 + \alpha) + \dfrac{Y_{12}}{2}\sin(\xi_1 - \alpha).
\end{array} \tag{18}$$

The solutions of (17) can be parameterized using Lemma 1 in the Appendix, whose proof is inspired by the standard parametrization of hyperbola. The lemma imposes the condition $s(\xi_1) > 0$. If $P_1 > P_2$ the condition can be satisfied with a sufficiently small β. On the other hand, if $P_2 > P_1$ we can make the independent term negative with a small β. This case can also be treated with a slight modification to the lemma. Therefore, without loss of generality we will assume

[3] The third and fourth equations in (3) are satisfied with a suitable definition of the control v.

in the sequel that $P_1 > P_2$ and $s(\xi_1) > 0$. In this case, direct application of Lemma 1 proves that all solutions of (17) can be parameterized as

$$\begin{bmatrix} \pi_3(\xi) \\ \pi_4(\xi) \end{bmatrix} = \sqrt{s} \begin{bmatrix} \frac{1}{\sqrt{G_{11}}}\cosh(\rho) - \frac{a_{12}}{G_{11}\sqrt{\frac{a_{12}^2}{G_{11}}+G_{22}}}\sinh(\rho) \\ \frac{1}{\sqrt{\frac{a_{12}^2}{G_{11}}+G_{22}}}\sinh(\rho) \end{bmatrix},$$

with $\rho(\xi)$ a free function.

To simplify our derivations it is convenient to make $\rho(\xi)$ function only of ξ_1—in this case π_3 and π_4 are also functions of ξ_1 only—and such that $\pi_4(\xi)$ is a constant that, in view of (16), has to be $\pi_4 = x_4^\star$. That is, we select

$$\rho(\xi_1) = \sinh^{-1}\left(x_4^\star\sqrt{\frac{a_{12}^2(\xi_1)+G_{11}G_{22}}{s(\xi_1)G_{11}}}\right),$$

where we recall that $\sinh$ is a globally invertible function. This yields, after some simple calculations, the expression

$$\pi_3(\xi_1) = \frac{1}{G_{11}}\left(\sqrt{sG_{11}+(x_4^\star)^2(a_{12}^2+G_{11}G_{22})} - a_{12}x_4^\star\right), \tag{19}$$

where we have used the identity

$$\cosh(\sinh^{-1}(\theta)) = \sqrt{1+\theta^2}.$$

Remark 3 It is important to underscore that the equation (17), evaluated at the equilibrium $\xi^\star$, is precisely the equilibrium equation of $\dot{x}_2$. Since $x_2 = \omega_1 - \omega_2$, and it is assumed that the open–loop system (9) has an equilibrium, we conclude that (17) has, at least, a *local solution*. Although it is shown above that the equation has a global solution, the remark will be important for the n–machine extension of Section V.

D. Implicit manifold condition

Once the mapping $\pi(\xi)$ has been defined, we proceed to verify condition (4). From (15) and $\pi_4 = x_4^\star$ it is clear that the mapping $\phi(x)$ is defined as

$$\phi(x) \quad = \quad \begin{bmatrix} x_3 - \pi_3(x_1) \\ x_4 - x_4^\star \end{bmatrix} \tag{20}$$

E. Manifold attractivity and trajectory boundedness

It only remains to verify condition (H4). Towards this end, let $z := \phi(x)$ denote the off–the–manifold coordinate. Then, we have that

$$\begin{aligned} \dot{z}_1 &= \dot{x}_3 - \dot{\pi}_3(x_1) \\ &= -a_1x_3 + b_1x_4\cos(x_1+\alpha) + \\ &\quad + \frac{1}{\tau_1}(E_{F_1}^\star + \psi_1(x,z)) - \pi_3'(x_1)x_2, \end{aligned}$$

while

$$\begin{aligned} \dot{z}_2 &= \dot{x}_4 \\ &= -a_2x_4 + b_2x_3\cos(-x_1+\alpha) + \frac{1}{\tau_2}(E_{F_2}^\star + \psi_2(x,z)) \end{aligned}$$

where we have substituted $\dot{x}_3$, $\dot{x}_4$, from (11), and we recall that $\psi_1(x,z)$ and $\psi_2(x,z)$ are the actual controllers that we apply. Selecting the globally defined functions

$$\begin{aligned} \frac{\psi_1(x,z)}{\tau_1} &= a_1x_3 - b_1x_4\cos(x_1+\alpha) - \frac{E_{F_1}^\star}{\tau_1} + \pi_3'(x_1)x_2 - \gamma z_1 \\ \frac{\psi_2(x,z)}{\tau_2} &= a_2x_4 - b_2x_3\cos(-x_1+\alpha) - \frac{E_{F_2}^\star}{\tau_2} - \gamma z_2, \end{aligned} \tag{21}$$

with $\gamma > 0$, yields the exponentially stable system $\dot{z} = -\gamma z$.

It only remains to prove that the state of the system (11) in closed–loop with $v = \psi(x,z)$ is bounded. First, from (18), (19) we see that $\pi_3(x_1)$ is bounded. Since z_1 is, clearly, also bounded we conclude that x_3 is bounded. The same argument, using z_2, establishes that x_4 is bounded. Now, $x_1 \in \mathbb{S}$, hence, is bounded. Finally, from the second equation of (11) we see that x_2 is the output of the stable filter $\frac{1}{s+D}$ with a bounded input, consequently it is also bounded.

F. Global synchronization of the 2–machine power system

The derivations above established the following result.

Proposition 1 Consider the 2–machine system (9) verifying (10) and $P_1 > P_2$.[4] Fix the positive constants $c_{12}, E_1^\star, E_2^\star$. Let the control be given by $v = \psi(x, \phi(x))$, which is defined by (12), (18), (19), (20) and (21). The power system globally synchronizes, that is, the set

$$\mathcal{S}_2 = \{\delta_1 - \delta_2 = c_{12}, \omega_1 = \omega_2, E_1 = E_1^\star, E_2 = E_2^\star\}$$

is globally attractive.

Remark 4 Global attractivity is, of course, not an issue in practical applications. Actually, the coordinates E_j in the model (7) are (physically) restricted to be positive and the behavior of δ_j is of interest only in the set $|\delta_i - \delta_j| \leq \frac{\pi}{2}$.

V. I&I CONTROL FOR n–MACHINE SYSTEM

In this subsection we show that the I&I procedure applied in Subsection II can be directly extended to the n–machine case yielding a locally synchronizing controller. The only difference is that, in this case, instead of one algebraic equation in two unknowns, we end up with $(n-1)$ algebraic equations in n unknowns, for which no explicit solution has been found. However, in the light of Remark 3, we will prove the existence of a local solution.

Again, we consider the case of uniform relative damping, that is, (10) holds for all j. We are interested in the angular and velocity differences with respect to a reference machine that, without loss of generality, we select to be the n–th machine. After redefining the notation like in Section IV,

[4] As explained in Subsection IV-C the latter assumption is done without loss of generality. Uniform relative damping is, on the other hand, a critical nonphysically justified assumption. See the discussion in Section VII.

results in the $(3n-2)$ state space model

$$\begin{aligned}
\dot{\delta}_j - \dot{\delta}_n &= \omega_j - \omega_n, \\
\dot{\omega}_j - \dot{\omega}_n &= -D\left(\omega_j - \omega_n\right) + P_{m_j} - G_{jj}E_j^2 - \\
&\quad -E_j \sum_{k=1,k\neq j}^{n} E_k Y_{jk} \sin(\delta_j - \delta_k + \alpha_{jk}) - \\
&\quad -P_{m_n} + G_{nn}E_n^2 - \\
&\quad -E_n \sum_{k=1}^{n-1} E_k Y_{nk} \sin(\delta_n - \delta_k + \alpha_{nk}), \\
\dot{E}_j &= -a_j E_j + b_j \sum_{k=1,k\neq j}^{n} E_k \cos(\delta_j - \delta_k + \alpha_{jk}) + \\
&\quad + \tfrac{1}{\tau_j}\left(E^{\star}_{F_j} + v_j\right).
\end{aligned} \tag{22}$$

Mimicking the derivations of the previous section, choose the target system as $(n-1)$ coupled pendula with damping $R_i = D$ and potential energies $V_i(\xi)$ verifying

$$(c_{12}, 0, \ldots, c_{(n-1)n}, 0) = \text{arg min} \sum_{i=1}^{n-1} V_i(\xi).$$

Select the mapping $\pi(\xi)$ as

$$\pi(\xi) := \text{col}(\xi_1, \xi_2, \cdots, \xi_{2n-2}, \pi_{2n-1}(\xi), \cdots, \pi_{3n-2}(\xi)).$$

It is easy to see that the immersion condition (3) reduces to the algebraic equations

$$\begin{aligned}
& P_{m_i} - G_{ii}\pi^2_{i+2n-2} - P_{m_n} + G_{nn}\pi^2_{3n-2} \\
- \quad & \pi_{i+2n-2} \sum_{k=1,k\neq i}^{n} \pi_{k+2n-2} Y_{ik} \sin(\xi_i - \xi_k + \alpha_{ik}) \\
- \quad & \pi_{3n-2} \sum_{k=1}^{n-1} \pi_{k+2n-2} Y_{nk} \sin(-\xi_k + \alpha_{nk}) = -\frac{\partial V_i(\xi)}{\partial \xi_i},
\end{aligned} \tag{23}$$

with $i = 1, ..., n-1$. These are $(n-1)$ quadratic equations in the n unknowns $\pi_{2n-1}(\xi), \cdots, \pi_{3n-2}(\xi)$ for which, in view of Remark 3, a local solution is insured. Proceeding exactly as done in Section IV we can establish the following result.

Proposition 2 Consider the n–machine system (7) verifying (10). Fix the positive constants $c_{ij}, E^{\star}_j$. There exists a static state–feedback control v that locally synchronizes the power system, that is, that renders the set $\mathcal{S}_n$, defined in (8), locally attractive.

VI. A Benchmark Simulation Example

We consider here the classical 2-machine system considered in [2]. We analyze the response of (9) to a short circuit which consists of a zero-impedance three phase fault. The parameters of the model (9) are given in Table I, and the equilibrium point is $x^{\star} = (x_1^{\star}, x_2^{\star}, x_3^{\star}, x_4^{\star}) = (\delta_{12}^{\star}, \omega_{12}^{\star}, E_1^{\star}, E_2^{\star}) = (-0.15, 0, 1, 1.08)$. The fault is introduced at $t = 0.5$ sec and removed after a certain time (called the clearing time and denoted t_{cl}), after which the system is back to its pre-disturbance topology. During the fault the trajectories tend away from the equilibrium. The largest time interval "before instability" called the critical clearing time (t_{cr}), is determined via simulation. This system has a critical clearing time $t_{cr} = 0.1$ sec in open loop. With the proposed controller, and with a suitable selection of the parameters γ, and β, this time could be increased to 1 sec; a value that is far beyond the time scale of interest in this problem. The tuning parameter β decides the shape of the energy function for the closed-loop system, and γ decides the rate at which the closed-loop system trajectories come closer to the desired trajectories.

Parameter	Gen 1 (pu)	Gen 2 (pu)
a	17.71225	14.6054
b	11.8321	10.0303
D	0.2	0.2
P	52.2526	48.4902
G_{ii}	30.7858	19.0708
Y	54.6097	39.007
α	0.5226	0.5226
$E_F^{\star}$	5.8103	7.9279

TABLE I
PARAMETERS OF THE 2–MACHINE SYSTEM

Figures 1 and 2 depict the transient response of system (9) to a short-circuit with clearing time $t_{cl} = 0.8$ s, $\gamma = 140$, and $\beta = 4$ where the open loop system is unstable. As it can be seen, the controller is able to significantly improve the transient stability of the system and to enlarge its domain of attraction. Figure 3 shows the total energy function of the target system (13) with the potential energy (14) during the perturbation.

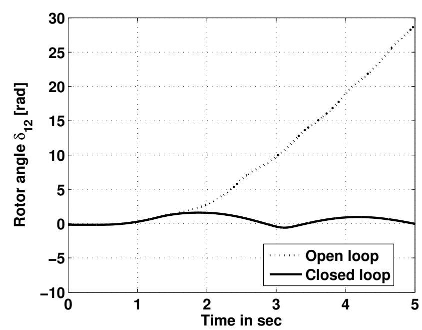

Fig. 1. Response of the 2-machine system (9) with the I&I control laws (21) : Dotted line (open loop response), solid line (closed-loop response with $\gamma = 140$, $\beta = 4$ and t_{cl}=0.8 sec).

VII. Concluding Remarks

Some preliminary results on synchronization of power systems have been reported. Besides the restrictive assumptions that all generators are actuated and the availability of full–state measurement, the main drawback of our result is the condition of uniform relative damping. This condition cannot be physically justified and is imposed to translate the problem of stabilization of the set $\mathcal{S}_n$ for the system (7) into a standard equilibrium stabilization problem of a reduced system. Research is under way to relax this assumption.

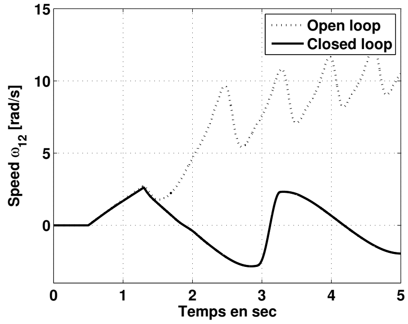

Fig. 2. Response of the 2-machine system (9) with the I&I control laws (21) : Dotted line (open loop response), solid line (closed-loop response with $\gamma = 140$, $\beta = 4$ and t_{cl}=0.8 sec).

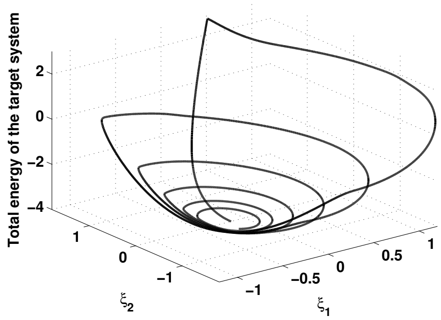

Fig. 3. Total energy of the pendulum target system (13) with the potential energy (14), and with $\gamma = 140$, $\beta = 4$ and t_{cl}=0.8 sec.

We are pursuing our research along three directions.

- The derivation of explicit controller expressions for the n–machine case. Some preliminary calculations using "dynamic solutions" of the algebraic equations are very encouraging.
- Study the effect of the choice of the target dynamics on the solvability and complexity of the algebraic equations and the definition of the control. Regarding the latter, we have presented here a "partially linearizing" scheme that cancels the potential energies of the target system and yields a linear decoupled dynamics to the off–the–manifold coordinate z. However, we believe that incorporating physical intuition a more clever option should be available.
- The study of the interaction of the network topology and the machines dynamics. In particular, it is of interest to identify generators (or transmission lines) where the addition of control actions would have a major impact in the transient stability improvement.

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APPENDIX

Lemma 1 Consider the quadratic form $y^\top A y = s$, with $y \in \mathbb{R}^2$,

$$A = \begin{bmatrix} a & b \\ b & -c \end{bmatrix},\ a, c, s > 0.$$

All solutions of the quadratic form can be parameterized as

$$y = \sqrt{s} \begin{bmatrix} \frac{1}{\sqrt{a}}\cosh(\rho) - \frac{b}{a\sqrt{\frac{b^2}{a}+c}}\sinh(\rho) \\ \frac{1}{\sqrt{\frac{b^2}{a}+c}}\sinh(\rho) \end{bmatrix},$$

with ρ a free constant.

Proof First, note that the congruence transformation $T^\top A T := \tilde{A}$ with

$$T = \begin{bmatrix} 1 & \frac{-b}{a} \\ 0 & 1 \end{bmatrix},$$

yields

$$\tilde{A} = \begin{bmatrix} a & 0 \\ 0 & -\left(\frac{b^2}{a} + c\right) \end{bmatrix}.$$

Defining $\tilde{y} := T^{-1} y$ and replacing in the quadratic form yields $\tilde{y}^\top \tilde{A} \tilde{y} = s$. The proof is completed selecting

$$\tilde{y} = \sqrt{s} \begin{bmatrix} \frac{1}{\sqrt{a}}\cosh(\rho) \\ \frac{1}{\sqrt{\frac{b^2}{a}+c}}\sinh(\rho) \end{bmatrix},$$

and using the fact that $\cosh^2(\rho) - \sinh^2(\rho) = 1$.