2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

A Finite-Time Tracker for Nonholonomic Systems Using Recursive Singularity-Free FTSM

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***Abstract*—A novel recursive singularity free FTSM (Fast Terminal Sliding Mode) strategy for finite time tracking control of nonholonomic systems is proposed. As a result, the singularity problem around the origin resulting from the fractional power of conventional terminal sliding mode is resolved. Simulation results are given for two benchmark examples of extended chained-form nonholonomic systems: a wheeled mobile robot and an underactuated surface vessel. The results show the effectiveness of the proposed strategy.**

***Index Terms*—Nonholonomic systems, fast terminal sliding mode (FTSM), finite-time tracking.**

I. Introduction

THE problem of stabilizing nonholonomic systems has received much attention in the past few years [1]. Control of the nonholonomic systems presents greater challenges than systems without constraints. This kind of nonlinear systems is very common in practical applications such as wheeled mobile robot, surface vessel and space robots. These systems are not linearly controllable around any of equilibrium points and do not satisfy Brockett's necessary smooth feedback stabilization condition as shown in [2]. Then, nonholonomic systems cannot be stabilized by smooth feedback control laws [3]. Nevertheless, several control methods have been proposed to stabilize this class of systems using discontinuous controllers based on sliding modes, hybrid control and time-varying strategies [1, 4]. Thus, the stabilization problem of such systems becomes a focus of research which still remains to be an interesting topic [5]. Compared with the stabilization problem, the tracking problem sometimes called stabilization of trajectories has received less consideration [6].

The stabilization of the motion of dynamical systems in finite time is often desirable and has interested control engineers to study this problem. In particular, terminal sliding mode (TSM) method has shown some superior properties such as finite time convergence, high-precision control performance and easy to use characteristics [7]. Unlike conventional sliding mode method, TSM is based on a class of nonlinear differential equations including finite time solutions and recently there is more interest in the use of it [7-8].

Nevertheless, the conventional TSM control methods are very sensitive around the origin and because of the fractional power sliding modes and their derivatives can take unexpectedly large values, leading to the singularity problem [7]. Recently, a recursive singularity free TSM method has been proposed in [8].

In this paper, based on the recursive singularity free TSM and fast TSM (FTSM) control strategy proposed in [8- 9], we apply singularity free FTSM strategy to the control of nonholonomic systems. The new FTSM model that is able to combine the advantages of the TSM and conventional sliding-mode control together is proposed so that fast transient convergence in the vicinity of the equilibrium can be obtained.

The paper is organized as follows; section II contains a description of the nonholonomic system and presents dynamics equations of the system. In section III, we present the design procedure of the singularity free FTSM method for finite-time tracking of the system. Section IV implements the algorithm on two benchmark plants followed by conclusion in section V.

II. Problem Statement and Preliminaries

The nonholonomic systems in chained-form are described as [10]:

$$\begin{aligned} \dot{x}_1 &= u_1 \\ \dot{x}_2 &= u_2 \\ \dot{x}_3 &= x_2 u_1 \\ &\vdots \\ \dot{x}_n &= x_{n-1} u_1, \end{aligned} \tag{1}$$

where $x=(x_1, \ldots , x_n)^T$ is the state and u_1 and u_2 are two control inputs. Fig. 1 demonstrates the diagram of the general chained-form nonholonomic systems.

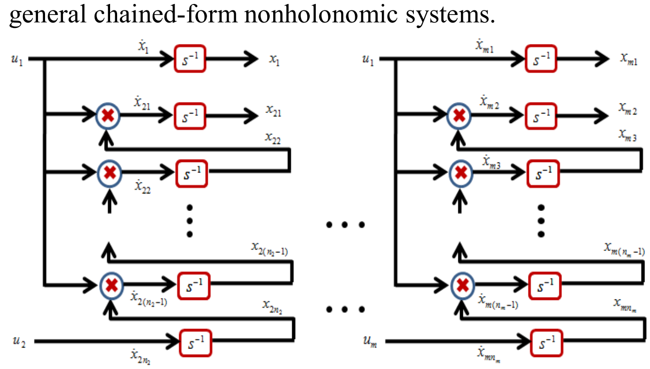

Fig. 1. Nonholonomic systems in chained-form.

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Now assume that the desired trajectory $x_d=(x_{1d}, \ldots, x_{nd})^T$ is generated using the following equations:

$$\begin{aligned}\dot{x}_{1d} &= u_{1d} \\ \dot{x}_{2d} &= u_{2d} \\ \dot{x}_{3d} &= x_{2d}u_{1d} \\ &\vdots \\ \dot{x}_{nd} &= x_{(n-1)d}u_{1d},\end{aligned} \tag{2}$$

where u_{1d} and u_{2d} are the time-varying reference control inputs. The dynamics of the tracking errors defined as $x_e = x - x_d$ satisfy the following differential equations [10]:

$$\begin{aligned}\dot{x}_{1e} &= u_1 - u_{1d} \\ \dot{x}_{2e} &= u_2 - u_{2d} \\ \dot{x}_{3e} &= x_{2e}u_{1d} + x_2(u_1 - u_{1d}) \\ &\vdots \\ \dot{x}_{ne} &= x_{(n-1)e}u_{1d} + x_{n-1}(u_1 - u_{1d}).\end{aligned} \tag{3}$$

The dynamics of system (1) is extended by the following equations:

$$\begin{aligned}\dot{u}_1 &= v_1, \\ \dot{u}_2 &= v_2,\end{aligned} \tag{4}$$

where v_1 and v_2 are the control inputs of the dynamic model.

Finite-time stability means that the state of the system converges to the desired target in a finite time. To demonstrate this concept clearly, let us introduce the following definition and lemma.

***Definition* 1** [11]**:** Consider the following dynamical system:

$$\dot{x}(t) = f(x), \tag{5}$$

where $x(t) \in R^n$ is the system state. If there exists a constant $T>0$ such that

$$\lim_{t\to T}\|x(t)\| = 0 \tag{6}$$

is satisfied and if $t \geq T$, we obtain $\|x(t)\| \equiv 0$, then the system (5) is finite time stable. Note that T may depend on the initial state $x(0)$.

***Lemma* 1** [12]**:** Assume that a continuous positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \leq -c.V^{\eta}(t) \qquad \forall t \geq t_0,\ V(t_0) \geq 0, \tag{7}$$

where $c>0$, $0 < \eta < 1$ are two constants. Then for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - c(1-\eta)(t - t_0), \quad t_0 \leq t \leq t_1 \tag{8}$$

and

$$V(0) \equiv 0, \qquad t \geq t_1 \tag{9}$$

Which the time t_1 is as follows:

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}. \tag{10}$$

The control task is to design the controllers v_1 and v_2 such that $x(t)$ tracks the desired trajectory $x_d(t)$ in finite-time. The control goal of this paper is to make the tracking error x_e converges to zero in finite-time.

III. Finite Time Tracking Using Singularity Free FTSM

Considering (3) and (4), the system dynamics can be divided into two subsystems. Firstly, the following subsystem is considered:

$$\begin{aligned}\dot{x}_{1e} &= u_1 - u_{1d} \\ \dot{u}_1 &= v_1.\end{aligned} \tag{11}$$

The control law v_1 for this subsystem is chosen as [10, 13]:

$$v_1 = \dot{u}_{1d} - 3x_{1e}^{-2/5}(u_1 - u_{1d}) - 5\left(u_1 - u_{1d} + 5x_{1e}^{3/5}\right)^{3/5}. \tag{12}$$

where the control law (12) guarantees that x_{1e} reaches zero in a certain finite time T_0 and keeps zero afterwards. Secondly, rest of the system is considered as:

$$\begin{aligned}\dot{x}_{2e} &= u_2 - u_{2d} \\ \dot{x}_{3e} &= x_{2e}u_{1d} \\ &\vdots \\ \dot{x}_{ne} &= x_{(n-1)e}u_{1d} \\ \dot{u}_2 &= v_2\end{aligned} \tag{13}$$

where for the above subsystem, first changes of coordinates are defined as $y_1 = x_{ne}, y_2 = x_{(n-1)e}, \ldots, y_{n-1} = x_{2e}, y_n = u_2 - u_{2d}$, then the system (13) can be transformed into the following form:

$$\begin{aligned}\dot{y}_1 &= u_{1d}y_2 \\ \dot{y}_2 &= u_{1d}y_3 \\ &\vdots \\ \dot{y}_{n-1} &= y_n \\ \dot{y}_n &= v_2 - \dot{u}_{2d}\end{aligned} \tag{14}$$

Now, for designing the control law v_2 to make y_i (i=1, 2, …, n) become zero in finite time, we take the following recursive FTSM structure:

$$\begin{aligned}s_0 &= y_1 \\ s_1 &= \dot{s}_0 + \alpha_0 s_0 + \beta_0|s_0|^{n-\frac{1}{a}}\operatorname{sgn}(s_0) \\ s_2 &= \dot{s}_1 + \alpha_1 s_1 + \beta_1|s_1|^{n-1-\frac{1}{a}}\operatorname{sgn}(s_1) \\ &\vdots \\ s_{n-1} &= \dot{s}_{n-2} + \alpha_{n-2}s_{n-2} + \beta_{n-2}|s_{n-2}|^{2-\frac{1}{a}}\operatorname{sgn}(s_{n-2})\end{aligned} \tag{15}$$

where $a>1$ and α_i and β_i are certain positive constants. From the structure of the above sliding surface, we obtain:

$$
\begin{aligned}
s_0 &= s_0(y_1) \\
s_1 &= s_1(t, y_1, y_2) \\
s_2 &= s_2(t, y_1, y_2, y_3) \\
&\vdots \\
s_{n-1} &= s_{n-1}(t, y_1, \cdots, y_n).
\end{aligned} \tag{16}
$$

where differentiating s_{n-1} along the solutions of (14), we have:

$$
\dot{s}_{n-1} = \frac{\partial s_{n-1}}{\partial t} + \sum_{i=1}^{n-2} \frac{\partial s_{n-1}}{\partial y_i} u_{1d} y_{i+1} + \frac{\partial s_{n-1}}{\partial y_{n-1}} y_n + \frac{\partial s_{n-1}}{\partial y_n}\left(v_2 - \dot{u}_{2d}\right). \tag{17}
$$

***Lemma* 2:** For the given s_i (i=1, …, n-1), the following equation is satisfied:

$$
\frac{\partial s_{n-1}}{\partial y_n} = u_{1d}^{n-1}(t). \tag{18}
$$

***Proof*:** From (16), we have $\partial s_1 / \partial y_2 = u_{1d}(t)$. Suppose that for $1 \le i \le n-2$, $\partial s_i / \partial y_{i+1} = u_{1d}^{i}(t)$ is true. Now using the principle of induction, we compute $\partial s_{i+1} / \partial y_{i+2}$ as follows:

Since we have $s_{i+1} = \dot{s}_i + \alpha_i s_i + \beta_i |s_i|^{n-i-\frac{1}{a}} \operatorname{sgn}(s_i)$ and $s_i = s_i(t, y_1, \cdots, y_{i+1})$, then terms $(\partial s_{i+1} / \partial y_{i+2}) = (\partial \dot{s}_i / \partial y_{i+2})$ and $\dot{s}_i = (\partial s_i / \partial t) + \sum_{k=1}^{i} (\partial s_i / \partial y_k) u_{1d} y_{k+1} + (\partial s_i / \partial y_{i+1}) u_{1d} y_{i+2}$ are satisfied. Now we obtain:

$$
\frac{\partial s_{i+1}}{\partial y_{i+2}} = \frac{\partial s_i}{\partial y_{i+1}} u_{1d} = u_{1d}^{i+1}(t), \tag{19}
$$

where for i=n-2, it yields:

$$
\frac{\partial s_{n-1}}{\partial y_n} = u_{1d}^{n-1}(t), \tag{20}
$$

which completes the proof of the lemma.

Then by the lemma 2 and letting the control input as

$$
v_2 = -\frac{1}{u_{1d}^{n-1}}\left(\frac{\partial s_{n-1}}{\partial t} + \sum_{i=1}^{n-2} \frac{\partial s_{n-1}}{\partial y_i} u_{1d} y_{i+1} + \frac{\partial s_{n-1}}{\partial y_{n-1}} y_n + k.\operatorname{sgn}(s_{n-1})\right) + \dot{u}_{2d}, \tag{21}
$$

finally (17) can be transformed as:

$$
\dot{s}_{n-1} = -k.\operatorname{sgn}(s_{n-1}). \tag{22}
$$

Therefore the sliding surface s_{n-1}=0 is reached in finite time. After satisfying s_{n-1}=0, from (15) we have

$$
\dot{s}_{n-2} + \alpha_{n-2} s_{n-2} + \beta_{n-2} |s_{n-2}|^{2-\frac{1}{a}} \operatorname{sgn}(s_{n-2}) = 0 \tag{23}
$$

where after rearranging (23), the original FTSM is achieved as

$$
\dot{s}_{n-2} = -\alpha_{n-2} s_{n-2} - \beta_{n-2} |s_{n-2}|^{2-\frac{1}{a}} \operatorname{sgn}(s_{n-2}). \tag{24}
$$

Thus according to the finite-time convergence property of FTSM, the sliding surface s_{n-2}=0 will be reached in finite time. Furthermore, s_{n-2}=0 means that the following equation is satisfied:

$$
\dot{s}_{n-3} + \alpha_{n-3} s_{n-3} + \beta_{n-3} |s_{n-3}|^{3-\frac{1}{a}} \operatorname{sgn}(s_{n-3}) = 0 \tag{25}
$$

where with the same procedure as (23) and (24), s_{n-3}=0 will also be reached in finite time. Then recursively s_1 and s_0=y_1 will reach zero in finite time and we can get $y_1 \equiv \cdots \equiv y_n \equiv 0$ and $v_2 \equiv 0$ in finite time. That is $x_{2e} \equiv \cdots \equiv x_{ne} \equiv 0$ and $u_2 \equiv u_{2d}$ are reached in finite time.

According to (15) and the condition a>1, no negative fractional powers appear in derivatives of the recursive FTSM surfaces in every step, and then the singularity problem doesn't occur in this procedure.

***Theorem* 1:** For the closed loop system (1)-(4), the control laws v_1 and v_2 are chosen as (12) and (21), then the switching surface s=0 is reached in finite time. Therefore the tracking errors x_{1e}, x_{2e}, …, x_{ne} and control laws v_1 and v_2 will reach zero in finite time.

***Proof*:** The derivatives of the switching surfaces are

$$
\begin{aligned}
\dot{s}_{n-1} &= -k.\operatorname{sgn}(s_{n-1}), \\
\dot{s}_{n-2} &= -\alpha_{n-2} s_{n-2} - \beta_{n-2} |s_{n-2}|^{2-\frac{1}{a}} \operatorname{sgn}(s_{n-2}), \\
\dot{s}_{n-3} &= -\alpha_{n-3} s_{n-3} - \beta_{n-3} |s_{n-3}|^{3-\frac{1}{a}} \operatorname{sgn}(s_{n-3}), \\
&\vdots \qquad \vdots \\
\dot{s}_0 &= -\alpha_0 s_0 - \beta_0 |s_0|^{n-\frac{1}{a}} \operatorname{sgn}(s_0).
\end{aligned} \tag{26}
$$

Now consider the following candidate Lyapunov function:

$$
V = \frac{1}{2} s^T s = \frac{1}{2} \|s\|^2, \tag{27}
$$

where the derivatives of the Lyapunov function along the switching surfaces (26) are as follows:

$$
\begin{aligned}
\dot{V}_{n-1} &= s_{n-1}^T \dot{s}_{n-1} = -k|s_{n-1}| \le -c.V_{n-1}^{\eta} \\
\dot{V}_{n-2} &= s_{n-2}^T \dot{s}_{n-2} = -\alpha_{n-2} s_{n-2}^T s_{n-2} - \beta_{n-2} |s_{n-2}|^{(3-\frac{1}{a})} \\
&\le -2^{\frac{(3-1/a)}{2}} \beta_{n-2} V_{n-2}^{\frac{(3-1/a)}{2}} \\
&\vdots \qquad \vdots \\
\dot{V}_0 &= s_0^T \dot{s}_0 = -\alpha_0 \|s_0\|^2 - \beta_0 |s_0|^{(n-\frac{1}{a}+1)} \\
&\le -2^{\frac{(n+1-1/a)}{2}} \beta_0 V_0^{\frac{(n+1-1/a)}{2}}
\end{aligned} \tag{28}
$$

Then from lemma 1, the states will move toward the sliding surface s=0 in a finite time. Therefore, the tracking errors will reach zero in finite time and it can be easily derived that the time of the tracking errors to reach zero is calculated as follows:

From (15) and for s_{n-1}=0, we have:

$$
\dot{s}_{n-2} + \alpha_{n-2} s_{n-2} + \beta_{n-2} |s_{n-2}|^{2-\frac{1}{a}} \operatorname{sgn}(s_{n-2}) = 0 \tag{29}
$$

where after rearranging (32), one obtains:

$$\left|s_{n-2}\right|^{-(2-\frac{1}{a})}\dot{s}_{n-2}+\alpha_{n-2}s_{n-2}{}^{1-(2-\frac{1}{a})}+\beta_{n-2}=0 \tag{30}$$

and:

$$\frac{\left|s_{n-2}\right|^{-(2-\frac{1}{a})}ds_{n-2}}{\alpha_{n-2}\left|s_{n-2}\right|^{1-(2-\frac{1}{a})}+\beta_{n-2}}=-dt \tag{31}$$

then the settling time can be given by:

$$t_{n-1}=\frac{1}{\alpha_{n-2}(\frac{1}{a}-1)}\ln\left(\frac{\alpha_{n-2}\left|s_{n-2}\right|^{\frac{1}{a}-1}+\beta_{n-2}}{\beta_{n-2}}\right). \tag{32}$$

With the same procedure as (29)-(32), recursively s_{n-2}, s_{n-3}, ..., s_0 will reach zero in finite time and the spent time can be calculated. Then, the tracking total time is as follows:

$$T=\sum_{i=1}^{n-1}t_i =\sum_{i=1}^{n-1}\frac{1}{\alpha_{i-1}(i-n+\frac{1}{a})}\ln\left(\frac{\alpha_{i-1}\left|s_{i-1}(t_i)\right|^{i-n+\frac{1}{a}}+\beta_{i-1}}{\beta_{i-1}}\right). \tag{33}$$

IV. Simulation Results

In this section, the FTSM tracking method is illustrated to apply on two nonholonomic systems: wheeled mobile robot and underactuated surface vessel.

A. Wheeled Mobile Robot

Consider the simulation example of the wheeled mobile robot in the below extended chained form [14]:

$$\begin{aligned}&\dot{x}_1=u_1,\quad \dot{x}_2=u_2,\quad \dot{x}_3=x_2u_1\\ &\dot{u}_1=v_1,\quad \dot{u}_2=v_2,\end{aligned} \tag{34}$$

where state and control transformations are defined by

$$\begin{aligned}x_1&=\theta\\ x_2&=-x\sin\theta+y\cos\theta\\ x_3&=x\cos\theta+y\sin\theta\\ v_1&=w_2\\ v_2&=w_1-(x\cos\theta+y\sin\theta)w_2\end{aligned} \tag{35}$$

where (x, y), θ, w_1 and w_2 denote the coordinates of the center of mass, the heading angle measured from x-axis, forward velocity of the center of mass, and the angular velocity of the robot, respectively.

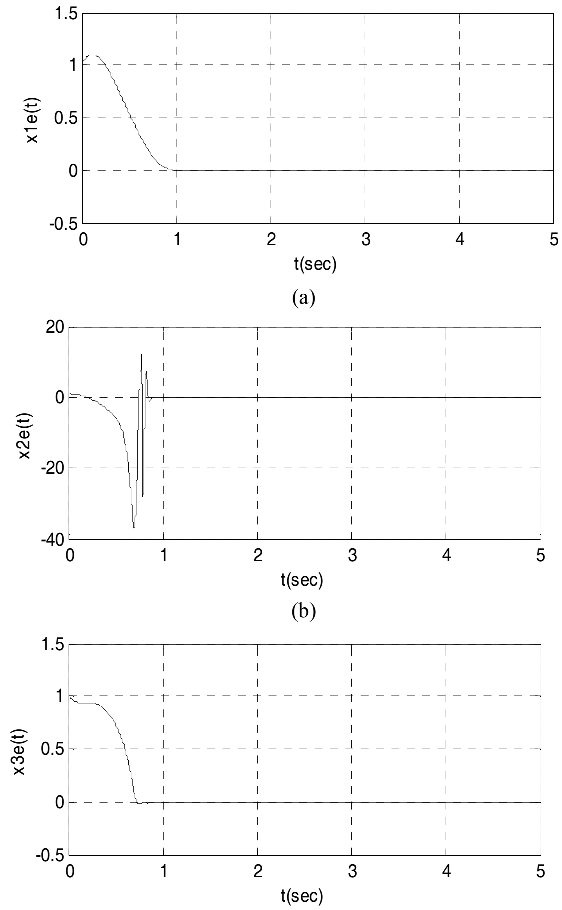
Assume that the reference model is considered as:

$$\begin{aligned}\dot{x}_{1d}&=u_{1d}\\ \dot{x}_{2d}&=u_{2d}\\ \dot{x}_{3d}&=x_{2d}u_{1d}.\end{aligned} \tag{36}$$

where taking $x_{ie}=x_i-x_{id}$, $i=1,2,3$, the tracking error model is resulted as follows:

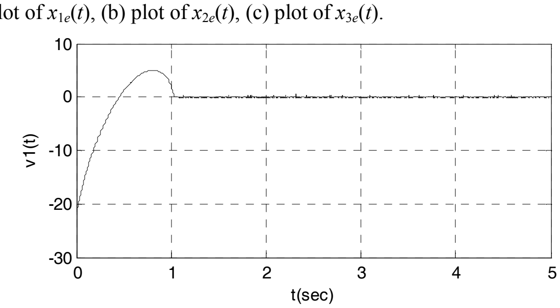
$$\begin{aligned}\dot{x}_{1e}&=u_1-u_{1d}\\ \dot{x}_{2e}&=u_2-u_{2d}\\ \dot{x}_{3e}&=x_{2e}u_{1d}+x_2(u_1-u_{1d})\\ \dot{u}_1&=v_1\\ \dot{u}_2&=v_2.\end{aligned} \tag{37}$$

For simulation use, take β=2, α=2, a=10, k=50, $u_{1d}(t)$=1 and $u_{2d}(t)$=0. Also initial conditions are chosen as: $x_{1e}(0)$=1, $x_{2e}(0)$=1, $x_{3e}(0)$=1, $u_1(0)$=3 and $u_2(0)$=1. Fig. 2 shows the trajectory of the tracking errors. Also, control performance of the system is demonstrated in fig. 3.

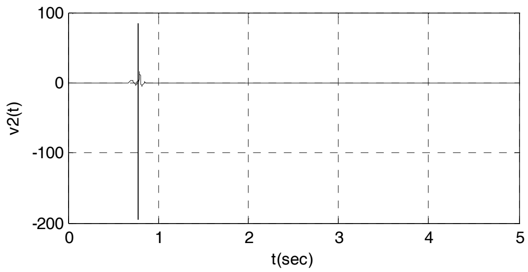


(c)

Fig. 2. Trajectory of the tracking errors of the wheeled mobile robot. (a) plot of $x_{1e}(t)$, (b) plot of $x_{2e}(t)$, (c) plot of $x_{3e}(t)$.



(a)



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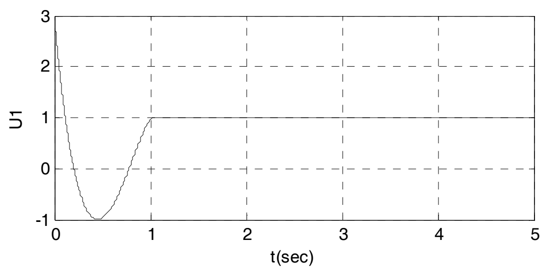

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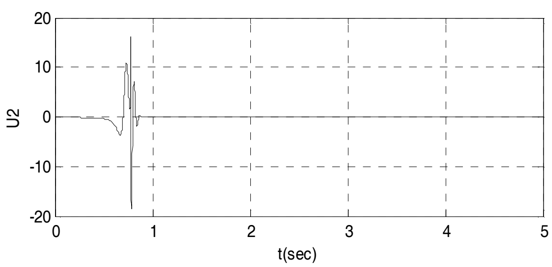


(d)

Fig. 3. Control performance of the wheeled mobile robot. (a) plot of $v_1(t)$, (b) plot of $v_2(t)$, (c) plot of $u_1(t)$, (d) plot of $u_2(t)$.

Figs. 2 and 3 show that the proposed method has superior convergence efficiency. It can be seen that the tracking errors quickly converge to zero in a few seconds as well as control laws.

B. Underactuated Rigid Body

Consider the kinematics of the attitude of a rigid body in the below Euler angles form [15]:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ \sin\theta\tan\phi & 1 & -\cos\theta\tan\phi \\ -\sin\theta\sec\phi & 0 & \cos\theta\sec\phi \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \tag{38}$$

where θ, ϕ, ψ are Euler angles and w_1, w_2, w_3 are the angular velocities corresponding to the body frame. Now assume that one of the actuators is failed and w_3 is constrained to be zero. Then the system is underactuated and has the following form:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta\tan\phi & 1 \\ -\sin\theta\sec\phi & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{39}$$

It has been shown in [15-16] that the following coordinate and input transformations convert the underactuated rigid body (39) to the chained form. Then introducing the variables,

$$\begin{aligned} x_1 &= \ln|\sec\phi + \tan\phi|, \\ x_2 &= -\psi, \\ x_3 &= \theta, \\ u_1 &= w_1 \sec\phi, \\ u_2 &= w_1 \sin\theta\tan\phi + w_2, \end{aligned} \tag{40}$$

the system can be rewritten as:

$$\begin{aligned} \dot{x}_1 &= u_1 \cos x_3, \\ \dot{x}_2 &= u_1 \sin x_3, \\ \dot{x}_3 &= u_2. \end{aligned} \tag{41}$$

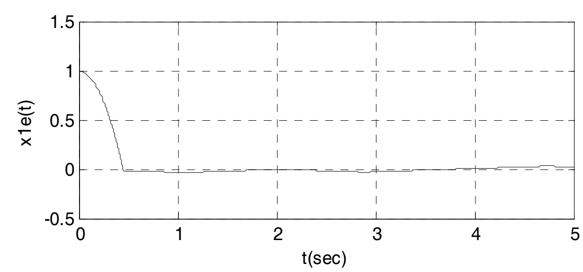
Assume that the reference model is considered as:

$$\begin{aligned} \dot{x}_{1d} &= u_{1d}.\cos x_{3d}, \\ \dot{x}_{2d} &= u_{1d}.\sin x_{3d}, \\ \dot{x}_{3d} &= u_{2d}, \end{aligned} \tag{42}$$

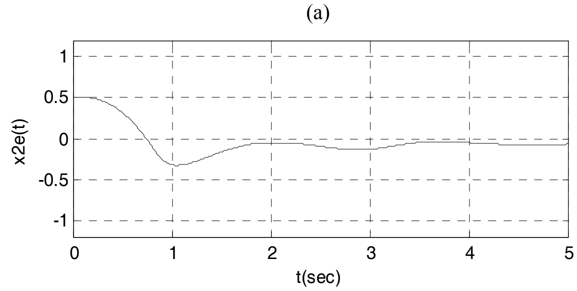
where taking $x_{ie} = x_i - x_{id}$, $i = 1, 2, 3$, the tracking error model is resulted as follows:

$$\begin{aligned} \dot{x}_{1e} &= u_1 \cos x_3 - u_{1d} \cos x_{3d} \\ \dot{x}_{2e} &= u_1 \sin x_3 - u_{1d} \sin x_{3d} \\ \dot{x}_{3e} &= u_2 - u_{2d} \\ \dot{u}_1 &= v_1 \\ \dot{u}_2 &= v_2. \end{aligned} \tag{43}$$

For simulation use, take β=30, α=10, a=2, k=10, x_{1d}=0, x_{2d}=0, x_{3d}=0, $u_{1d}(t)$=1 and $u_{2d}(t)$=0. Also initial conditions are chosen as: $x_1(0)$=1, $x_2(0)$=0.5, $x_3(0)$=0, $u_1(0)$=1 and $u_2(0)$=2. Fig. 4 illustrates the trajectory of the tracking errors. Also, control performance of the system is shown in fig. 5.



(a)



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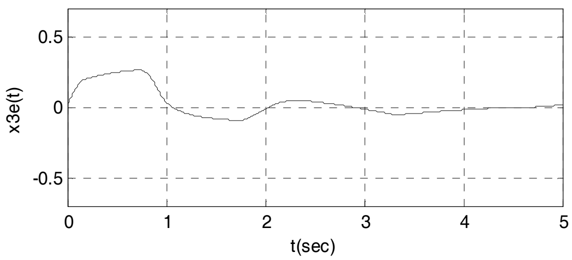
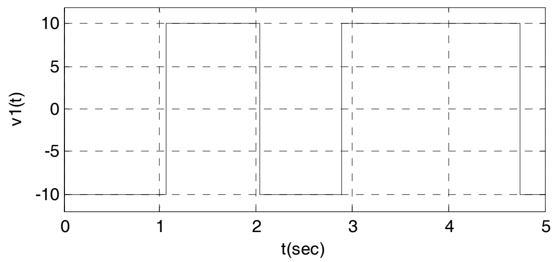
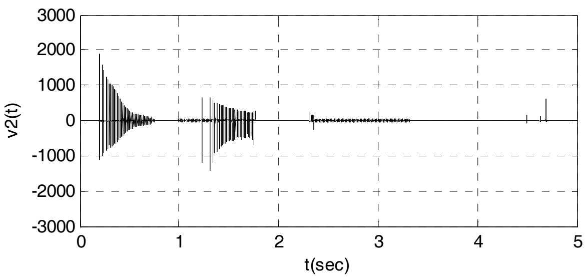


(c)

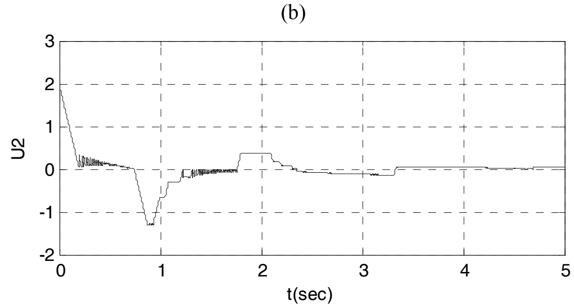
Fig. 4. Trajectory of the tracking errors of the underactuated rigid body. (a) plot of $x_{1e}(t)$, (b) plot of $x_{2e}(t)$, (c) plot of $x_{3e}(t)$.



(a)



(b)



(c)

Fig. 5. Control performance of the underactuated rigid body. (a) plot of $v_1(t)$, (b) plot of $v_2(t)$, (c) plot of $u_2(t)$.

From the above figures, one can see that the underactuated rigid body is stabilized in a finite time by the proposed nonsingular fast terminal sliding mode control. Then, this controller ensures that the tracking errors converge quickly to zero in finite time so that the complete tracking is realized.

V. Conclusion

This paper presents a singularity free recursive FTSM method for finite time tracking control of a class of nonholonomic systems. Simulation results are illustrated for two benchmark examples of extended chained form nonholonomic systems. The results demonstrate the efficiency of proposed control method. The control strategy developed in this paper can be applied in a high-precision tracking performance to a wide range of problems such as chaos synchronization and filter design. Designing of FTSM controller for general class of nonholonomic systems with disturbances and drift uncertainties can be the topic of our future research.

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