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Calibrating Energy Conversion Networks for Utility Optimization and Risk Management

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***Abstract*— We demonstrate modeling and parameter estimation of flexible energy systems used in campus and municipal scale utilities with complex energy requirements, multiple fuel sources, and requiring substantial operational flexibility. Useful models should accurately predict the work production of these flexible utilities. A framework for such a model is a class of bilinear models for estimating the efficiency of complex and flexible energy utilities [9]. This framework, which we call Energy Conversion Networks (ECN), may be used to determine financially optimal operating conditions and opportunities for financial and operational hedging by the utility operator [8], [10].**

Given an ECN model, we compute a unique input-output mapping from the decision variables and heat input to work output.

Multiple network realizations may be possible from a given input/output model. Work output from these models are expressed as rational functions of entropy flux with parameters of temperatures, thermal conductances, and engine efficiencies.

Using plant data from a report published by the California Energy Commission [6], we demonstrate fitting this data on several network realization examples. We use measured work output as a function of heat input to calibrate a model. Heat rate curves are the most common way these data are typically available. We developed a data fitting procedure to determine the parameters resulting in the best calibrated model given a particular network model and data set.

I. Introduction

Management of energy utilities is challenging due to the uncertainties and limited operational flexibility. Sources of uncertainties include market prices for fuels and purchased electricity and the demand for heating, cooling, and electricity. An operator also has limited operational flexibility such as deciding whether to generate or purchase options and futures on fuels, purchase and store certain fuels, and enter into interruptible contracts for fuel and electricity. Financial optimization of energy utilities must incorporate these uncertainties and flexibilities to determine an optimal strategy.

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The focus of our research is on developing models, techniques, and providing insight in characterizing the financial operation of flexible fuel energy systems. Integration of the energy utility operations, financial instruments, and contractual elements must be incorporated to maximize the value of energy production. Our group [1], [2], [8], [10] and others [3], [4], [11] have shown the value of integrating hedging with operational decisions in energy applications. In particular, we previously introduced a class of bilinear models for Energy Conversion Networks (ECN) that incorporate principles from finite-time thermodynamics [5]. In this paper we show how to calibrate these models for use in the economic optimization and risk management of flexible fuel utilities.

This paper seeks development and calibration of the bilinear models capturing the complex behavior between heat input and work output in an energy conversion utility. The data for the heat input and work output behavior are often captured in the form of heat rate curves.

A network realization consists of several heat nodes and at least one engine connected by thermal links. Multiple network realizations are possible for a given heat input/work output

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model. The heat input and work output from these models can be expressed as rational functions of entropy flux σ and the parameters.

Parameters are estimated for a given network realization using the least squares method. Financial optimization with a calibrated model allows for risk management analysis to be done.

The following sections of this paper include a review of the ECN framework, network realization illustration that demonstrates that heat input and work output can be expressed as rational functions of entropy flux σ, followed by parameter estimation to plant data using the least squares fit method.

II. Energy Conversion Network Framework

The basic concept behind the ECN framework is to break down the energy utility into simple building blocks. These building blocks include heat nodes, heat transport via thermal links, heat engines, heat pumps, and work nodes. The collection of these building blocks form a class of bilinear models that incorporates the first and second laws of finite-time thermodynamics and are used to estimate the efficiency and predict the performance of real energy utilities.

Heat Node: Heat nodes (Fig. 1) are the most common building blocks of the models. Each heat node is characterized by a temperature T_i and external heat flux Q_i. Heat nodes are connected by thermal links, heat engines, or heat pumps. A heat node accepting a heat flux represents the energy flux or power entering the node while one emitting a heat flux represents the rejection of heat.

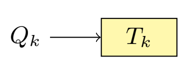

Fig. 1. External heat transport to a heat node

Heat Transport and Thermal Links: Heat transport between two heat nodes is represented by a thermal link (Fig. 2). We use Newton's law for heat transport in this model (Eq. 1).

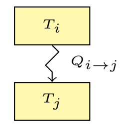

Fig. 2. Heat Transport between two heat nodes

$$Q_{i\to j} = K_{ij}\left(T_i - T_j\right) \tag{1}$$

Heat Engines: Equipment producing work are represented by heat engines in our framework (Fig. 3). Each heat engine is modeled between two heat nodes and is assumed to be externally adiabatic $(Q^+ = Q^- + W)$ and is characterized by an isentropic efficiency η, work flux W, and entropy flux σ. The equations for isentropic efficiency, entropy flux, heat fluxes, and work output for the m^{th} heat engine (m indexing all heat engines and pumps) are (Eq. 2-6):

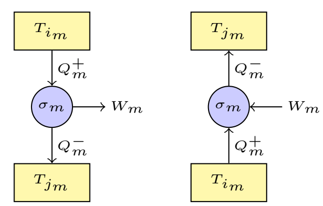

Fig. 3. Heat Engine and Heat Pump

$$\eta_m = \frac{W_m}{W_m^{Rev}} \tag{2}$$

$$\sigma_m = \frac{Q_m^+}{T_{i_m}} \tag{3}$$

$$Q_m^+ = \sigma_m T_{i_m} \tag{4}$$

$$W_m = \eta_m \sigma_m \left(T_{i_m} - T_{j_m}\right) \tag{5}$$

$$Q_m^- = \left(1 - \eta_m\right)\sigma_m T_{i_m} + \eta_m \sigma_m T_{j_m} \tag{6}$$

Heat Pumps: Equipment that consumes work is represented by heat pumps in our framework (Fig. 3). Work flux is negative for a heat pump but positive for a heat engine. The modeling and characterization of heat pumps in this framework are the same as with heat engines with each heat pump being characterized by an isentropic efficiency η, a work flux W, and entropy flux σ and modeled between two heat nodes assumed to be externally adiabatic. The equations for heat fluxes and work output for the m^{th} heat pump are (Eq. 7-11):

$$\eta_m = \frac{W_m^{Rev}}{W_m} \tag{7}$$

$$\sigma_m = \frac{Q_m^+}{T_{i_m}} \tag{8}$$

$$Q_m^+ = \sigma_m T_{i_m} \tag{9}$$

$$W_m = \frac{1}{\eta_m}\sigma_m \left(T_{i_m} - T_{j_m}\right) \tag{10}$$

$$Q_m^- = \left(1 - \frac{1}{\eta_m}\right)\sigma_m T_{i_m} + \frac{1}{\eta_m} T_{j_m} \tag{11}$$

Work Nodes: Work nodes combine work output and input from heat engines and pumps to provide a cumulative work flux. This allows work of the same value to be combined. Conservation of work at a work node is required. This is particularly important when a heat pump requires work from a heat engine to operate; a heat pump can only use the amount of work generated by the heat engine, a constraint captured by the work node in between them. Work nodes are represented by a small circle where work fluxes are combined.

Bilinear Models: The collection of these simple building blocks form bilinear models. These models are combined with a financial objective function to form an optimization problem, given by Eq. 12a - 12d:

$$\min_{T,\sigma} \quad f \quad = R^{\intercal} q - S^{\intercal} w \tag{12a}$$

$$s.t. \quad q \quad = \quad \left(K + \sum_m E_m \sigma_m\right) T \tag{12b}$$

$$w \quad = \quad \left(\sum_m W_m \sigma_m\right) T \tag{12c}$$

$$\begin{array}{ccccc} T^L & \leq & T & \leq & T^U \\ \sigma^L & \leq & \sigma & \leq & \sigma^U \\ q^L & \leq & q & \leq & q^U \\ w^L & \leq & w & \leq & w^U \end{array} \tag{12d}$$

This optimization problem has the following structure. N, M, and P are the number of heat nodes, engines, work nodes, respectively. q is a length N vector of heat flux and w is a length $M + P$ vector of work flux. R is a length N vector of heat input (fuel) costs and S is a length $M + P$ vector of work values. K is a $M \times M$ matrix of thermal links. E_m is a $N \times N$ matrix containing the isentropic efficiency (η_m) for the m^{th} engine and is used for the heat flux calculation. W_m is a $(M+P) \times N$ matrix containing isentropic efficiency for the m^{th} engine and is used for the work flux calculation.

This optimization problem is bilinear in nature since both q and w have the term σT present. We reported a solution technique to this bilinear optimization problem in a previous paper [9] but it will not be discussed in this paper.

III. Network Realization Illustration

We review three network realizations for use in fitting heat input/work output data. These are a two heat node/one engine realization (Fig. 4), a three heat node/one engine realization (Fig. 5), and a four heat node/one engine realization (Fig. 6).

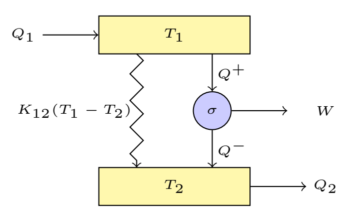

Fig. 4. 2 heat nodes/one engine

The two heat node/one engine system has the following equations for the heat nodes and work:

$$Q_1 = K_{12}\left(T_1 - T_2\right) + \sigma T_1 \tag{13}$$

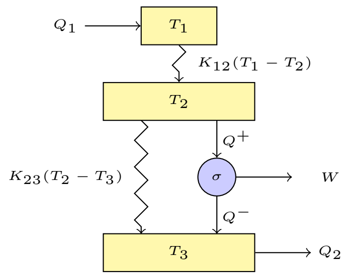

Fig. 5. 3 heat nodes/one engine

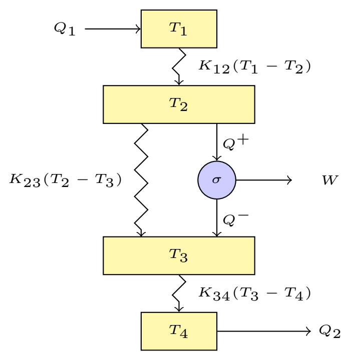

Fig. 6. 4 heat nodes/one engine

$$\begin{aligned} Q_2 = &-K_{12}\left(T_1 - T_2\right) \\ &- \left(1-\eta\right)\sigma T_1 - \eta\sigma T_2 \end{aligned} \tag{14}$$

$$W = \eta\sigma\left(T_1 - T_2\right) \tag{15}$$

Since we assume T_1 is an unknown constant, T_1, η, and K_{12} are parameters and T_2 is a know constant, Eq. 13 and Eq. 15 are the rational functions of σ and the parameters for Q_1 and W, respectively for the 2 heat node/one engine realization.

The three heat node/one engine system has the following equations for heat nodes and work:

$$Q_1 = K_{12}\left(T_1 - T_2\right) \tag{16}$$

$$\begin{aligned} 0 = &-K_{12}\left(T_1 - T_2\right) \\ &+ K_{23}\left(T_2 - T_3\right) + \sigma T_2 \end{aligned} \tag{17}$$

$$\begin{aligned} Q_2 = &-K_{23}\left(T_2 - T_3\right) \\ &- \left(1-\eta\right)\sigma T_2 - \eta\sigma T_3 \end{aligned} \tag{18}$$

$$W = \eta\sigma\left(T_2 - T_3\right) \tag{19}$$

Using these equations and solving for W in terms of σ and the parameters result in Eq. 20:

$$W = \eta\sigma\left(\frac{-T_3\sigma + K_{12}\left(T_1 - T_3\right)}{\sigma + K_{12} + K_{23}}\right) \tag{20}$$

Solving for Q_1 in terms of σ and the parameters result in Eq. 21:

$$Q_1 = \frac{K_{12}T_1\sigma + K_{12}K_{23}\left(T_1 - T_3\right)}{\sigma + K_{12} + K_{23}} \tag{21}$$

The equations for the four heat node/one engine and work for this system are:

$$Q_1 = K_{12}\left(T_1 - T_2\right) \tag{22}$$

$$\begin{aligned} 0 = &-K_{12}\left(T_1 - T_2\right) \\ &+ K_{23}\left(T_2 - T_3\right) + \sigma T_2 \end{aligned} \tag{23}$$

$$\begin{aligned} 0 = &K_{23}\left(T_2 - T_3\right) + K_{34}\left(T_3 - T_4\right) \\ &- \left(1-\eta\right)\sigma T_2 - \eta\sigma T_3 \end{aligned} \tag{24}$$

$$Q_2 = -K_{34}\left(T_3 - T_4\right) \tag{25}$$

$$W = \eta\sigma\left(T_2 - T_3\right) \tag{26}$$

Using these equations and solving for W in terms of σ and the parameters results in Eq. 27:

$$W = \eta\sigma\left(\frac{f\left(\sigma\right)}{g\left(\sigma\right)}\right) \tag{27}$$

where $f\left(\sigma\right)$ and $g\left(\sigma\right)$ are given by Eq. 28 and Eq. 29, respectively:

$$\begin{aligned} f\left(\sigma\right) = &\left(-K_{12}T_1 - K_{34}T_4\right)\sigma \\ &+ K_{12}K_{34}\left(T_1 - T_4\right) \end{aligned} \tag{28}$$

$$\begin{aligned} g\left(\sigma\right) = &-\eta\sigma^2 + \left(K_{34} - \eta K_{12}\right)\sigma \\ &+ K_{12}K_{23} + K_{12}K_{34} + K_{23}K_{34} \end{aligned} \tag{29}$$

Solving for Q_1 in terms of σ and the parameters results in Eq. 27:

$$Q_1 = \frac{h\left(\sigma\right)}{g\left(\sigma\right)} \tag{30}$$

where $g(\sigma)$ is given above (Eq. 29) and $h(\sigma)$ is given by Eq. 31:

$$h(\sigma) = -\eta K_{12} T_1 \sigma^2 + K_{12} K_{34} T_1 \sigma + K_{12} K_{23} K_{34} (T_1 - T_4) \quad (31)$$

Eq. 20 and Eq. 27 are not equivalent, thus the three heat node/one engine and four heat node/one engine systems are not equivalent. However, one simplifying case where these models are equivalent is when the four heat node/one engine system has the following property:

$$K_{34} \to \infty \quad (32)$$

This can be shown by dividing both the numerator and denominator of Eq. 30 by K_{34}. As $K_{34} \to \infty$, this manipulation will make all terms except those containing K_{34} in this equation to go towards zero. The remaining terms produce Eq. 20.

IV. Parameter Estimation

The following example uses data from a California Energy Commission report from 1998 [7]. The report contains detailed heat input/work output data for the Moss Landing 7 and Hunters Point 3 units. This example focuses on these units to demonstrate parameter estimation using the network realizations introduced above.

Given a set a work input/heat output data, we seek a network realization that captures the behavior of the data. More complex network realizations help to fit the data better but also requires more parameters to estimate.

One method of finding estimates of these parameters is by using the least squares fit. If p represents the parameters to estimate and Q_i and W_i are the heat input and work output, respectively, for the i^{th} data point, the least squares method is given by Eq. 33.

$$\min_p \sum_i \left(\hat{W}(p, Q_i) - W_i \right)^2 \quad (33)$$

Using each set of network realizations and each set of data, parameters were estimated using the least squares fit. We assumed the initial temperature T_1 is an unknown constant for all realizations and the final heat node (T_2 in the two heat node/one engine, T_3 in three heat node/one engine model, T_4 in the four heat node/one engine model) has a temperature of 277.6 K. For each simulation, the parameters to estimate were T_1, all thermal links, and η (the isentropic efficiency of the engine).

Fig. 7 summarizes the residual using each of these models on the Moss Landing 7 unit. Fig. 8 summarizes the residuals using each of these models on the Hunters Point 3 unit. There is a vast improvement in fit going from a two heat node/one engine to a three heat node/one engine network realization. There is very little improvement in fit going to the four heat node/one engine from the three heat node/one engine network realization.

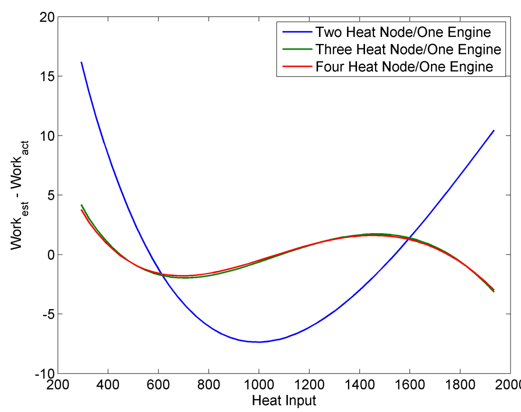

Fig. 7. Residuals for each model on Moss Landing 7 data

Table I and II summarize the results for both network realizations.

Parameter	2 Heat Node	3 Heat Node
K_{12}	N/A	41.50 MW/K
K_{23}	0.469 MW/K	0.745 MW/K
T_1	562.1 K	518.0 K
η	0.822	1.0
error^2	1958.9	118.0

TABLE I

Moss Landing 7 parameter estimates

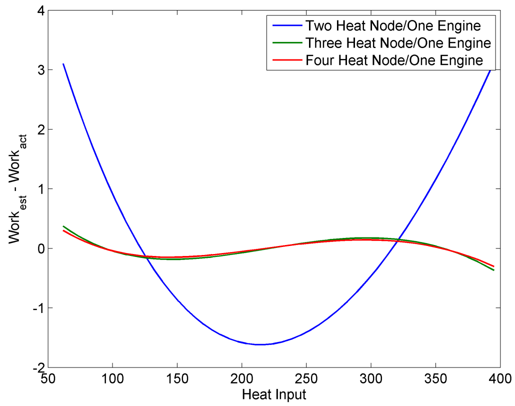

Fig. 8. Residuals for each model on Hunters Point 3 data

Parameter	3 Heat Node	4 Heat Node
K_{12}	N/A	10.61 MW/K
K_{23}	0.103 MW/K	0.218 MW/K
T_1	439.9 K	426.9 K
η	0.789	1.0
error^2	103.0	1.143

TABLE II
HUNTERS POINT 3 PARAMETER ESTIMATES

V. CONCLUSIONS

Three network realization were reviewed and equations for the heat input and work output as rational functions of entropy flux and the parameters for each realization were determined. These network realizations were used to fit plant data. Going from a two heat node/one engine to a three heat node/one engine network realization resulted in a significant fit improvement. The improvement in fit of going to a four heat node/one engine network realization is minimal.

REFERENCES

[1] Anees Attarwala. Optimal process operations: Valuing real options. Master's thesis, University of Notre Dame, 2008.

[2] Anees Attarwala and Jeffrey C. Kantor. On the integration of finance and process operations. In *AIChE Annual Meeting*, number 440a, 2007.

[3] S.J. Deng and S.S. Oren. Electricity derivatives and risk management. *Energy*, 31(6-7):940–953, 2006.

[4] Alexander Eydeland and Krzysztof Wolyniec. *Energy and Power Risk Management: New Developments in Modeling, Pricing and Hedging*. Wiley, 2003.

[5] Jeffrey C. Kantor and Patrick Mousaw. Models for the optimization and risk management of energy conversion networks. In *2009 American Control Conference*, number WeA20.6, pages 671–676, June 2009.

[6] Joel B. Klein. The use of heat rates in production cost modeling and market modeling. Technical Report HR-DESC.DOC 4/17/98, Electricity Analysis Office, California Energy Commission, 1998.

[7] Joel B. Klein. The use of heat rates in production cost modeling and market modeling. Technical report, California Energy Commission, 1998.

[8] Patrick Mousaw and Jeffrey Kantor. Valuation of flexibility in energy conversion networks. In *AIChE 2009 Paper 302e: Session "Control for Business and Financial Objectives"*. American Institute of Chemical Engineers, 2009.

[9] Patrick Mousaw and Jeffrey C. Kantor. Models for the optimization and risk management of energy conversion networks. In *Proceedings of the 2009 American Control Conference*, 2009.

[10] Patrick Mousaw and Jeffrey C. Kantor. Optimal hedging for flexible fuel energy conversion networks. In *2010 American Control Conference*, 2010.

[11] Dragana Pilipovic. *Energy Risk: Valuing and Managing Energy Derivatives*. McGraw-Hill, second edition edition, 2007.