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Optimal Control of Hybrid Electric Vehicles with Power Split and Torque Split Strategies: A Comparative Case Study

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Abstract— **In designing control strategies to optimize fuel consumption, driveability and other objectives for hybrid electric vehicles (HEVs), one can choose to use either power split or torque split as one of the control variables. While both approaches have been employed and documented, no systematic study has been reported that illuminates what implications this choice might have in terms of HEV performance, system robustness, and control strategy design and implementation complexity. This work aims to develop a case study that explores this degree of design freedom and to quantify any differences that this control design selection might impart on a given HEV architecture. Using a validated HEV model, we will derive optimal operating strategies using dynamic programming for two cases: one uses power split and other torque split. Performance metrics of fuel consumption as well as the computational complexity associated with the two different strategies will be assessed.**

I. INTRODUCTION

Hybrid Electric Vehicles (HEVs), which combine an Internal Combustion Engine (ICE) and one or more Electric Motors (EMs), have drawn much attention in the past two decades [1], [2], [3], [4]. The interest and success of HEVs have been driven largely by concerns about oil savings and air quality. Since the first HEV was introduced to the market, up to 50% improvement in fuel economy and 33% reduction in CO has been reported [1]. Increased fuel efficiency and enhanced potential to use alternative fuels of HEV also contribute to significant reductions in emissions of carbon dioxide (CO_2, a key greenhouse gas), thereby addressing the concern of global warming [5].

Power management, whose functions include coordinating multiple power sources to minimize fuel consumption while satisfying the driver's demand, is a key element in HEVs development [6], [7]. Most power management strategies for HEVs are based on heuristic techniques such as rule-based methods, fuzzy logic, or neural networks [2], [8], [6], [9]. Heuristic techniques require intensive tuning and therefore their development is time consuming and vehicle dependent. Optimization based approach, on the other hand, is a systematic design approach which has also been developed. Using HEV models, optimization techniques help to determine the proper power split between the two energy sources through a systematic development process. Optimization based approaches can be characterized into three groups: static optimization methods [10], analytical dynamic optimization methods [11], [12] and numerical dynamic optimization methods [13], [15]. In static optimization methods, electric power is generally expressed as an equivalent steady-state fuel rate in order to minimize an overall energy cost. Dynamic optimization approaches consider the dynamic nature of the system components and formulate the energy management problem as a nonlinear optimization problem.

An analytical solution to such a nonlinear constrained optimization problem does not exist in general. However, approximations of the original optimization problem based on the minimum principle [11], [12] can result in a closed-form solution. The computation burden is dramatically reduced by approximation and thus it is possible for realtime implementation. For most cases, the nonlinear constrained optimization problem is solved by using numerical solvers, such as Dynamic Programming (DP) [18].

DP is commonly used for finding an optimal trajectory of nonlinear dynamic system over a given time period [14], [15]. When analytical closed form solution is not available, this method can be used to minimize the performance index in the presence of hard or soft constraints of the states and/or inputs. DP requires gridding of the state and time variables, and thus the optimal trajectory depends on the discretization of the inputs and states on time and value. Even though DP has several limitations, such as the well-known "curse of dimensionality", the need for knowing the entire drive cycle and the non-causal resulting controller, DP has been found to be an extremely useful tool to calculate a benchmark optimal control strategy and assess achievable performance of the HEV.

However, both the optimality and computational cost of DP are directly related to the number of grid points. Therefore it is worthwhile to find out the relationship between simulation accuracy and grid density. In addition, a different control variable may have different sensitivity over grid density. This paper is concerned with the sensitivity of the DP-based optimization strategy to the selection of control variables.

For HEVs, one can choose to use either power split or torque split as one of the control variables, where the former focuses on the power as the direct control variable while the latter used the torque in the optimization. While the two strategies could be equivalent in ideal situation (such as infinitely dense grid), they can lead to different results due to the following reasons: First, the choice of different control variables could lead to different constraint formulation. Second the finite grid points in state and input

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variables define that the numerical solution can vary for different input variables. Although both approaches have been employed and documented [21], [20], no systematic study has been reported that illuminates what implications this choice might have in terms of HEV performance, system robustness, and control strategy design and implementation complexity. This work aims to develop a case study that explores this degree of design freedom and to quantify any differences that this control design selection might impart on a given HEV architecture using DP based optimization.

The remainder of this paper is organized as follows: Section II introduces the HEVs model and formulates the problem. In Section III, DP implementation issues are discussed. Section IV presents simulation results and compares the power split and torque split strategies for different drive cycles. The conclusions are given in Section V.

II. HEV MODELING AND PROBLEM FORMULATION

To compare the differences between power split and torque split strategies of HEVs optimal control in terms of fuel economy performance, two HEV models used for DP optimization are described in this section.

A. HEV Models

In this study, we adopted the parallel HEV model of [22] and modified it for our special purpose. The given HEV model is a discrete-time model and constructed using a quasi-static approach, which takes battery state of charge (SOC) as the only state [2]. The vehicle speed and acceleration are included as part of the model to facilitate the implementation of DP method over a given drive cycle, and thus the model function has exogenous time varying input. The model can be found and downloaded at [23] and a more detailed description can be found in [21]. This paper will not cover the complete process of the vehicle modeling and only the modifications made by the authors to facilitate this case study will be discussed.

Several modifications are made to the original HEV model. The control variable for power management is changed to be the motor power P_m demand for power split strategy and the motor torque demand T_m for torque split strategy. Gear ratio K is added as an optimization variable. Consequently, the new models used for DP optimization can be described as following

$$x_{k+1} = f_k(x_k, u_k, K_k) + x_k, \qquad k = 0, 1, ..., N-1 \quad (1)$$

$$u_k = \begin{cases} P_{mk}, & \text{for power split strategy} \\ T_{mk}, & \text{for torque split strategy} \end{cases} \quad (2)$$

where control variables P_{mk}, T_{mk} and K_k have been defined above and state x_k is the battery SOC. The subscript k in f_k indicates that the model function is time-variant as explained before. All the assumptions made for the original HEV model are still applicable.

In addition, instead of using an affine Willans approximation (a linear engine modeling method that is used in [21]), an engine fuel efficiency map (scaled from a commercial engine

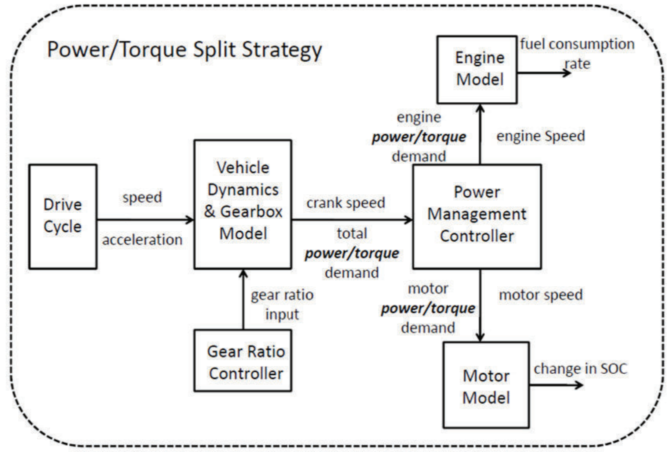

Fig. 1. HEV models used for the evaluation of power split and torque split strategies fuel consumption difference

model) is incorporated into the vehicle model to make the simulations more realistic and reliable. For this particular HEV designed for urban use, the engine and battery are sized such that the engine is never used for charging the battery, and the electric energy in battery is recovered from the regenerative braking system.

At each second, the total power demand P_{tot} and the torque demand T_{tot} on the powertrain are calculated for power split and torque split strategies respectively using the vehicle dynamics model. For the power split strategy, the DP determines the optimal motor power P_m, then the engine power demand P_e and engine torque demand T_e are calculated as

$$P_e = P_{tot} - P_m \quad (3)$$

$$T_e = P_e / \omega_e \quad (4)$$

where ω_e is the rotational speed of engine.

On the other hand, if the DP calculates the optimal motor torque T_m, the engine torque demand T_e for torque split strategy is directly computed in the following way

$$T_e = T_{tot} - T_m \quad (5)$$

Given engine torque input T_e and engine speed ω_e, the fuel consumption rate can therefore be computed by using the engine map.

A schematic of the HEV models and power management strategy is given for both power split and torque split strategies in Fig. 1, models are built based on the same vehicle architecture, and the only difference is the control variable used for power management.

B. DP Formulation

The DP optimization problem is formulated as the minimization of total fuel consumption for the given HEV model over a whole drive cycle and thus the cost function is defined as

$$J = \sum_{k=0}^{N-1} (\dot{m}_k \cdot T_s) \quad (6)$$

where $\dot{m}_k$ is the fuel consumption rate computed by the engine map and T_s is the time step. In this study, T_s is set to be one second.

A deterministic DP algorithm dpm is used to evaluate performance of both power management strategies. The dpm function is a generic Matlab based DP algorithm developed by Olle Sundström and Lino Guzzella at Swiss Federal Institute of Technology Zurich (ETH Zurich). A introductory tutorial of dpm function can be found in [22] and more related documents are available at [23].

Then the cost function of (6) is minimized subject to the dynamic equation (1), (2) and the following constraints

$$P_m \in \begin{cases} (0, \quad P_{max}), & \text{for } P_{tot} \geq 0 \\ (-P_{max}, 0), & \text{for } P_{tot} < 0 \end{cases} \tag{7}$$

$$T_m \in \begin{cases} (0, \quad T_{max}), & \text{for } T_{tot} \geq 0 \\ (-T_{max}, 0), & \text{for } T_{tot} < 0 \end{cases} \tag{8}$$

$$K \in (K_{min},\ K_{max}) \tag{9}$$

$$SOC \in (SOC_{min},\ SOC_{max}) \tag{10}$$

where P_{max} adn T_{max} are the maximum power and torque output of the motor respectively, K_{min}, K_{max} are gear ratio limits and SOC_{min}, SOC_{max} are the constraints of the battery SOC.

In dpm function, the grid points of inputs and states are evenly distributed within corresponding constraints and linear interpolation is used during forward simulation [22].

For the DP calculation, the gear ratio is assume to change continuously (by using linear interpolation) within the constraints so that the gearbox can be treated to be a continuous variable transmission (CVT). Since we only care about the sensitivity of DP algorithm regarding power management strategy, results will not be affected by using a CVT as the transmission. The values of gear ratio limits are tuned according to the vehicle size.

The values of battery SOC constraints are usually provided by manufacturers. Either empty or full charge would affect the battery lifecycle or damage the battery so that maintaining the SOC within the limits is crucial to extend the service lifetime as well as assure the performance of battery. In this study the SOC boundaries are set to be (0.4, 0.7). The upper limit 0.7 does not affect the results because the SOC never reaches 0.7.

It should be noted that P_m and T_m are positive during the acceleration to provide driving power and are negative during deceleration to provide regenerative braking. A symmetric electric motor efficiency map is used for the simulations. The HEV models are configured so that the sign of control variable P_m and T_m changes automatically in response to the vehicle driving condition at every time instant.

III. SENSITIVITY AND IMPLEMENTATION ISSUES OF DP

As discussed in Section I, DP optimization is a grid based numerical approach and the grid points of input and state have to be limited to contain the computational load. Consequently, the selection of control variables may cause differences in the results. Understanding the sensitivity of the grid-based DP solution to control variable choice is therefore of interest. For the implementation of DP in HEVs optimal control, the question of how the power split and torque split strategies affect the fuel economy performance on different drive cycles needs to be investigated, and the answer could shed light on control architecture design and implementation.

TABLE I
COMPUTATIONAL TIME WITH DIFFERENT GRID DENSITY

grid points (n)	11	21	31	41	51	101	501	1001
time (s)	46	57	68	83	92	167	1274	2420

The power and torque output of electric motor has the relationship

$$P_m = T_m \cdot \omega_m \tag{11}$$

where ω_m is the rotational speed of motor. Then because of the time-variant motor speed, power split and torque split strategies would search around different operating points of the motor even given the same resolution of grid points.

Because of the equivalent physical properties in both HEV models, power split and torque split strategies are expected to have the same performance with sufficient grid density. However the computational time is usually restricted and thus the number of grid points is constrained. Hence it is crucial to quantify the relationship between the fuel economy performance, the selection of power management strategies and the DP grid density so that proper decision can be made to minimize the negative impact of finite grid.

Before comparing the difference between power split and torque split strategies, the sensitivity of DP grid points is first explored by evaluating the fuel consumptions on a US city drive cycle with respect to the grid density n ($n = \{11, 21, 31, 41, 51, 101, 501, 1001\}$). The fuel consumption computed at $n = 1001$ appears to be the limit of fuel consumption on a given drive cycle so that M_{1001} is used as benchmark M_∞ throughout the study. Therefore the relative fuel consumption is defined as

$$rM_{fuel} = \frac{M_{fuel}}{M_\infty} \cdot 100\% \tag{12}$$

where the M_{fuel} is the absolute value of fuel consumption of a DP solution for a give grid selection.

The representative results along with the drive cycle are shown in Fig. 2. The corresponding computational time (recorded on a 64-bit Intel Pentium 4 3.4GHz with 2.0 GB RAM) are shown in Table I and Fig. 3. It is clear that the value of relative fuel consumption rM_{fuel} reduces when grid density n increases from 11 to 101. From $n = 101$ to $n = 1001$, the fuel consumptions remain close and thus the curves become flat. The results indicate that the DP algorithm has a high sensitivity with respect to grid points when $n \leq 101$ and the computational time is approximately proportional to the number of grid points.

Based on this result, the fuel economy performance of the HEV models with grid density $n = \{11, 21, 31, 41, 51, 101\}$

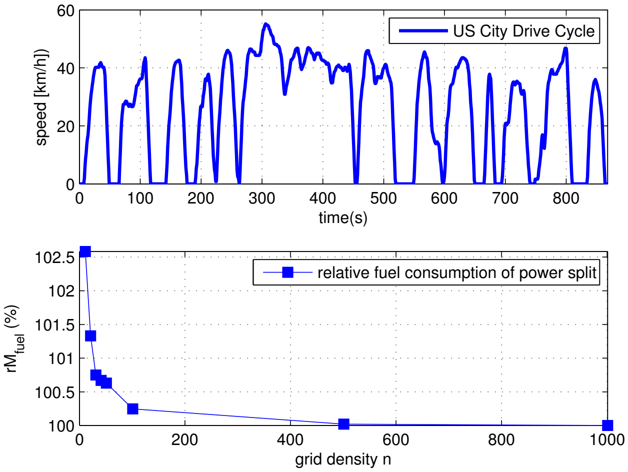

Fig. 2. US city drive cycle and its relative fuel consumptions

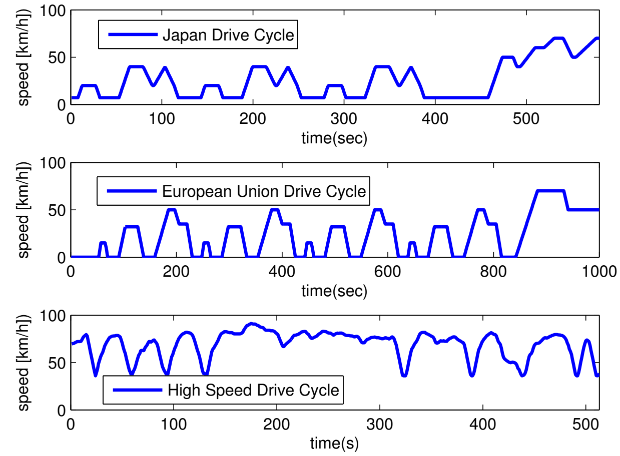

Fig. 4. Additional drive cycles examined for the study

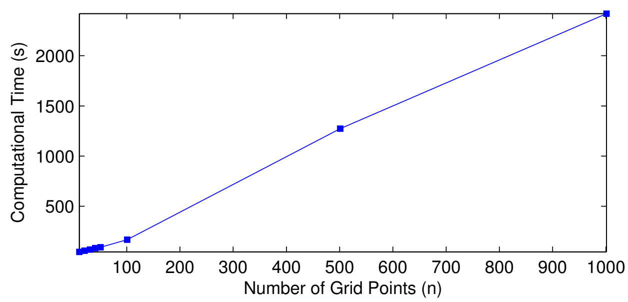

Fig. 3. Computational time comparison on US city drive cycle with the power split strategy

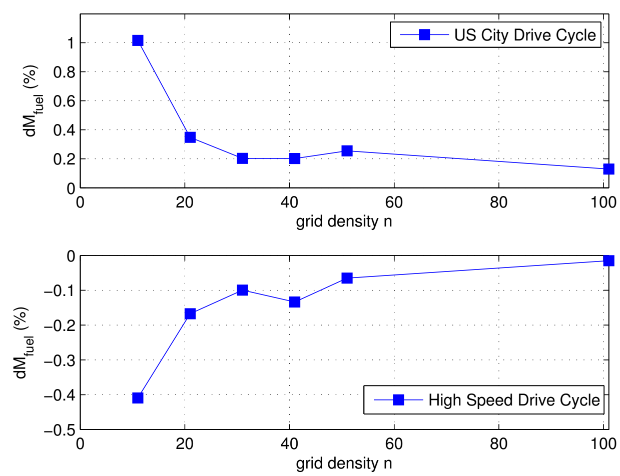

Fig. 5. The percentage difference of fuel consumption between power split and torque strategies over given drive cycles

are simulated with both power split and torque split strategies and the results are analyzed in the next section.

IV. RESULTS AND DISCUSSIONS

To understand the sensitivities of the DP results to grid selection and their dependence to the choice of the control variables, we considered several representative drive cycles (shown in Fig. 4) and define the following quantity (13) to measure the difference between the power split and torque split,

$$dM_{fuel} = \frac{(M_{ps} - M_{ts})}{M_{\infty}} \cdot 100\% \qquad (13)$$

where dM_{fuel} is the percentage difference of the fuel consumptions, M_{ps} is the fuel consumption of the power split strategy and M_{ts} is the fuel consumption of the torque split strategy on a given drive cycle.

The Japan drive cycle and European Union drive cycle shown in Fig. 4 give similar results compared to the US city drive cycle in Fig. 2 in terms of fuel consumption difference. Hence we decide to take the US city drive cycle and high speed drive cycle (which is scaled from the city drive cycle) shown in Fig. 4 for our case study since the results are more comparative.

The results of both drive cycles are shown in Fig. 5. It is also noteworthy to see that the torque split strategy gives a better fuel economy performance on the US city drive cycle while the power split strategy wins over the high speed drive cycle.

In order to explain the difference in the fuel consumptions of the two strategies, we choose to analyze the simulation results at $n = 11$. The resulting SOC trajectories for both strategies over two drive cycles are plotted and shown in Fig. 6. In both cases, the starting value of SOC is set to be the lower boundary (which is 0.4 in this study) and final state constraints are not implemented. This setting gives a more obvious performance difference between the two strategies and allows us to make a clearer comparison. As a result, both the SOC trajectories converge to 0.4 at the end of the drive cycle so that the optimal fuel economy results are achieved. In other words, the energy recuperated from the regenerative braking system is fully utilized over the entire drive cycle.

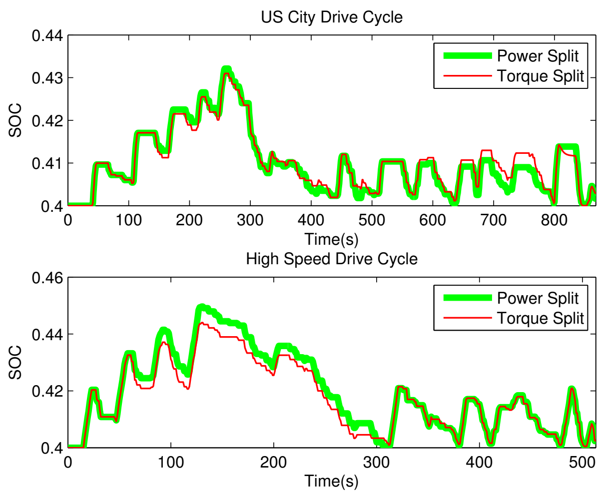

Fig. 6. State of Charge comparison with the grid density $n = 11$

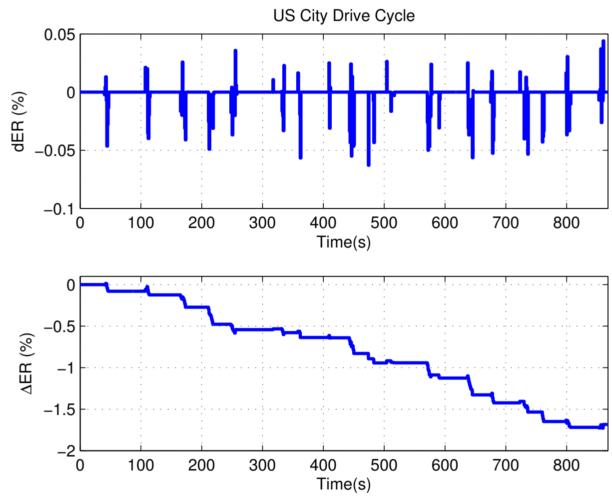

Fig. 7. Energy recuperation comparison with the grid density $n = 11$

The SOC trajectories show slight deviations between the two power management strategies over both drive cycles and the primary physical explanation can be found in the differences in regenerative braking energy recuperation (engine is not used for battery charging as discussed in Section II). Therefore the following quantities are defined to capture the energy recuperation effectiveness,

$$\Delta SOC = x_{k+1} - x_k \tag{14}$$

$$ER = \begin{cases} \Delta SOC, & \text{for } \Delta SOC \geq 0 \\ 0, & \text{for } \Delta SOC < 0 \end{cases} \tag{15}$$

$$dER = \frac{ER_{ps} - ER_{ts}}{0.4} \cdot 100\% \tag{16}$$

$$\Delta ER = \sum_{k=0}^{N-1} (dER_k \cdot T_s) \tag{17}$$

where ER is the instant energy recuperation, subscripts ps and ts represent power split strategy and torque split strategy respectively, ΔSOC is the change of SOC within one time step T_s, dER is the instant energy recuperation difference between two power management strategies, 0.4 is the starting value of SOC and ΔER is total difference of energy recuperation over a drive cycle.

The energy recuperation difference are shown in Fig. 7 and Fig. 8. As expected, the results are consistent with the plots in Fig. 6 that more energy recuperation reduces engine load and therefore leads to better fuel economy.

In order to further explain the energy recuperation difference from the perspective of DP algorithm, the control variables and their numerical distributions are analyzed and shown in Table II as well as in Fig. 9. As defined in equation (12), rM_{fuel} represents the relative fuel consumption compared to the bench mark M_∞. Mean values and standard deviations of u/u_{max}, where u is control variable and u_{max} is the corresponding maximum value, are tabulated.

Based on the definition of standard deviation (STD), a larger STD means a wider spread of data. As shown in Fig. 9,

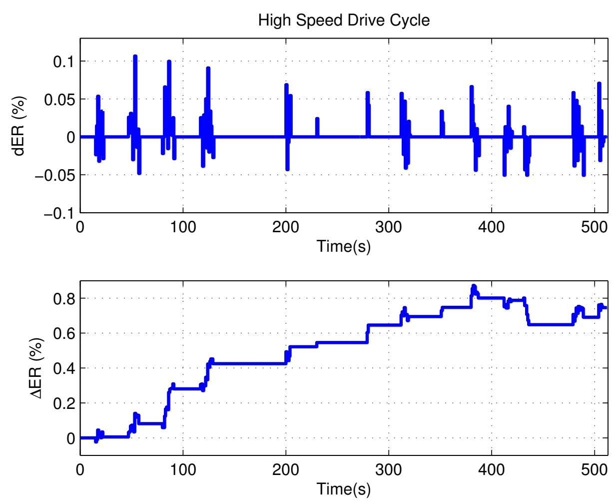

Fig. 8. Energy recuperation comparison with the grid density $n = 11$

the occurance frequencies of control variables corresponding to each grid point are normalized into the range of (0, 1). It is clear that the values of the control variable of torque split strategy are distributed more evenly than power split strategy over the US city drive cycle, while for the high speed drive cycle, the opposite result is observed. Accordingly, the results indicate that a wider functional searching range of control variables results in a better fuel economy as shown in Table II. Consequently, a conclusion can be drawn that neither power split nor torque split is guaranteed to be a better power management strategy and the selection of control variables should be determined in association with the characteristics of the drive cycles.

In addition, a preliminary study has been conducted on the sensitivity of gear ratio selection. The results show that even though the absolute value of fuel consumption varies with the change of the gear ratio limits, the trajectories resulting from DP optimization has almost the same sensitivity to control

TABLE II

NUMERICAL RESULTS COMPARISON OF POWER SPLIT (PS) AND TORQUE SPLIT (TS) STRATEGIES WITH THE GRID DENSITY $n = 11$

		PS(%)	TS(%)	PS-TS(%)
US City	rM_{fuel}	102.59	101.57	1.02
	mean(u/u_{max})	22.89	30.18	-7.29
Drive Cycle	std(u/u_{max})	17.98	22.08	-4.10
High Speed	rM_{fuel}	102.08	102.49	-0.41
	mean(u/u_{max})	39.68	33.90	5.78
Drive Cycle	std(u/u_{max})	28.35	24.90	3.45

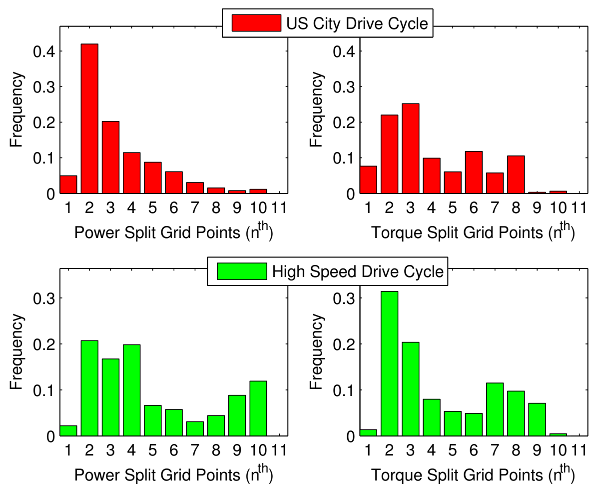

Fig. 9. Comparison of control variables distribution

variable selection for different gear ratios. However, if we keep the same density of grid points (i.e. $n = 11$) and change the grid distribution, a different result is expected.

V. CONCLUSIONS

A comparative study of HEVs optimal control with power split and torque split strategies is discussed in this paper. The difference is quantified by comparing simulation results of a representative HEV design for different drive cycles, where the fuel consumptions on given drive cycles are evaluated. The results show that the maximum difference in fuel consumptions is about 1% and the difference converges to zero as the grid density is increased. This number may suggest that the difference between the power split and torque split is inconsequential. However, our analysis shows that this difference is well associated with the distribution of the control variables. For a fixed number of grid points, selecting the variable that spreads out over the entire range as the control variable leads to better results. For the case study performed here, we concluded that the high speed drive cycle benefits from power split strategy while the low speed drive cycle could gain from the torque split strategy. Further analysis to extend the results to other HEV configurations will be performed to fully understand the two power management strategies and their implications on other performance indices such as drivability.

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