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A Connectivity Preserving Containment Control Strategy for a Network of Single Integrator Agents

Amir Ajorlou, Ahmadreza Momeni, and Amir G. Aghdam

Abstract— **This paper is concerned with the distributed control strategies for a group of single integrator agents with static leaders. The information flow graph of the underlying multi-agent network is assumed to be static, undirected, and connected. A bounded control law is proposed which is shown to have strict connectivity preservation property. The containment of the followers by leaders is subsequently proved by using the LaSalle invariance principle. The effectiveness of the proposed strategy is demonstrated by simulations.**

I. Introduction

Cooperative control of a network of autonomous agents has been a topic of considerable research interest in recent years [1], [2], [3]. In this line of research, it is aimed to achieve a global objective, such as consensus, flocking, formation, and containment, by applying an appropriate distributed control strategy [2], [4], [5].

One of the important issues in cooperative control of a multi-agent system is to maintain network connectivity during the operation. This problem has been thoroughly studied in the literature for various global objectives. As an effective control design methodology for this type of system, one can search for potential functions or nonlinear weights that tend to infinity when an edge in the network is about to lose connectivity [6], [7], [8], [9]. However, this method may not be feasible in general, as the actuators can only handle finite force or torque. To remedy this shortcoming, bounded distributed connectivity preserving control strategies are proposed in [10], [11].

In the containment problem, it is desired that a subset of the agents, called followers, converge to the convex hull formed by the rest of the agents, called leaders, which could be stationary or moving. A hybrid Stop-Go policy is presented in [5], which is shown to be convergent if the leaders are stationary and the interaction graph is connected. The work [12] proposes a containment control strategy for unicycle agents, where the leaders are supposed to converge to a predefined formation.

In this work, bounded connectivity preserving control laws are presented for multi-agent systems in order to achieve containment. It is assumed that the system consists of single integrator agents with static leaders. The information exchange between the agents is constrained, and is specified mathematically by an information flow graph, which is assumed to be static, undirected, and connected. The results from the authors' previous work on connectivity preservation of single integrator agents is first used to show the strict connectivity preservation for the proposed control law. LaSalle invariance principle is subsequently used to prove the convergence of the followers to the convex hull of the leaders. The proposed control strategy is verified by simulations.

The authors are with the Department of Electrical and Computer Engineering, Concordia University, 1455 De Maisonneuve Blvd. W., Montréal, Québec, H3G 1M8, Canada {a_ajor, a_momeni, aghdam}@ece.concordia.ca

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II. Problem Formulation

Consider a set of n single integrator agents in a plane with the dynamics of the form

$$\dot{q}_i(t) = u_i \tag{1}$$

where $q_i(t)$ denotes the position of agent i in the plane at time t. Denote with $G = (V, E)$ the information flow graph, where $V = \{1, \dots, n\}$ is the set of vertices, and $E \subset V \times V$ is the set of the edges. It is assumed that the information flow graph G is connected and undirected, and that each agent is only allowed to incorporate the relative position of its neighbors in its control law. Denote the set of the neighbors of agent i in G with $N_i(G)$, and the degree of agent i in G with $d_i(G)$. Two agents i and j are said to be in the strict connectivity range if $\|q_i - q_j\| < d$, for a pre-specified positive real number d, where $\|\cdot\|$ denotes the Euclidean norm. It is assumed that all the agents in $N_i(G)$ are initially located in the strict connectivity range of agent i. It is also assumed that each agent either belongs to the set of leaders L or the set of followers F. In this paper only the case of static leaders is considered, i.e., the case when $u_i \equiv 0$, for $i \in L$; the case of moving leaders is left for future work.

The objective is to design a control strategy such that

- the followers converge to the convex hull of the leaders, i.e.
$$\lim_{t\to\infty} q_i(t) \in \mathrm{Conv}\{q_j | j \in L\}, \quad \forall i \in F \tag{2}$$
- strict connectivity is preserved in the sense that if $\|q_i(0) - q_j(0)\| < d$ for all $(i, j) \in E$, then $\|q_i(t) - q_j(t)\| < d$, for all $(i, j) \in E$ and all $t \geq 0$.

III. Main Results

For every agent i, define

$$\pi_i(t) := \frac{1}{2} \prod_{j\in N_i(G)} (d^2 - \|q_i(t) - q_j(t)\|^2) \tag{3}$$

Consider the control inputs of the form

$$\dot{q}_i = \frac{\partial \pi_i}{\partial q_i}, \quad i \in F \tag{4}$$

The aim of this section is to show that under the control law given by (4), the followers converge to the convex hull of the leaders while preserving strict connectivity as defined earlier. Define

$$\sigma_i(t) := \frac{1}{2} \sum_{j \in N_i(G)} \|q_i(t) - q_j(t)\|^2 \tag{5}$$

Let the notion of *connectivity preservation* be introduced as an alternative to the strict connectivity preservation. A control strategy is called connectivity preserving, if $\|q_i(0) - q_j(0)\| \leq d$ for all $(i,j) \in E$ implies that $\|q_i(t) - q_j(t)\| \leq d$, for all $(i,j) \in E$ and all $t \geq 0$.

Notation 1: For any given function $h(x,y)$, by $\frac{\partial h}{\partial y}h(x,0)$ we mean $\frac{\partial h}{\partial y}h(x,y)_{|y=0}$ (and similarly, $\frac{\partial h}{\partial x}h(0,y) = \frac{\partial h}{\partial x}h(x,y)_{|x=0}$). Notice that while this may be considered standard notation, it is emphasized here for the sake of clarity, and to avoid possible confusion.

To show the strict connectivity preservation for the proposed control law, the following theorem is borrowed from [11].

Theorem 1: Consider a set of n agents in the plane with the dynamics of the form (1), and control inputs $u_i = -\frac{\partial h_i(\sigma_i, \pi_i)}{\partial q_i}$. If h_i satisfies the following conditions

$$\frac{\partial h_i}{\partial \sigma_i}(\sigma_i, 0) = 0, \; \frac{\partial h_i}{\partial \pi_i}(\sigma_i, 0) < 0, \qquad \forall \sigma_i \in \mathbb{R}^+ \tag{6}$$

then the resultant control law is connectivity preserving.

By choosing $h_i = -\pi_i$ for $i \in F$, the control law given in (4) satisfies the statement of Theorem 1 and also clearly meets the conditions given in (6). However, the connectivity preservation cannot be deduced directly from Theorem 1 since the control inputs of the leaders ($u_i \equiv 0$ for $i \in L$) do not satisfy the conditions given in (6). It is shown in the sequel how by using a simple trick connectivity preservation is ensured even though conditions (6) hold only for the followers.

Construct a new graph $\bar{G}$ from G as follows. For any $i \in L$, consider two *virtual agents* i_1 and i_2 initially located at a distance d from each other and from i. Add these two new vertices to $V(G)$, and all possible edges between i, i_1, and i_2 to $E(G)$. Choose $h_i = -\pi_i$, $h_{i_1} = -\pi_{i_1}$, and $h_{i_2} = -\pi_{i_2}$ (which clearly satisfy conditions (6), and the statement of Theorem 1). Then connectivity preservation is guaranteed for $\bar{G}$, according to Theorem 1. To deduce connectivity preservation for G, it suffices to show that static leaders of G also stay fixed in $\bar{G}$. The control input of agent $i \in L$ in $\bar{G}$ (and similarly those of i_1 and i_2) can be written as

$$\begin{aligned} \dot{q}_i &= \frac{\partial \pi_i}{\partial q_i} \\ &= \sum_{j \in N_i(\bar{G})} \pi_{ij}(q_i - q_j) \end{aligned} \tag{7}$$

where π_{ij} for $i \in V(\bar{G})$ is defined as

$$\pi_{ij}(t) := \prod_{\substack{k \in N_i(\bar{G}) \\ k \neq j}} (d^2 - \|q_i(t) - q_k(t)\|^2) \tag{8}$$

Since by assumption i, i_1, and i_2 are initially located at a distance d from each other, all the coefficients in (7) are initially 0 (similarly for i_1 and i_2), and hence these three agents stay fixed at a distance d from each other at all times. Therefore, connectivity preservation in the case of static leaders for G can be deduced from that for $\bar{G}$.

Using this connectivity preservation property, it is now desired to show the strict connectivity preservation as well as the convergence of the followers to the convex hull of the leaders under the control law given in (4). Consider the function $\pi(t)$ defined by

$$\pi(t) = \prod_{\substack{(i,j) \in E(G) \\ i<j}} (d^2 - \|q_i(t) - q_j(t)\|^2) \tag{9}$$

Note that

$$\begin{aligned} \dot{\pi} &= \sum_{i \in F} \dot{q}_i^T \frac{\partial \pi}{\partial q_i} \\ &= \sum_{i \in F} \dot{q}_i^T \frac{\partial \pi_i}{\partial q_i} \bar{\pi}_i \\ &= \sum_{i \in F} \bar{\pi}_i \|\dot{q}_i\|^2 \end{aligned} \tag{10}$$

where $\bar{\pi}_i$ is the product of those terms in π which do not appear in π_i (i.e. $\pi = \pi_i \bar{\pi}_i$). It results from connectivity preservation that $\bar{\pi}_i \geq 0$ for $t \geq 0$, and hence $\dot{\pi} \geq 0$ for $t \geq 0$, implying that π is a non-decreasing function of time. To prove the strict connectivity preservation property, it is enough to note that if $\pi(0) > 0$, then $\pi(t) > 0$ for all $t \geq 0$ since π is a non-decreasing function of time.

To show the convergence of the followers to the convex hull of the leaders, first note that $0 < \pi < d^{|E(G)|}$ and $\dot{\pi} \geq 0$. Therefore, using LaSalle's invariance principle one can conclude the convergence of the agents to the largest invariant set in $\dot{\pi} = 0$ (which is $\dot{q}_i = 0$ for all $i \in F$, i.e. an equilibrium of the system), on noting that the coefficients $\bar{\pi}_i$'s are positive due to strict connectivity preservation. Therefore, it suffices to show that at equilibrium, there exists at least one leader at each vertex of the convex hull of the agents. Assume that there is a follower i at a vertex of the convex hull of the agents coinciding with no leader. It is implied from $\dot{q}_i = 0$ that $\sum_{j \in N_i(G)} \pi_{ij}(q_i - q_j) = 0$. Solving this for q_i, one arrive at

$$q_i = \sum_{j \in N_i(G)} \alpha_{ij} q_j \tag{11}$$

where $\alpha_{ij} = \frac{\pi_{ij}}{\sum_{j \in N_i(G)} \pi_{ij}}$. Clearly, $0 < \alpha_{ij} < 1$ (from strict connectivity preservation) and $\sum_{j \in N_i(G)} \alpha_{ij} = 1$. This means that q_i is in the convex hull of its neighbors. Thus, for q_i to be at a vertex of the convex hull of the agents, it should coincide with all of its neighbors in $N_i(G)$. Repeating the same argument, one can conclude that q_i should coincide with the agents reachable from i in G. This is a contradiction

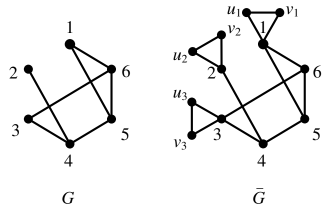

Fig. 1: The information flow graph G along with $\bar{G}$

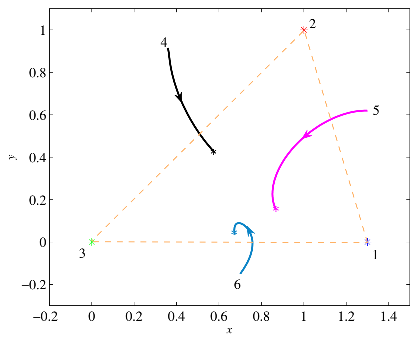

Fig. 2: The agents' planar motion

as every leader is reachable from i since G is connected. This completes the proof of the convergence of the followers to the convex hull of the leaders.

IV. Simulation Results

Consider a team of 3 static leaders and 3 followers, with the information flow graph given in Fig. 1. Let d be equal to 1, and the initial position of each agent be marked by its index, as shown in Fig. 2. The graph $\bar{G}$ (obtained by adding the virtual agents to G), is also depicted in Fig. 1.

The motion of the agents, the relative distances, and the control inputs of the followers are depicted in Figs. 2, 3, and 4, respectively. As can be seen from these figures, the followers converge to the triangle created by the static leaders while preserving the strict connectivity.

V. Conclusions

This work presents a distributed control strategy for the containment problem. The important characteristics of the proposed control law are boundedness and connectivity preservation. It is assumed that the followers have single integrator dynamics, and that the leaders are static. The convergence of the followers to the convex hull of the leaders is shown using the LaSalle invariance principle, based on the strict connectivity preservation property of the proposed control strategy.

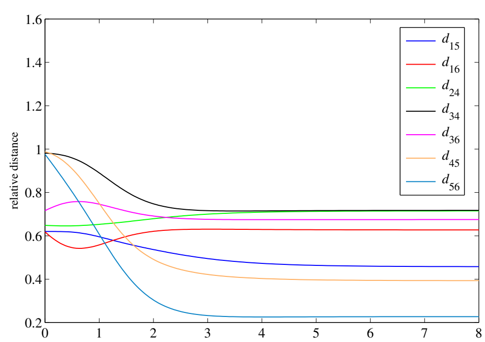

Fig. 3: The relative distances

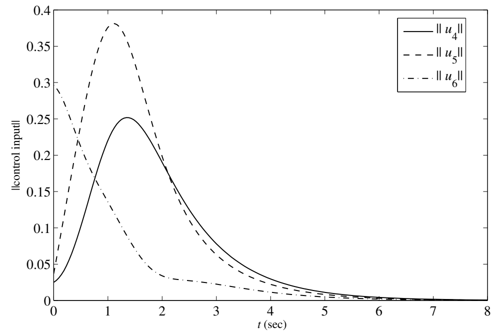

Fig. 4: The norms of the control inputs

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