2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

A simple PI^{2}D output feedback controller for the Permanent Magnet Synchronous Motor

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Abstract— **In this paper it is presented a PI control scheme with guaranteed stability properties for the Permanent Magnet Synchronous Motor (PMSM). The proposed controller solves the speed tracking control problem and is of the output–feedback type, since it is assumed that only stator currents and the rotor position are available for measurement, i.e. it is avoided the use of (noisy) speed sensors. The algorithm is equipped with an adaptation mechanism that compensates a (constant) unknown load torque. From a theoretical perspective its more valuable characteristic lies in the fact that Uniform Global Exponential Stability (UGES) of the desired operation regime is achieved. The usefulness of the proposed scheme is evaluated in a numerical setting.**

I. INTRODUCTION

Control of electrical machines is a topic that has been deeply studied both from a practical and from a theoretical perspective since a long time ago [1], [2]. As a result of this research, currently it is possible to find in the literature several control design methodologies under which different alternatives have been proposed with the purpose to improve the performance achieved by these devices. On the one hand, the Field Oriented Control (FOC) is the preferred scheme in industrial applications, due to its structure based on nested PI loops [3], while from the control theory community the proposals that have captured more the attention of practitioners are those obtained from a passivity–based perspective [4] and from a feedback linearization approach [5].

Some efforts have been devoted to reduce the gap between the theoretical and the practical approaches, namely: In [6] it has been shown the existence of a downward compatibility between a passivity–based control for induction motors and its corresponding FOC. Some PI tuning rules for the FOC of induction motors have been proposed in [7] exploiting its passivity–properties, and in [8] a feedback linearization controller was proposed exploiting the stability properties of the FOC for induction motors. Nevertheless, the goal of designing PI controllers with proved stability properties is a topic that has not been received the attention that it deserves.

In this paper the problem of designing a PI control scheme for speed tracking with guaranteed stability properties for the Permanent Magnet Synchronous Motor (PMSM) is approached. The motivation for dealing with this control problem comes, first, from the remarkable features exhibited by this machine concerning dynamic behavior and, second, to the fact that in spite of the very valuable attempts reported from the control theory community [9], [10], [11], it is the author's belief that the structural advantages of this machine can be further exploited.

The controller presented in this paper belongs to the class of output–feedback schemes since it considers that only the stator currents and the rotor position are available for measurement. Its structure was inspired by the controller reported for robot manipulators presented in [12] and from a stability point of view its more valuable characteristic lies in the fact that Uniform Global Exponential Stability (UGES) of the desired operation regime is achieved. The attractiveness of the contribution from a practical viewpoint appears from the elimination of (noisy) speed sensors and the necessity for knowing the load torque, which is viewed as a constant unknown perturbation that is compensated by including an adaptation mechanism in the controller. The usefulness of the proposed scheme is evaluated in a numerical setting.

The rest of the paper is organized as follows: In Section II, the considered model for the PMSM together with the control problem formulation is presented. The proposed output feedback controller is introduced in Section III with a brief discussion about its structure while the formal proof of its stability properties is developed in Section IV. The aforementioned numerical evaluation is carried out in Section V. Section VI is devoted to state some concluding remarks.

II. PROBLEM FORMULATION

In this section, after presenting the model considered for the PMSM and identifying the complications that arise for its control, the problem solved in this paper is presented.

A. PMSM model

Consider the well–known dq model of the non–salient PMSM given by the system [5]

$$\begin{aligned} \mathbf{L}\frac{di}{dt} &= -\mathbf{R}_s i - \omega\Phi\mathbf{J}\rho - \omega\mathbf{J}\mathbf{L}i + V \\ J\dot{\omega} &= n_p\Phi i_2 - \tau_L \\ \dot{\theta} &= \omega \end{aligned}$$

where $i = [i_d, i_q]^T$ and $V = [u_d, u_q]^T$ are the stator currents and voltages, respectively, θ and ω are the mechanical (position and speed) variables, τ_L is the load torque, $\mathbf{L} = L_d\mathbf{I}$ is the inductance matrix with L_d the proper inductance of the stator windings, $\mathbf{R}_s = R\mathbf{I}$ contains the stator resistances, Φ is the magnetic field, J is the moment of inertia and n_p

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is number of pole pairs. Here $\mathbf{I}$ is the 2×2 identity matrix while

$$\mathbf{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}; \quad \rho = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

It is easy to see that for solving a control problem (without lost of generality), the system above can be written as

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_2 x_3 + u_1 && (1) \\ \dot{x}_2 &= -x_2 - x_1 x_3 - \gamma x_3 + u_2 && (2) \\ \dot{x}_3 &= \sigma x_2 - \tau_L && (3) \\ \dot{x}_4 &= x_3 && (4) \end{aligned}$$

where they have been retained only the terms related with the magnetic field Φ while the other parameters have been normalized. In this new representation, it has been defined $x_1 = i_d$, $x_2 = i_q$, $x_3 = \omega$, $x_4 = \theta$, $u_1 = u_d$ and $u_2 = u_q$.

Remark. One advantage of system (1-4) that has been exhaustively exploited for controller design purposes is that the first three equations are decoupled from the fourth one, i.e. they are position decoupled. In this case however, this mechanical variable is explicitly included since it will be considered as the mechanical measurable variable. This decision does not complicate the controller structure, since the mechanical position must be known in order to recover the implementable stator variables (via the celebrated Park's transformation) but, as will be seen later, allows for eliminating the use of speed sensors.

Remark. In spite of the above advantage, one complication that must be noticed is that the steady state behavior of the stator current x_2 is completely determined by (3), since

$$x_2^* = \frac{1}{\sigma}\left[\dot{x}_3^* + \tau_L\right] \qquad (5)$$

In this sense, if the load torque is unknown then this value is also unknown.

B. Output feedback speed tracking control problem

In the context presented in the last section, the control problem approached in this paper can be formulated as

Consider the system (1-4) with measurable output $y = [x_1, x_2, x_4]^T$, unmeasurable state x_3 and unknown (constant) perturbation τ_L. Design a dynamic output feedback control law

$$\begin{aligned} \mathbf{U} &= \Gamma_1(\xi, y, \mathbf{U}) \\ \dot{\xi} &= \Gamma_2(\xi, y, \mathbf{U}) \end{aligned}$$

such that

$$\lim_{t\to\infty}(x_3 - x_3^*) = 0$$

with x_3^* a twice differentiable function that states the desired behavior for the mechanical speed, guaranteeing internal stability.

III. Position feedback control

The design of the proposed controller departs from the ideal structure given by

$$\begin{aligned} u_1 &= x_1^* + \dot{x}_1^* - x_2 x_3^* - k_1' e_1 && (6) \\ u_2 &= \gamma x_3^* + x_1 x_3^* + x_2^* + \dot{x}_2^* - k_2' e_2 && (7) \end{aligned}$$

where x_2^* is computed as in (5) and the error terms are given by $e_i = x_i - x_i^*$. Unfortunately, this scheme is not implementable due to the fact that the load torque τ_L is not known and therefore several modifications are introduced.

First, instead of expression (5), the set point is defined as

$$\begin{aligned} x_2^* &:= \frac{1}{\sigma}[\nu + \dot{x}_3^* + v_3] && (8) \\ v_3 &= -k_p e_4 - k_d \vartheta, \quad k_p,\, k_d > 0 && (9) \end{aligned}$$

where ν is a parameter to compensate for the unknown load τ_L, to be updated online via

$$\dot{\nu} = -k_i(e_4 - \vartheta) \qquad (10)$$

The rational behind this modification is the PID structure imposed to the reference value, since the variable ν indeed is given by the integral of the position error while the variable v_3 clearly exhibits a proportional–derivative structure over this variable. The limitation for implementing a classical PID control comes from the fact that the time derivative of the position error is not measurable. Hence it is necessary to introduce the dirty-derivative filter

$$\begin{aligned} \dot{q}_c &= -a(q_c + b e_4) && (11) \\ \vartheta &= q_c + b e_4, \quad a,\, b > 0 && (12) \end{aligned}$$

which completely depends on available information.

Unfortunately, it must be noticed that the time derivative of the reference value (8) is given by

$$\begin{aligned} \dot{x}_2^* = & \frac{1}{\sigma}[\dot{\nu} + \ddot{x}_3^* + \dot{v}_3] \\ = & \frac{1}{\sigma}[\dot{\nu} + \ddot{x}_3^* + a k_d \vartheta] - \frac{1}{\sigma}(k_p + b k_d) e_3 && (13) \end{aligned}$$

where the last term is not available since e_3 is unmeasurable. This problem is avoided by, instead of directly using $\dot{x}_2^*$ in the control law, the first three terms in the expression of $\dot{x}_2^*$, namely,

$$\frac{1}{\sigma}[\dot{\nu} + \ddot{x}_3^* + a k_d \vartheta] = \frac{1}{\sigma}(k_p + b k_d) e_3 + \dot{x}_2^*$$

are implemented.

Under these conditions, the implementable controller is given by (6) and

$$\begin{aligned} u_2 &= \gamma x_3^* + x_1 x_3^* + x_2^* + v_2 + \rho - k_2' e_2 && (14) \\ \rho &= \frac{1}{\sigma}[\dot{\nu} + \ddot{x}_3^* + a k_d \vartheta] \end{aligned}$$

where v_2 is an stabilizing term (to be defined later).

The proposed scheme leads to the error equations for the stator currents given by

$$\begin{aligned} e_1 &= -k_1 e_1 + x_2 e_3; \quad k_1 = k_1' + 1 \\ \dot e_2 &= -k_2 e_2 + \Delta e_3 + v_2; \quad k_2 = k_2' + 1 \\ \Delta &= \frac{1}{\sigma}(k_p + b k_d) + (\gamma - x_1) \end{aligned}$$

On the other hand, the error equation for the rotor speed can be written as

$$\dot x_3 - \dot x_3^* = \sigma x_2 - \tau_L - \dot x_3^* \pm \sigma x_2^*$$

which is equivalent to

$$\dot e_3 = \sigma e_2 + \tilde\nu - k_p e_4 - k_d \vartheta\,. \tag{15}$$

Further, if as in [12] it is defined

$$\begin{aligned} z &= \tilde\nu - \frac{k_i}{\varepsilon} e_4, \quad 0 < k_i < \varepsilon \ll 1 & (16) \\ k_p' &= k_p - \frac{k_i}{\varepsilon} & (17) \end{aligned}$$

with $\tilde\nu = \nu - \tau_L$, then

$$\dot e_3 = \sigma e_2 - k_p' e_4 - k_d\vartheta + z \tag{18}$$

while

$$\dot z = -k_i(e_4 - \vartheta) - \frac{k_i}{\varepsilon} e_3 \tag{19}$$

The overall closed-loop system yields

$$\begin{aligned} \dot e_1 &= -k_1 e_1 + x_2 e_3 & (20) \\ \dot e_2 &= -k_2 e_2 + \Delta e_3 + v_2 & (21) \\ \dot e_3 &= \sigma e_2 - k_p' e_4 - k_d \vartheta + z & (22) \\ \dot e_4 &= e_3 & (23) \\ \dot\vartheta &= -a\vartheta + b e_3 & (24) \\ \dot z &= -k_i(e_4 - \vartheta) - \frac{k_i}{\varepsilon} e_3 & (25) \end{aligned}$$

IV. Stability analysis

Consider the following functions which, as the reference x_2^* are proposed following the steps of [12]

$$\begin{aligned} V_1 &= \frac{1}{2}(e_1^2 + e_2^2) & (26) \\ V_2 &= \frac{1}{2}\left(e_3^2 + k_p' e_4^2 + \frac{k_d}{b}\vartheta^2 + \frac{\varepsilon}{k_i} z^2\right) & (27) \\ V_3 &= \varepsilon e_3 (e_4 - \vartheta), \qquad \varepsilon \ll 1 & (28) \\ \dot V &:= V_1 + V_2 + V_3\,. & (29) \end{aligned}$$

1) Positivity of V: Using the triangle inequality it is clear that

$$-\frac{1}{2}\varepsilon\left(e_3^2 + e_4^2 + \vartheta^2\right) \le V_3 \le \frac{1}{2}\varepsilon\left(e_3^2 + e_4^2 + \vartheta^2\right) \tag{30}$$

therefore, given any control gains, there always exists a constant $1 \gg \varepsilon > 0$ and for such ε, positive reals α_1, α_2 such that the function V satisfies

$$\alpha_1 |x|^2 \le V(x) \le \alpha_2 |x|^2 \quad \forall\, x \in \mathbb{R}^6 \tag{31}$$

where we defined the closed-loop state $x = [e_1, e_2, e_3, e_4, \vartheta, z]$. Hence, V is positive definite and proper.

2) Negativity of $\dot V$: The total time derivative of V_1 along the trajectories of the closed-loop system yields

$$\dot V_1 = -k_1 e_1^2 + x_2 e_1 e_3 - k_2 e_2^2 + \Delta e_2 e_3 + v_2 e_2; \tag{32}$$

while the derivative of V_2 is given by

$$\dot V_2 = \sigma e_2 e_3 - \frac{k_d a}{b}\vartheta^2 - \varepsilon z(e_4 - \vartheta) \tag{33}$$

and the derivative of V_3 satisfies

$$\begin{aligned} \dot V_3 &= \varepsilon(e_4 - \vartheta)[\sigma e_2 - k_p' e_4 - k_d\vartheta + z] \\ &\quad + \varepsilon e_3[e_3 - (-a\vartheta + b e_3)] \\ &= -\varepsilon k_p' e_4^2 - \varepsilon k_d e_4 \vartheta + \varepsilon(e_4 - \vartheta) z + \varepsilon e_3^2 + \varepsilon a e_3 \vartheta \\ &\quad -\varepsilon b e_3^2 + \varepsilon k_p' e_4 \vartheta + \varepsilon k_d \vartheta^2 \pm \frac{k_d a}{2b}\vartheta^2 \\ &\quad + \varepsilon(e_4 - \vartheta)\sigma e_2 \\ &= \frac{k_d a}{2b}\vartheta^2 - \frac{\varepsilon b'}{2} e_3^2 - \frac{\varepsilon k_p'}{2} e_4^2 + \varepsilon(e_4 - \vartheta)(z + \sigma e_2) \\ &\quad - \frac{1}{2}\begin{bmatrix} e_3 \\ e_4 \\ \vartheta \end{bmatrix}^\top \begin{bmatrix} \varepsilon b' & 0 & -\varepsilon a \\ 0 & \varepsilon k_p' & \varepsilon(k_p' - k_d) \\ -\varepsilon a & \varepsilon(k_p' - k_d) & k_d\left(\frac{a}{b} - 2\varepsilon\right) \end{bmatrix} \begin{bmatrix} e_3 \\ e_4 \\ \vartheta \end{bmatrix} \end{aligned}$$

where we used $b' := b - 1$. The matrix above is positive semidefinite if

$$\begin{aligned} & k_d\left(\frac{a}{b} - 2\varepsilon\right) \ge \frac{(\varepsilon a)^2}{\varepsilon b'} + \frac{\varepsilon^2 (k_p' - k_d)^2}{\varepsilon k_p'} \\ \iff \quad & \frac{k_d a}{b} \ge 2 k_d \varepsilon + \frac{\varepsilon a^2}{b'} + \frac{\varepsilon (k_p' - k_d)^2}{k_p'} \\ \iff \quad & \frac{k_d a}{b} \ge \varepsilon\left[2 k_d + \frac{a^2}{b} + \frac{(k_p' - k_d)^2}{k_p'}\right] \qquad (34) \end{aligned}$$

which holds for sufficiently small values of ε. Note that this restricts the choice of k_i, by definition, but not the other control gains.

Next, define the control term

$$v_2 = -\varepsilon\sigma(e_4 - \vartheta) \tag{35}$$

then, under the condition (34) the total time derivative of V satisfies

$$\begin{aligned} \dot V &\le -k_1 e_1^2 + x_2 e_1 e_3 - k_2 e_2^2 + \Delta e_2 e_3 - \varepsilon\sigma(e_4 - \vartheta) e_2 \\ &\quad \sigma e_2 e_3 - \frac{k_d a}{b}\vartheta^2 - \varepsilon z(e_4 - \vartheta) \\ &\quad \frac{k_d a}{2b}\vartheta^2 - \frac{\varepsilon b'}{2} e_3^2 - \frac{\varepsilon k_p'}{2} e_4^2 + \varepsilon(e_4 - \vartheta)(z + \sigma e_2) \\ &\le -k_1 e_1^2 + x_2 e_1 e_3 - k_2 e_2^2 + \Delta e_2 e_3 \\ &\quad + \sigma e_2 e_3 - \frac{k_d a}{2b}\vartheta^2 - \frac{\varepsilon b'}{2} e_3^2 - \frac{\varepsilon k_p'}{2} e_4^2 \\ &\le -\frac{1}{2}\left[k_1 e_1^2 + k_2 e_2^2 + \frac{k_d a}{2b}\vartheta^2 + \varepsilon b' e_3^2 + \frac{\varepsilon k_p'}{2} e_4^2\right] \\ &\quad -\frac{1}{2}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}^\top \begin{bmatrix} k_1 & x_2 & 0 \\ x_2 & k_2 & (\Delta + \sigma) \\ 0 & (\Delta + \sigma) & \varepsilon b' \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}. \end{aligned}$$

The matrix in the expression above is positive semidefinite for positive values of all the control gains and if

$$k_1 k_2 \geq x_2^2 \tag{36}$$
$$k_1 k_2 \varepsilon b' \geq (\Delta+\sigma)^2 k_1 + \varepsilon b' x_2^2 \,. \tag{37}$$

which hold if and only if

$$k_2 \geq \frac{(\Delta+\sigma)^2}{\varepsilon b'} + \frac{x_2^2}{k_1} \tag{38}$$

condition that holds for sufficiently large constant values of k_1 and k_2.

Under the previous conditions we obtain the existence of a constant $\alpha_3 > 0$ such that, defining $y := [e_1,\, e_2,\, e_3,\, e_4,\, \vartheta]$

$$\dot{V}(x) \leq -\alpha_3 |y|^2 \leq 0 \quad \forall\, x \in \mathbb{R}^6 \tag{39}$$

that is, V is negative semidefinite and the origin of the system, $x = 0$, is uniformly globally stable. In particular, the solutions are uniformly globally bounded and the origin is Lyapunov stable. These properties *together*, constitute what is called uniform global stability –see [16].

3) Uniform global exponential stability: The proof of exponential stability follows invoking [17, Lemma 3] which we recall below for sake of completeness.

Lemma 1. Let $F : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}^n$ be continuous. If, for the system $\dot{x} = F(t, x)$, there exist constants $c_1,\, c_2 > 0$, $p \in [1, \infty)$ such that for all $t_\circ \in \mathbb{R}_{\geq 0}$, $x_\circ \in \mathbb{R}^n$, all solutions $x(\cdot, t_\circ, x_\circ)$ satisfy the:

uniform $\mathcal{L}_\infty$ bound $\quad \sup_{t \geq t_\circ} |x(t)| \leq c_1 |x_\circ|, \quad (40)$

uniform $\mathcal{L}_p$ bound $\quad \left(\lim_{t\to\infty} \int_{t_\circ}^t |x(t)|^p\right)^{1/p} \leq c_2 |x_\circ|$

then, the origin of the system $\dot{x} = F(t, x)$ is uniformly globally exponentially stable.

To invoke Lemma 1 we need to compute "uniform $\mathcal{L}_p$ bounds" on the system's trajectories. From (39) we see that

$$\dot{V}(x(t)) \leq -\alpha_3 |y(t, t_\circ, x_\circ)|^2 \leq 0 \quad \forall\, t_\circ \in \mathbb{R}_{\geq 0},\ x_\circ \in \mathbb{R}^6$$

which is equivalent to

$$V(x(t)) - V(x(t_\circ)) \leq -\alpha_3 \int_{t_\circ}^t |y(s, t_\circ, x_\circ)|^2 ds \quad x(t_\circ) = x_\circ$$

hence, in view of the positivity and boundedness of V –see (31) it is obtained that for all $t \geq t_\circ \geq 0$ and all $x_\circ \in \mathbb{R}^6$

$$\int_{t_\circ}^t |y(s, t_\circ, x_\circ)|^2 ds \leq \frac{\alpha_2}{\alpha_3} |x_\circ|^2 \,. \tag{41}$$

Furthermore, we also have

$$\alpha_1 |x(t)|^2 \leq V(x(t)) \leq V(x(t_\circ)) \leq \alpha_2 |x_\circ|^2 \tag{42}$$

hence, for all $t \geq t_\circ \geq 0$ and all $x_\circ \in \mathbb{R}^6$,

$$|x(t)| \leq c_1 |x_\circ| \qquad c_1 := \sqrt{\frac{\alpha_2}{\alpha_1}} \tag{43}$$

hence (40) holds. It is left to find a uniform $\mathcal{L}_2$ bound on $z(t)$. For this, consider the function

$$V_4 = e_3 z \,. \tag{44}$$

Its total time derivative along the closed-loop trajectories yields

$$\dot{V}_4 = -z^2 - z(\sigma e_2 - k_d \vartheta - k_p' e_4) - e_3(-k_i(e_4 - \vartheta) - \frac{k_i}{\varepsilon} e_3) \tag{45}$$

which after the triangle inequality, satisfies

$$\dot{V}_4 \leq -z^2 + \frac{1}{2}\left[z^2 + d_1 |y|^2\right] \tag{46}$$

for an appropriate choice of $d_1 \gg 1$. Integrating the above along the trajectories, from $t_\circ$ to t we obtain

$$2e_3(t)z(t) - 2e_3(t_\circ)z(t_\circ) \leq -\int_{t_\circ}^t z(s)^2 ds + \int_{t_\circ}^t d_1 |y(s)|^2 ds \,. \tag{47}$$

Now, although of undefinite sign, the terms on the left hand side of the inequality are bounded by $c_1 |x_\circ|^2$ hence, using (41) we obtain

$$\int_{t_\circ}^t z(s)^2 ds \leq \left[\frac{d_1 \alpha_2}{\alpha_3} + c_1\right] |x_\circ|^2 \tag{48}$$

Uniform global exponential stability follows from (41), (48) and (43) by invoking Lemma 1.

V. Simulation results

The usefulness of the proposed control scheme was evaluated through numerical simulations. To this end the considered model parameters were set to $\sigma = 0.51$ and $\gamma = 0.17$ which corresponds to $\Phi = 0.17Wb$ and $n_p = 3$ as reported in [13]. The experiment consisted in imposing a speed reference that was inspired in the signal profile proposed as a benchmark by the French Working Group *Commande des Entraînements Electriques* that can be consulted in `http://www2.irccyn.ec-nantes.fr/CE2/`, namely: The desired motor speed started in zero, increasing with a slope of 5.25 $\frac{rad}{s^2}$ until reaching a value of 5.25$\frac{rad}{s}$ at $t = 1s$. This value was kept constant until $t = 3s$ when its value was increased again, this time with a slope of 3.675 $\frac{rad}{s^2}$, to achieve a value of 12.6$\frac{rad}{s}$ during 2 seconds, to be decreased (with a slope of 6.3 $\frac{rad}{s^2}$), remaining at zero for the rest of the simulation (whose total length was 14 seconds). During all this time the applied load torque was equal to 1 Nm.

Following field orientation ideas, the desired value for x_1 was set to zero while the controller gains were $k_1 = 4$, $k_2 = 75$, $k_p = 5$, $k_d = 10$, $k_i = 0.01$, $a = 50$, $b = 50$ and $\epsilon = 0.02$. It was considered that the motor was at standstill at the beginning of the experiment, i.e. all the motor states were set to zero, and in a similar way both the initial value of the derivative filter state q_c and the estimated load torque were considered zero, the latter to include the worst case regarding the knowledge of the actual load torque.

The reference speed profile together with the actual speed response are shown in Figure 1 where, besides the achievement of the stabilization objective predicted by the theory, it can be observed the remarkable performance exhibited by proposed controller. This behavior is verified in a more detailed way in Figure 2 where the speed error is presented.

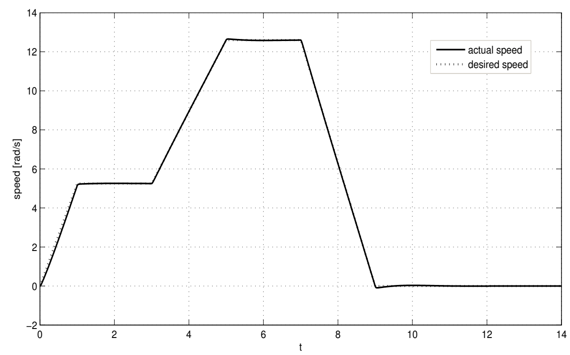

Fig. 1. Actual (continuous line) and desired (dashed line) speeds.

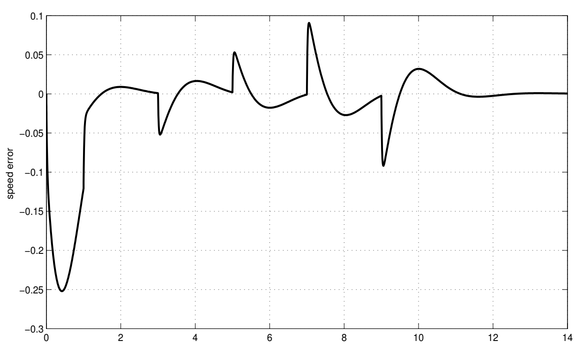

Fig. 2. Speed error.

With the aim to show how the reference values for the other states of the machine are also reached under the control scheme, in Figure 3, Figure 4 and Figure 5 the behavior of the stator currents and the rotor position, respectively, are depicted comparing their evolution with their corresponding reference values. Here it must be noticed that in the latter a small steady state error can be observer, which is due to the fact that the convergence rate of this variable is lower with respect to the other states.

In order to illustrate in a complete way the internal closed–loop stability of the system, in Figure 6 the estimated load torque is exhibited. In this case, due to the small value of the estimation gain k_i required to assure the stability of the closed–loop system, the convergence of this variable to its actual value is quite slow. However, the basic boundedness requirement was also satisfied.

Finally, in Figure 7 and Figure 8 the behavior of the derivative filter state and the stator voltages, respectively, are presented, where it can be verified their boundedness

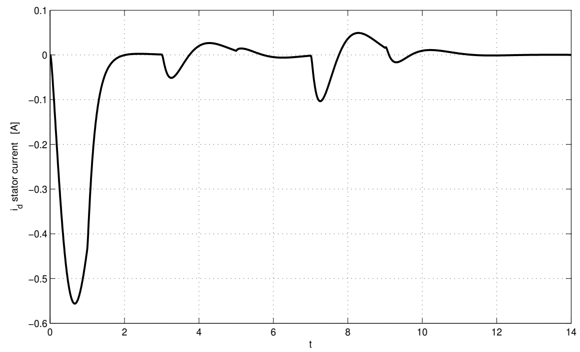

Fig. 3. Actual i_d stator current with a desired value equal to zero.

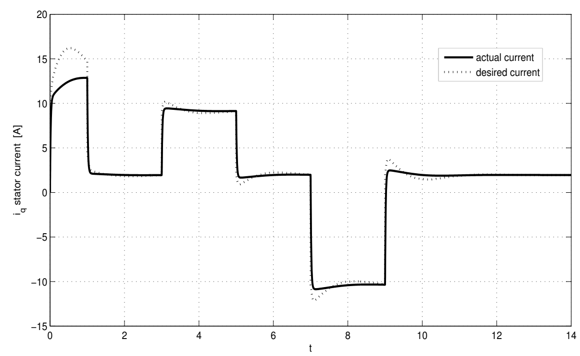

Fig. 4. Actual (continuous line) and desired (dashed line) i_q stator current.

during the experiment. Concerning the control input u_2 some spikes are present due to the sudden change in the first and second derivatives of the reference speed. These spikes are not present if a smooth reference is designed but with the aim to evaluate the controller under stringer conditions the discontinuities were not avoided during the experiment.

VI. Concluding Remarks

An output (position) feedback controller of the PID type that solves the speed tracking control problem for PMSM is presented in this paper. The proposed scheme enjoys the simple structure characteristic of this kind of schemes but, in contrast to the major part of these algorithms, its stability properties were formally proved. Specifically, it was shown that the desired speed behavior is Uniformly Globally Asymptotically Stable. From a practical perspective, besides the remarkable dynamic performance achieved by the closed–loop system, the proposed design avoids the use of (noisy) speed sensor and does not rely in the knowledge of the load torque, which is considered as a constant unknown perturbation. The controller was evaluated via numerical simulations exhibiting an operation that allows for expecting very good results in an experimental setting.

References

[1] Meisel, J.: **Principles of Electromechanical Energy Conversion**, Mc. Graw Hill, 1961.

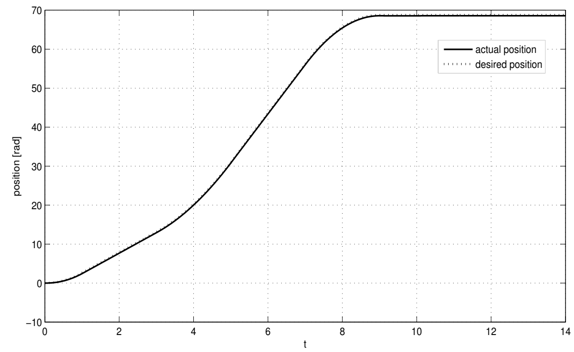

Fig. 5. Actual (continuous line) and desired (dashed line) rotor position.

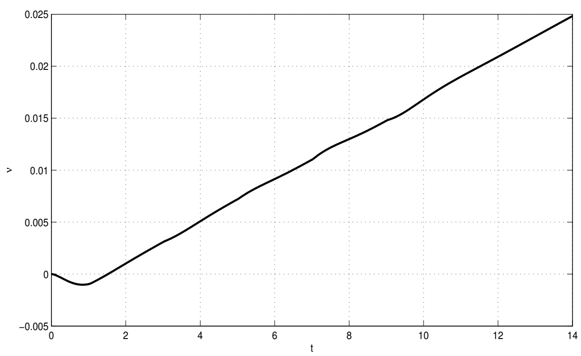

Fig. 6. Estimated load torque.

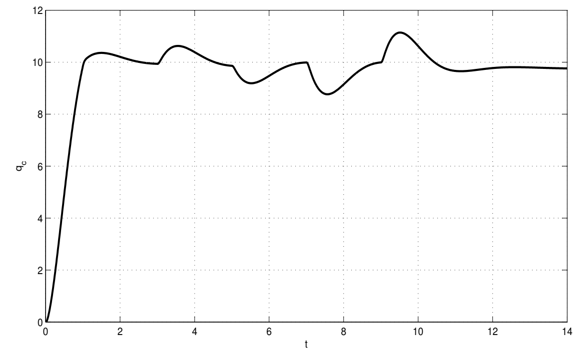

Fig. 7. Internal derivative filter state.

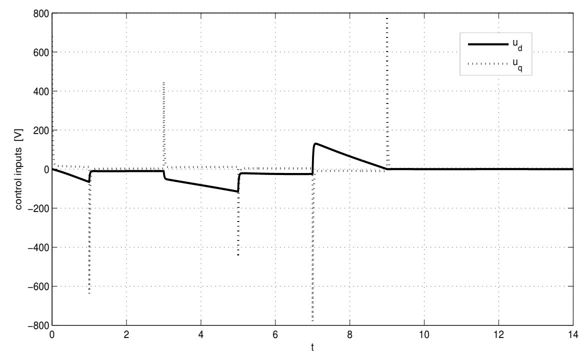

Fig. 8. Stator voltages.

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