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Neuroadaptive Fault-tolerant Control of High Speed Trains with Input Nonlinearities and Actuator Failures

Q. Song and Y. D. Song

Abstract—**This work investigates the position and velocity tracking control problem of high speed trains with multiple vehicles connected through couplers. A dynamic model coupled with nonlinear and elastic impacts between adjacent vehicles as well as traction/braking nonlinearities and actuation faults is derived. Neuroadaptive fault-tolerant control algorithms are developed to deal with various factors such as input nonlinearities, actuator failures, and uncertain impacts of in-train forces in the system. The effectiveness of the proposed approach is also confirmed through numerical simulation.**

I. Introduction

Various control techniques for automatic train operation have been reported in the literature [1]-[7]. It is noted that most methods are based on single point-mass model, and the coupling elastic dynamics among the vehicles are ignored. Multiple point-mass model is more practical, and several researchers have tackled the control issues of cargo and passenger trains based on multiple point-mass model coupled with in-train forces in the past few years [8]-[13]. It should be mentioned that modeling or measuring the impacts of in-train forces is fairly difficult in practice. Furthermore, the basic resistive forces become increasingly significant as the train speed increases. Also, as the traction/braking notches are used in train operation, the underlying dynamics contain input nonlinearities.

This work is concerned with movement control of high speed trains in which the factors of input nonlinearities, actuator failures and in-train forces are explicitly addressed in control design and stability analysis. A multiple point-mass model considering traction/braking notches is derived and neuroadaptive fault-tolerant control schemes are developed. The salient feature of the proposed control method lies in its simplicity in design and effectiveness in dealing with various uncertainties and nonlinearities or even faults occurring during system operation.

II. Modeling and Problem Statement

Prior to designing the neuroadaptive scheme, a multiple point-mass model considering traction/braking notches is introduced, followed by the problem statement.

A. Multiple Point-mass Model Considering Traction and Braking Notches

Consider a train consisting of n vehicles (q locomotives and p carriages) connected by $n-1$ nonlinear and elastic couplers and draft gears. The multiple point-mass model can be derived as [11, 14]

$$diag(m_i)\begin{bmatrix}\ddot{x}_1\\ \ddot{x}_2\\ \vdots\\ \ddot{x}_n\end{bmatrix} = diag(\lambda_i)\begin{bmatrix}\Psi_1(F_1)\\ \Psi_2(F_2)\\ \vdots\\ \Psi_n(F_n)\end{bmatrix} + \begin{bmatrix} g_0\\ g_1(x_2,x_1,\dot{x}_2,\dot{x}_1,p_1)\\ \vdots\\ g_{n-1}(x_n,x_{n-1},\dot{x}_n,\dot{x}_{n-1},p_{n-1})\end{bmatrix} - \begin{bmatrix} g_1(x_2,x_1,\dot{x}_2,\dot{x}_1,p_1)\\ \vdots\\ g_{n-1}(x_n,x_{n-1},\dot{x}_n,\dot{x}_{n-1},p_{n-1})\\ g_n\end{bmatrix} - \begin{bmatrix} f_{d1}\\ f_{d2}\\ \vdots\\ f_{dn}\end{bmatrix} \quad (1)$$

where the in-train force $g_i(.)$ (the interaction between adjacent vehicles) is of the form

$$g_i(\cdot) = g_i(\dot{x}_{i+1}, \dot{x}_i, x_{i+1}, x_i, p_i) \quad (2)$$

which is essentially a nonlinear and uncertain function of $\dot{x}_i$, x_i, $\dot{x}_{i+1}$, x_{i+1} as well as the parameter vector p_i. ($g_0 = g_n = 0$ because there is no in-train force at the front of the first vehicle and the end of the last vehicle). The definitions of the other variable are: m_i is the mass of the i^{th} vehicle which might not be accurately available due to uncertain variation of passengers and loads (the number of passengers on board each vehicle is different and uncertain in general); x_i is defined as before; $\lambda_i > 0$ is a distribution constant determining the power/braking effort of the i^{th} vehicle; f_{di} denotes the resistive force acting on the i^{th} vehicle. Note that for safe and energy saving operation, different notches (scales) of traction/braking forces are required during different operating phases of the train, thus actuation saturation due to traction and braking notches are involved in the train system, which motivates the consideration of the above multiple point-mass model incorporated with nonlinear traction/braking notches and non-symmetric saturation.

Note that the resistive force f_{di} for each vehicle takes the form

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Q. Song is with the State Key Laboratory of Rail Traffic Control and Safety , Center for Intelligent Systems and Renewable Energy, Beijing, China (e-mail: 09111018@bjtu.edu.cn).

Y. D. Song is with the State Key Laboratory of Rail Traffic Control and Safety, Center for Intelligent Systems and Renewable Energy, Beijing, China (corresponding author: 010-51684432; fax: 010-51684432; e-mail: ydsong@bjtu.edu.cn).

$$f_{di} = a_{0i} + a_{1i}\dot{x}_i + a_{2i}\dot{x}_i^2 + f_{ri} + f_{ci} + f_{ti} \quad (3)$$

here a_{0i}, a_{1i}, and a_{2i} are the resistive coefficients for the i^{th} vehicle, f_{ri} is the ramp resistance due to the track slope, f_{ci} is the curve resistance due to railway curvature, and f_{ti} is the tunnel resistance, all acting on the i^{th} vehicle; The traction/braking force considering the notch effect is,

$$\Psi_i(F_i) = \begin{cases} F_i^{r\max}, & F_i > F_i^{rn} \\ k_i^{rj}F_i + \zeta_i^{rj}, & \begin{pmatrix} F_i^{r(j-1)} \le F_i \le F_i^{rj} \\ j = 2,\cdots,n \end{pmatrix} \\ k_i^{r1}F_i, & 0 \le F_i \le F_i^{r1} \\ k_i^{l1}F_i, & F_i^{l1} \le F_i \le 0 \\ k_i^{lj}F_i + \zeta_i^{lj}, & \begin{pmatrix} F_i^{lj} \le F_i \le F_i^{l(j-1)} \\ j = 2,\cdots,n \end{pmatrix} \\ F_i^{l\max}, & F_i < F_i^{ln} \end{cases} \quad (4)$$

where ζ_i^{rj} and ζ_i^{lj} $(i = 1, 2, \cdots, n;\ j = 2, \cdots, n)$ are the vertical-intercept of the i^{th} notch. They are zero when $F_i > F_i^{rn}$ and $F_i < F_i^{ln}$ as well as $0 \le F_i \le F_i^{r1}$ and $F_i^{l1} \le F_i \le 0$, while F_i is the variable to be designed, F_i^{rj} and F_i^{lj} are the neighboring notch values, k_i^{rj} and k_i^{lj} are the slope of the i^{th} notch [14].

Remark 1: It should be stressed that the in-train forces involved in the model are extremely difficult to model or measure precisely due to the nonlinear and elastic nature of the couplers connecting the vehicles. In most existing works such in-train forces are either ignored or approximated with a linear model [8, 12]. In this work, the in-train force of the form (2) is considered without linearization to better reflect the practical situation.

Remark 2: The model considered here is more effective in characterizing the dynamic behavior of a train as compared with the single point-mass model or multiple point-mass model with linear approximation commonly used in the literature.

B. Problem Statement

Let $X = [x_1, \cdots, x_n]^T$ be the displacement vector and $\dot{X} = [\dot{x}_1, \cdots, \dot{x}_n]^T$ be the velocity vector, the control objective is to design control force $F = [F_1, \cdots, F_n]^T$ so that for any given desired $\dot{X}^* - X^*$ pair, we have $E \to 0$ and $\dot{E} \to 0$ as $t \to \infty$, where $E = X - X^*$ and $\dot{E} = \dot{X} - \dot{X}^*$ denote the position tracking error and velocity tracking error, respectively, $\dot{X}^*$ and X^* are the desired velocity and position, which, together with the desired acceleration $\ddot{X}^*$, are ensured to be smooth and bounded.

The problem can be solved by using several existing methods [9]-[13] if m_i, f_{di} and $g_i(.)$ are available precisely. However, the mass of each vehicle of the train (m_i), the resistance coefficients (a_{0i}, a_{1i}, and a_{2i}), and other resistance (f_{ri}, f_{ci}, and f_{ti}) for each vehicle cannot be obtained accurately in practice, and some of them might even vary with operation conditions. Furthermore, even if those coefficients or resistance forces are obtainable, it is still a painful task to determine the in-train force $g_i(.)$. The typical way to address this difficulty is to use the largely simplified linear/approximation model [8, 9] [11-13] for control design.

In this study, we propose a solution to this problem by neuroadaptive control scheme, in which no specific information on the total mass of the train, the resistance coefficients, or any other resistance forces is required. There is no need for precise measurement or computation of the in-train forces. Meanwhile, input nonlinearity and actuator failure are considered, as detailed in next section.

III. Control Design and Stability Analysis

Note that (4) can be equivalently expressed as

$$\Psi_i(F_i) = \tau_i(\cdot)F_i + \zeta_i \quad (5)$$

where $\tau_i(\cdot) > 0$ is some unknown positive parameter. Obviously, $|\zeta_i| \le \zeta_i^m < \infty$ for some constant $\zeta_i^m \ge 0$. For the traction and braking systems under consideration, there exist some positive constants $k_i^{l\min}$, $k_i^{l\max}$, $k_i^{r\min}$ and $k_i^{r\max}$ such that

$$0 < k_i^{l\min} \le k_i^{lj} \le k_i^{l\max}, \quad F_i^{lj} \le F_i \le F_i^{l(j-1)}$$
$$0 < k_i^{r\min} \le k_i^{rj} \le k_i^{r\max}, \quad F_i^{r(j-1)} \le F_i \le F_i^{rj}$$

Therefore, it can be established that for all $F_i \in R$ there exists some constant $\tau_i^{\min} > 0$ such that $\tau_i^{\min} \le \tau_i$, $(i = 1, 2, \cdots, n)$. Then we can get that $0 < \tau_{\min} \le \tau_i(\cdot)\ (i = 1, 2, \cdots, n)$, and $\tau_{\min} = \min\{\tau_1^{\min}, \cdots, \tau_i^{\min}, \cdots, \tau_n^{\min}\}$. For simple notation, we rewrite (1) as

$$\begin{aligned} M\ddot{X} &= \Lambda\Psi(F) - F_d + (T - I)G_{in} \\ \Psi(F) &= \tau(\cdot)F + \zeta \end{aligned} \quad (6)$$

which, in light of (5), can be further expressed as

$$\begin{aligned} M\ddot{X} &= \Lambda\Gamma F + \Upsilon_d(\cdot) \\ \Upsilon_d(\cdot) &= \Lambda\zeta - F_d + (T - I)G_{in} \end{aligned} \quad (7)$$

where $M = diag(m_i)$, $\Lambda = diag(\lambda_i)$, $\Gamma = diag(\tau_i)$, $\zeta = [\zeta_1 \ \cdots \ \zeta_n]^T$, $T = \begin{bmatrix} 0_{1\times(n-1)} & 1 \\ I_{(n-1)\times(n-1)} & 0_{(n-1)\times 1} \end{bmatrix}_{n\times n}$, with I being the identity matrix. ($g_0 = g_n$ is employed here to represent the in-train force as $(T\text{-}I)G_{in}$), $F_d = [f_{d1}, f_{d2}, \cdots, f_{dn}]^T$ and $G_{in} = [g_1, g_2, \cdots, g_n]^T$ are the vectors of resistance and in-train forces, respectively, $\dot{X}$ is the velocity vector, $\ddot{X}$ is the acceleration vector, and Λ is the power/braking effectiveness distribution matrix.

To facilitate the control design, we define a filtered variable $S \in R^n$, in terms of tracking errors:

$$S = \dot{E} + BE \tag{8}$$

with $B = diag(\beta_i) \in R^n \times R^n$, where β_i is a free positive parameter chosen by the designer/user. Based on (7) and (8), we can re-express (6) as

$$M\dot{S} = \Lambda\Gamma F + L_d(\cdot) \tag{9}$$

$$L_d(\cdot) = \Lambda\zeta - F_d + (T\text{-}I)G_{in} - M\ddot{X}^* + MB\dot{E} \tag{10}$$

Examining (10) reveals that $L_d(\cdot) \in R^{n\times 1}$ is nonlinear and difficult to obtain precisely. In other words, the above dynamic model contains significant nonlinearities and uncertainties. Thus it is highly desirable to develop a control scheme that does not rely on $L_d(\cdot)$ directly. The control schemes presented in next subsection utilize the available "core" information of $L_d(\cdot)$ [19-21], and NNs (Neural Networks) are employed to on-line approximate the nonlinear function.

Three neuroadaptive control schemes are developed based on different treatment of $L_d(\cdot)$, with the first one to deal with the vector $L_d(\cdot)$ directly, the second one to cope with the norm bound on $L_d(\cdot)$ and the third one to accommodate $L_d(\cdot)$ and possible actuator faults.

A. Neuroadaptive Control

As the first step, we reconstruct $L_d(\cdot)$ via an NN unit as:

$$L_d = W^T\Phi(z) + \kappa(z) \tag{11}$$

where $W \in R^{l\times n}$ is the weight matrix, $\Phi(\cdot) = [\phi_1(z) \ \cdots \ \phi_l(z)]^T$ is the basis function vector, $\kappa(\cdot) = [\varepsilon_1(\cdot) \ \cdots \ \varepsilon_n(\cdot)]^T$ is the reconstruction error, and $z = [z_1, \cdots, z_n]^T$, $z_i = [x_i, \dot{x}_i, x_{i+1}, \dot{x}_{i+1}, \ddot{x}_i^*, \dot{e}_i]^T$ $i = (1, 2, \cdots, n)$. By the universal approximation theory [15-18], it is reasonable to assume that the NN reconstruction error $\|\kappa\| \le \varepsilon_0$, where ε_0 is an unknown positive constant.

Theorem 1

Consider the train dynamics with in-train force and traction/braking notches as described by (7). If the following control algorithm is applied,

$$F = -\Lambda^T\left(k_0 S + \hat{W}^T\Phi - u_c\right) \tag{12a}$$

$$u_c = -\hat{a}\frac{\left(\left\|\hat{W}^T\Phi\right\| + 1\right)S}{\|S\|} \tag{12b}$$

with $\dot{\hat{W}} = \Phi S^T$, $\dot{\hat{a}} = \sigma_1\left(\left\|\hat{W}^T\Phi\right\| + 1\right)\|S\|$, where $\hat{a}$ is the estimation of a and $a = \max\{\|I - \Lambda\Gamma\Lambda^T\|, \varepsilon_0\}$, $\sigma_1 > 0$ $k_0 > 0$ are two free design parameters, then asymptotically stable position and velocity tracking are ensured.

Proof:

With the proposed control (12), one gets the closed-loop error dynamic equation from (9)

$$M\dot{S} = -k_0\Lambda\Gamma\Lambda^T S - \Lambda\Gamma\Lambda^T\hat{W}^T\Phi + \Lambda\Gamma\Lambda^T u_c + L_d(\cdot) \tag{13}$$

Using (14), it is straightforward to get

$$M\dot{S} = -k_0\Lambda\Gamma\Lambda^T S + \tilde{W}^T\Phi + \Lambda\Gamma\Lambda^T u_c + \gamma_d(\cdot) \tag{14}$$

with $\gamma_d(\cdot) = \left(I - \Lambda\Gamma\Lambda^T\right)\hat{W}^T\Phi + \kappa$ (15)

where $\tilde{W} = W - \hat{W}$. Let $a = \max\{\|I - \Lambda\Gamma\Lambda^T\|, \varepsilon_0\}$, it is seen that

$$\|\gamma_d(\cdot)\| \le a\left(\left\|\hat{W}^T\Phi\right\| + 1\right) \tag{16}$$

Consider the Lyapunov function candidate

$$V = \frac{1}{2}S^T M S + \frac{tr\left(W - \hat{W}\right)^T\left(W - \hat{W}\right)}{2} + \frac{(a - \lambda_m\hat{a})^2}{2\lambda_m\sigma_1} \tag{17}$$

where $0 < \lambda_m \le \lambda_{\min}(\Lambda\Gamma\Lambda^T)$ and $\lambda_{\min} > 0$ is the minimum eigenvalue of $\Lambda\Gamma\Lambda^T$. Using (14), it follows that

$$\begin{aligned} \dot{V} = &-k_0 S^T\Lambda\Gamma\Lambda^T S + S^T\tilde{W}^T\Phi + S^T\Lambda\Gamma\Lambda^T u_c + S^T\gamma_d \\ &+ tr\left(W - \hat{W}\right)^T\left(-\dot{\hat{W}}\right) + (a - \lambda_m\hat{a})\frac{\left(-\dot{\hat{a}}\right)}{\sigma_1} \end{aligned} \tag{18}$$

Noting that $S^T\tilde{W}^T\Phi$ is a scalar and hence $tr\left(S^T\tilde{W}^T\Phi\right) = tr\left(\tilde{W}^T\Phi S^T\right)$. Using the updating algorithms for $\hat{W}$ and $\hat{a}$ as well as the compensating unit u_c, one gets from (18) that

$$\begin{aligned} \dot{V} \le &-k_0\lambda_m\|S\|^2 + tr\tilde{W}^T\left(\Phi S^T - \dot{\hat{W}}\right) + (a - \lambda_m\hat{a})\frac{\left(-\dot{\hat{a}}\right)}{\sigma_1} \\ &- \lambda_m\hat{a}\left(\left\|\hat{W}^T\Phi\right\| + 1\right)\|S\| + a\left(\left\|\hat{W}^T\Phi\right\| + 1\right)\|S\| \le -k_0\lambda_m\|S\|^2 \le 0 \end{aligned}$$

Therefore we have $V \in \ell_\infty$, which ensures that $S \in \ell_\infty$, $\hat{a} \in \ell_\infty$, hence $\dot{E} \in \ell_\infty$, $E \in \ell_\infty$, $u_c \in \ell_\infty$, $F \in \ell_\infty$. Then it is readily shown that $\dot{S} \in \ell_\infty$, i.e., S is uniformly continuous, which, together with the fact that $\int_0^\infty k_0\lambda_m\|S\|^2 dt \le V(0) < \infty$, allows the Barbalat lemma to be used to conclude that $\lim_{t\to\infty} S = 0$, therefore $\dot{E} \to 0, E \to 0$ as $t \to \infty$ by the definition of S.

Remark 3: It should be pointed out that the estimation method of $\hat{a}$ as presented in Theorem 1 may cause parameter drift [23]. In order to avoid this problem, the following PI (Proportion Integration) estimate algorithm is used to make suitable corrections [22].

Theorem 2

Assume the same conditions as in Theorem 1. Let the control scheme be the same as in (12), where $k_0 \geq \dfrac{\sigma_0 a^2}{2\sigma_1 \lambda_m^2}$. If the following algorithm for $\hat{a}$ is used,

$$\dot{\hat{a}} = -\sigma_0 \|S\|^2 \hat{a} + \sigma_1 \left(\left\|\hat{W}^T \Phi\right\| + 1\right)\|S\| - \sigma_2 \dot{\xi} \tag{19a}$$

$$\xi = \sigma_0 \|S\|^2 \hat{a} - \sigma_1 \left(\left\|\hat{W}^T \Phi\right\| + 1\right)\|S\| \tag{19b}$$

where $\sigma_0 > 0$, $\sigma_1 > 0$, $\sigma_2 > 0$ are design parameters. Then asymptotically stable speed and position tracking are ensured.

Proof:

To prove the result, we need to modify the Lyapunov function candidate (17) to

$$V = \frac{1}{2} S^T M S + \frac{tr\left(W - \hat{W}\right)^T \left(W - \hat{W}\right)}{2} + \frac{\left(\tilde{a} + \lambda_m \sigma_2 \xi\right)^2}{2\lambda_m \sigma_1} \tag{20}$$

where $\tilde{a} = \lambda_m \hat{a} - a$. Then with (12a), (12b), (14) and (16), it is straightforward to show that

$$\begin{aligned} \dot{V} \leq & -k_0 \lambda_m \|S\|^2 - \tilde{a}\left(\left\|\hat{W}^T \Phi\right\| + 1\right)\|S\| + S^T \tilde{W}^T \Phi \\ & + tr\left(W - \hat{W}\right)^T \left(-\dot{\hat{W}}\right) + \frac{\left(\tilde{a} + \lambda_m \sigma_2 \xi\right)\left(\dot{\hat{a}} + \sigma_2 \dot{\xi}\right)}{\sigma_1} \end{aligned} \tag{21}$$

Upon using the updating algorithms (19a), (19b), it is not difficult to shown that

$$\begin{aligned} \dot{V} & \leq -k_0 \lambda_m \|S\|^2 - \tilde{a}\left(\left\|\hat{W}^T \Phi\right\| + 1\right)\|S\| + \frac{\left(\tilde{a} + \lambda_m \sigma_2 \xi\right)\left(\dot{\hat{a}} + \sigma_2 \dot{\xi}\right)}{\sigma_1} \\ & \leq -k_0 \lambda_m \|S\|^2 - \frac{\lambda_m \sigma_2 \xi^2}{\sigma_1} - \frac{\sigma_0 \|S\|^2 \tilde{a}^2}{2\lambda_m \sigma_1} + \frac{\sigma_0 \|S\|^2 a^2}{2\lambda_m \sigma_1} \\ & \leq -k_0 \lambda_m \|S\|^2 + \frac{\sigma_0 \|S\|^2 a^2}{2\lambda_m \sigma_1} \end{aligned} \tag{22}$$

Thus $\dot{V} \leq 0$ if $k_0 \geq \dfrac{\sigma_0 a^2}{2\sigma_1 \lambda_m^2}$. The result is established following the same argument as in the proof of Theorem 1.

Remark 4: It is noted that the proposed adaptation algorithm contains both integral term and proportional term. This PI estimate algorithm gives a better convergence property than using integral term alone.

B. Simplified Neuroadaptive Control

The previous neuroadaptive control scheme is based on using NN unit to deal with the lumped uncertain vector $L_d(\cdot)$ as defined in (10) directly. It is interesting to note that if the norm of $L_d(\cdot)$ is considered, a simpler neuroadative control can be developed, as detailed in what follows. First, note that

$$\|L_d\| \leq \eta_1(\cdot) + \eta_2(\cdot)$$

with $\eta_1(\cdot) = w^T j(z) + \varepsilon(z)$ and $\hat{\eta}_2(\cdot) = \hat{l}_1 + \hat{l}_2 \|\dot{X}\| + \hat{l}_3 \|\dot{X}\|^2$, where $\hat{l}_1$, $\hat{l}_2$, and $\hat{l}_3$ are the estimation of l_1, l_2, and l_3, $w \in R^p$ is the weight vector and $j(z) \in R^p$ is the basic function with $z = \left[\|X\|, \|\dot{X}\|\right]^T$ being the NN input, and $\varepsilon(z) \in R$ is the NN approximation error. Since the NN is used to cope with the scalar (rather than vector) unknown function, the corresponding control scheme turns out to be much simpler, as seen from the following development.

Theorem 3

Consider the train with the dynamics as given by (7). Assume that the approximation error $|\varepsilon| \leq \varepsilon_m < \infty$, where ε_m is an unknown constant. If the following NN based robust adaptive control law is applied,

$$F = -k_0 \Lambda^T S + u_a + u_b + u_c \tag{23a}$$

$$u_a = -\Lambda^T \left(\hat{l}_1 + \hat{l}_2 \|\dot{X}\| + \hat{l}_3 \|\dot{X}\|^2\right) \frac{S}{\|S\|} \tag{23b}$$

$$u_b = -\Lambda^T \hat{w}^T j \frac{S}{\|S\|} \quad \text{and} \quad u_c = -\Lambda^T \hat{\varepsilon}_m \frac{S}{\|S\|} \tag{23c}$$

with

$$\dot{\hat{w}} = \|S\| j, \quad \dot{\hat{\varepsilon}}_m = \sigma_1 \|S\| \tag{24a}$$

$$\dot{\hat{l}}_1 = \sigma_{l1} \|S\|, \ \dot{\hat{l}}_2 = \sigma_{l2} \|S\| \|\dot{X}\|, \ \dot{\hat{l}}_3 = \sigma_{l3} \|S\| \|\dot{X}\|^2 \tag{24b}$$

where $\sigma_1 > 0$, $\sigma_{l1} > 0$, $\sigma_{l2} > 0$, and $\sigma_{l3} > 0$ are some free design parameter, $\hat{\varepsilon}_m$ is the estimation of ε_m. Then asymptotically position and velocity tracking is ensured.

Proof:

Based on Eqs. (9) and (23a), we can get

$$M\dot{S} = -k_0 \Lambda \Gamma \Lambda^T S + \Lambda \Gamma u_a + \Lambda \Gamma u_b + \Lambda \Gamma u_c + L_d(\cdot) \tag{25}$$

Consider the Lyapunov function candidate

$$V = \frac{1}{2} S^T M S + \frac{\tilde{\varepsilon}_m{}^2}{2\lambda_m \sigma_1} + \frac{\tilde{l}_1^2}{2\lambda_m \sigma_{l1}} + \frac{\tilde{l}_2^2}{2\lambda_m \sigma_{l2}} + \frac{\tilde{l}_3^2}{2\lambda_m \sigma_{l3}} + \frac{\tilde{w}^T \tilde{w}}{2\lambda_m} \tag{26}$$

where $\tilde{\bullet} = \bullet - \lambda_m \hat{\bullet}$ ("$\bullet$" denotes ε_m, l_1, l_2, l_3 and w) is the generalized estimate error and $\lambda_m > 0$ is defined as before. It can be shown with the control algorithms (23a) - (24b) that

$$\begin{aligned} \dot{V} \leq & -k_0 \lambda_m \|S\|^2 - \lambda_m \left(\hat{l}_1 + \hat{l}_2 \|\dot{X}\| + \hat{l}_3 \|\dot{X}\|^2\right)\|S\| - \lambda_m \|S\| \hat{w}^T j \\ & - \lambda_m \hat{\varepsilon}_m \|S\| + \|S\|\left(l_1 + l_2 \|\dot{X}\| + l_3 \|\dot{X}\|^2\right) + \|S\| w^T j + \|S\| \varepsilon_m \end{aligned}$$

$$- \tilde{\varepsilon}_m \|S\| + \tilde{l}_1 \left(-\|S\|\right) + \tilde{l}_2 \left(-\|S\|\|\dot{X}\|\right) + \tilde{l}_3 \left(-\|S\|\|\dot{X}\|^2\right)$$
$$+ \tilde{w}^T \left(-\|S\|\phi\right) \le -k_0 \lambda_m \|S\|^2 \le 0$$

The result is then established using the same argument as in the proof of Theorem 1.

C. Neuroadaptive Fault-tolerant Control

As faults may occur during the system operation, it is important to address the fault-tolerant control issue explicitly. Here we consider the situation that the powering unit or braking unit (called actuator hereafter) of some vehicle fails to work properly in that actuation/braking capabilities are fading. The fault-tolerant control objective here is to maintain safe operation and avoid any possible operation accident whenever a failure occurs. Note that in such situation, the actual traction/braking force F_a and the designed traction/braking force F are not identical anymore, instead, they are related to through

$$F_a = \gamma F + R(\cdot) \tag{27}$$

where $\|R(\cdot)\| \le r_c < \infty$ is the uncertain part caused by the actuator failure, $\boldsymbol{\gamma}(\cdot) = diag\{\gamma_1(\cdot), \gamma_2(\cdot), ..., \gamma_n(\cdot)\}$ is a diagonal matrix of powering and braking effectiveness, with $0 \le \gamma_i(\cdot) \le 1$ being the "powering/braking health indicator" [20] for the i^{th} vehicle. The case of $\gamma_i(\cdot) = 0$ implies that the i^{th} vehicle totally loses its traction or braking capability; $0 < \gamma_i(\cdot) < 1$ corresponds to the case that the i^{th} vehicle partially loses its traction or braking effectiveness; and $\gamma_i(\cdot) = 1$ implies that the powering/braking system of the i^{th} vehicle is healthy.

In order for the system to admit a feasible control solution, one must assume that the remaining functional actuators are able to generate sufficient power to move forward or stop the train whenever it is necessary. To address the control design problem, we consider the case that some or all actuators suffer from partial actuation failures, i.e., $0 < \gamma_i(\cdot) < 1 \ (i = 1, \cdots, n)$. Note that $\gamma(\cdot)$ is a diagonal but uncertain actuation effectiveness matrix.

Now we develop an neuroadaptive fault-tolerant control to cope with the actuation faults and the uncertain in-train forces as well as the non-symmetric input nonlinearities. To this end, we combine (9), (10) and (27) get the closed-loop error dynamics

$$M\dot{S} = \Lambda\Gamma\gamma F + H_d(\cdot) \tag{28}$$

$$H_d(\cdot) = \Lambda\Gamma R(\cdot) + \Lambda\zeta - F_d + (T - I)G_{in} - M\ddot{X}^* + MB\dot{E} \tag{29}$$

The idea is to use an NN unit of the form $W^T\Phi$ to counteract the lumped uncertain term $H_d(\cdot)$ and incorporate a robust unit to compensate the NN reconstruction error $\|\kappa\| \le \varepsilon_0 < \infty$. This leads to the following result.

Theorem 4

Consider the train with actuation faults satisfying the condition as imposed in (27). If the control force F is designed as in (12a) and (12b), then asymptotic velocity and position tracking is achieved.

$$F = -\Lambda^T \left(k_0 S + \hat{W}^T \Phi - u_c\right) \tag{30a}$$

$$u_c = -\hat{a} \frac{\left(\|\hat{W}^T \Phi\| + 1\right) S}{\|S\|} \tag{30b}$$

where

$$\dot{\hat{W}} = \Phi S^T \tag{31a}$$

$$\dot{\hat{a}} = \sigma_1 \left(\|\hat{W}^T \Phi\| + 1\right) \|S\| \tag{31b}$$

Proof:

With the proposed control (30), the closed-loop error dynamics become

$$M\dot{S} = -k_0 \Lambda\Gamma\gamma\Lambda^T S - \Lambda\Gamma\gamma\Lambda^T \hat{W}^T \Phi + \Lambda\Gamma\gamma\Lambda^T u_c + \mathrm{H}_d(\cdot) \tag{32}$$

Using the NN unit, the lumped uncertain term $H_d(\cdot)$ is now replaced by $W^T\Phi(z) + \kappa(z)$. As a result, (32) can be readily expressed as

$$M\dot{S} = -k_0 \Lambda\Gamma\gamma\Lambda^T S + \tilde{W}^T \Phi + \Lambda\Gamma\gamma\Lambda^T u_c + \gamma_d(\cdot) \tag{33}$$

with $$\gamma_d(\cdot) = \left(I - \Lambda\Gamma\gamma\Lambda^T\right) \hat{W}^T \Phi + \kappa \tag{34}$$

where $\tilde{W} = W - \hat{W}$. Let $a = \max\{\|I - \Lambda\Gamma\gamma\Lambda^T\|, \varepsilon_0\}$, it holds that

$$\|\gamma_d(\cdot)\| \le a\left(\|\hat{W}^T \Phi\| + 1\right) \tag{35}$$

Consider the Lyapunov function candidate

$$V = \frac{1}{2} S^T M S + \frac{tr\left(W - \hat{W}\right)^T \left(W - \hat{W}\right)}{2} + \frac{\left(a - \lambda_m \hat{a}\right)^2}{2\lambda_m \sigma_1} \tag{36}$$

where $0 < \lambda_m \le \lambda_{\min}(\Lambda\Gamma\gamma\Lambda^T)$ and $\lambda_{\min} > 0$ is the minimum eigenvalue of $\Lambda\Gamma\gamma\Lambda^T$. The rest of the proof follows the same lines as in the proof Theorem 1 (with the update algorithms (31a) and (31b)).

Remark 5: It is noted that the control schemes (12), (23), and (30) do not require detail information of the system, nor the need for estimating $\lambda_{\min}$, thus the implementation is fairly easy. Also note that these schemes nvolve the switch function $S/\|S\|$ which might cause chattering as $\|S\|$ approaches to zero. A simple and effective solution is to replace it by $S/(\|S\| + \Delta)$, where Δ is a small constant.

IV. Simulation

To test the performance of the proposed control strategies, simulation tests are carried out on a train similar to CRH-5 with eight vehicles (i.e., 4 locomotives with both motoring and braking capabilities and 4 carriages with braking capabilities). The travel distance tested in the simulation is $68.958 km$ which covers two acceleration phases, four cruise phases and three braking phases. The goal is to make the actual velocity $\dot{X}$ and position X track the desired velocity $\dot{X}^*$ and desired position X^* with high precision, respectively. With three fading actuators, the fault-tolerant control algorithms (30a)-(31b) are tested and the results are presented in Figures 1-2, from which one can observe that the proposed neuroadaptive fault-tolerant control scheme performs well even if some of the actuator lose their effectiveness during the system operation.

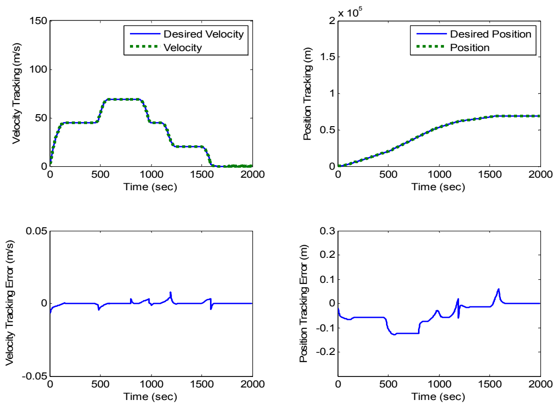

Fig. 1. Velocity and position tracking process and errors

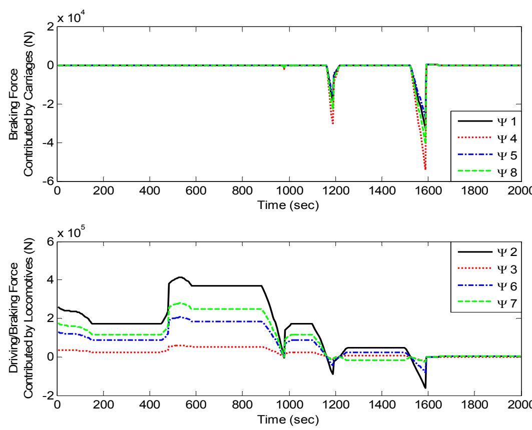

Fig. 2. Actual driving/braking force

V. CONCLUSIONS

Speed and position tracking control of high speed train with multiple vehicles is studied in this paper. Input nonlinearities, actuator failures, and in-train forces are considered explicitly in control design. Several control algorithms are developed based on Lyapunov stability theory. The salient feature of the proposed control lies in its simplicity in design and effectiveness in dealing with various uncertainties and nonlinearities or even actuation faults, as confirmed by theoretical analysis and numerical simulations.

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