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A Sub-optimal Sensor Scheduling Strategy using Convex Optimization

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***Abstract*— In this paper, we consider a sub-optimal off-line stochastic scheduling of a single sensor that visits (measures) one site, modeled as a discrete-time linear time-invariant (DTLTI) dynamic system, at each time instant with the objective to minimize certain measure of the estimation error. The objective of this paper is to search the optimal probability distributions under two cost functions. We show that the optimal scheduling distribution is computable by solving a quasi-convex optimization problem in the case we focus on the minimization of maximal estimate error among sites. When the cost function is the average estimate error of all sites, the scheduling problem for a set of special DTLTI systems can be casted and efficiently solved as a convex optimization problem by exploiting the structure of the underlying Riccati-like equation. Furthermore, we propose a deterministic scheduling strategy based on the optimal stochastic one. Finally, we show some simulation results to verify our strategies.**

***Index Terms*—** Kalman Filter, Quasi-convexity, Linear Matrix Inequality, Riccati-like Equation

I. INTRODUCTION

In this paper, we consider a situation where the evolution of N independent dynamic events (i.e. temperature, humidification, etc.) spatially distributed in an area need to be tracked (estimated) by a single entity, which we call sensor, like a mobile robot. We assume that the sensor has limited sensing range and therefore it needs to be in proximity of the dynamic events for obtaining measurements. The general question is to find the optimal site visiting strategy to minimize the estimation error. The assumption of a single sensor being used is motivated by some applications where the use of one sensor restricts the use of other sensors. One example, the implement of sonar range-finding devices, is presented in [1].

Because of the significance and wide applications, considerable research has focused on the sensor scheduling problems and its variations which include the sensor coverage problem [2], [3], [4], [5], [6] and the sensor selection problem [7], [8], [9]. In this paper, we assume that the dynamic events are the result of the evolution of DTLTI systems driven by Gaussian white noises, and the measurements are also affected by additive white Gaussian noise. When the systems dynamics and noise covariances are known, and the sensor has sufficient computational capabilities, the optimal scheduling that minimizes the tracking error covariance can in principle be derived at each step by running a tree-search algorithm in conjunction with Kalman filters [10]. This solution however is not practical. To deal with the complexity of a tree-search, many strategies have been proposed to prune the tree, i.e sliding-window, thresholding [7], relaxed dynamic programming [11], etc. However, these strategies may face difficulties again when some practical conditions (i.e measurement loss, special interest in certain event, etc) are considered.

In this paper, we concentrate on the search of off-line stochastic scheduling strategies. They are not only numerically more tractable than tree-search based strategies proposed in the literature above, but provide performance guarantees which are the upper bounds of the optimal scheduling strategy using Kalman filtering as well. There are other useful features as will be discussed later. For off-line strategies, the idea is that, at each time instant, the sensor visits site i according to some probability distribution to minimize certain measure of estimation error. Of course, the reduction of computational complexity but with guaranteed performance comes at the expenses of degradation of the ideal performance. However, in many situations the extra computational complexity cost may not be justified. Moreover the ideal performance is not easily computable. A setting similar to ours, which deals with the sensor selection problem, has been considered in [8] but no optimal solution method of the proposed strategy has not been established as instead we do in this paper.

The paper is organized as follows. In section II, we model and formulate the problem mathematically. In section III and IV, we develop results and algorithms for off-line stochastic scheduling under two cost functions, respectively. In section V, we give an approach to obtain a deterministic strategy based on the optimal stochastic one. At last, we present several simulations to verify our results.

Throughout the paper, A' is the transpose of matrix A. $Ones(n,n)$ implies a $n \times n$ matrix with 1 as all its entries. $Diag(V)$ denotes a diagonal matrix with vector V as its diagonal entries. $M \geq 0$ or $M \in S_+$ implies matrix M is positive semi-definite where S_+ represents the positive semi-definite cone.

II. MODELING AND PROBLEM FORMULATION

A. Modeling

Consider a set of N independent DTLTI systems to be measured evolving according to the equation

$$\begin{aligned} x_i[k+1] &= A_i x_i[k] + w_i[k] \quad i = 1,2,\dots N \\ y_i[k] &= C_i x_i[k] + v_i[k] \end{aligned} \tag{1}$$

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where we assume that (A_i, C_i) is detectable. $x_i[k] \in \mathbb{R}^{n_i}$ is the process state vector and $y_i[k] \in \mathbb{R}^{p_i}$ is the output vector. $w_i[k] \in \mathbb{R}^{n_i}$ and $v_i[k] \in \mathbb{R}^{p_i}$ are the process noise and measurement noise which are white Gaussian and zero mean with covariance matrix Q_i, R_i, respectively. The initial state $x_i[0]$ is assumed to be a Gaussian zero mean random variable with covariance matrix $\pi_i[0]$. Note that the DTLTI system i modeled above may represent the local dynamic change of environment in its neighborhood. In the paper, without additional explanation, i takes values from the integer set $\{1,2,\cdots,N\}$. Since the case that A_i is stable (i.e all eigenvalues of A_i are less than or equal to unity in magnitude.) is very important in a large number of practical applications of estimation [8], we give the following assumption.

Assumption 1: All independent DTLTI systems to be estimated are stable.

We remark that all the exposition can be easily extended into the case of unstable DTLTI systems. Next, we give another assumption which a bit simplifies the model.

Assumption 2: All the modeled systems to be measured are evolving independently.

The more general coupled case will be explored in the future work. Our model allows a single sensor to keep hopping/switching among these N possible sites at which it takes measurements of the systems.

Assumption 3: The sensor at time instant k has only access to the i-th dynamic system measurement $\tilde{y}_i[k]$.

This assumption arises from the sensor's limited range of sensing and can be modeled mathematically as

$$\tilde{y}_i[k] = \xi_i[k] y_i[k]$$

where ξ_i is the indicator function indicating whether or not the sensor is at location i at time instant k and $\sum_{i=1}^{N} \xi_i[k] = 1$ since only one system can be measured by the single sensor at each time step k. Note that the real sensor scheduling problem is to decide on-line which site to visit at each time instant. Thus, the estimators implemented by the sensor are random but dependent on the current states. In this paper, we are interested in scheduling strategies which are simpler to implement. These strategies can be computed off-line and come with performance guarantees and thus, are useful as they provide tight upper bounds on the optimal scheduling strategy. Motivated by the above discussions, we remove the dependence and assume that ξ_i is an i.i.d Bernoulli random variable with distribution

$$\xi_i[k] = \begin{cases} 1 & \text{with probability } q_i \\ 0 & \text{with probability } 1-q_i \end{cases} \qquad i = 1,2,\ldots N$$

where q_i is the probability that the sensor chooses to visit system i at each time instant.

In this settings, for a given set of q_i's, the optimal estimator for each system is Kalman filter with intermittent observations. The covariance of the estimate error $P_i[k]$ for system i evolves according to the following Riccati-like equation.

$$\begin{aligned} P_i[k+1] =& A_i P_i[k] A_i' + Q_i - \\ & \xi_i[k] A_i P_i[k] C_i' (C_i P_i[k] C_i' + R_i)^{-1} C_i P_i[k] A_i' \end{aligned}$$

Since the sequence $\{\xi_i[k]\}_{k=0}^{\infty}$ is random, the above Riccati-like equation is stochastic and can not be determined off-line. Instead, we focus on the iteration of $\mathbb{E}[P_i[k]]$ where the expectation operator is taken over $\xi_i[k]$. Similar to [12], we define the following modified algebraic Riccati equation (MARE)

$$g_{q_i}(X_i) = A_i X_i A_i' + Q_i - q_i A_i X_i C_i' (C_i X_i C_i' + R_i)^{-1} C_i X_i A_i' \quad (2)$$

Note that the concavity of the MARE allows use of Jensen's inequality to find an upper bound on $\mathbb{E}[P_i[k]]$. That is,

$$\begin{aligned} \mathbb{E}[P_i[k+1]] \leq & A_i \mathbb{E}[P_i[k]] A_i' + Q_i - \\ & q_i A_i \mathbb{E}[P_i[k]] C_i' (C_i \mathbb{E}[P_i[k]] C_i' + R_i)^{-1} C_i \mathbb{E}[P_i[k]] A_i' \end{aligned}$$

As long as A_i is stable, MARE (2) converges to $\bar{X}_i$ (i.e. $\bar{X}_i = \lim_{k\to\infty} X_i[k]$ where $X_i[k+1] = g_{q_i}(X_i[k])$) for $\forall q_i \in \mathbb{R}_{[0,1]}$, where $\bar{X}_i$ is the unique positive-semidefinite fixed point of the MARE. Note that $\bar{X}_i$ is the upper bound of the steady-state value of $\mathbb{E}[P_i[k]]$.

B. Problem Formulation

Motivated by the above discussion, in this paper, we concentrate on minimizing some measure of $\bar{X}_i$ as a means to keep the steady-state value of $\mathbb{E}[P_i[k]]$ itself small. We consider two cost functions on $\bar{X}_i's$.

$$J_1(\bar{X}_1, \bar{X}_2, \cdots, \bar{X}_N) = \max_i f_i(\bar{X}_i)$$

$$J_2(\bar{X}_1, \bar{X}_2, \cdots, \bar{X}_N) = \frac{1}{N} \sum_{i=1}^{N} f_i(\bar{X}_i)$$

where f_i is assumed to be a monotone increasing linear mapping: $\mathbb{S}_+^{n_i} \to \mathbb{R}$,

$$X \geq Y \Rightarrow f_i(X) \geq f_i(Y) \quad \text{for} \quad X, Y \in \mathbb{S}_+^{n_i}$$

In the case where N systems are modeled for different phenomena (i.e. temperature, humidification, etc), these linear mappings allows us to uniform/rescale the measure of $\bar{X}_i$ such that the estimated states of N systems are in comparable unit. Based on previous assumptions and discussions, the objective of this paper is to search an optimal probability distribution (i.e q_i's) to minimize the cost function. The problem for our strategy is formulated as follows.

$$\begin{aligned} \underset{q_i, \bar{X}_i}{\text{minimize}} \quad & J \\ \text{subject to} \quad & \bar{X}_i = g_{q_i}(\bar{X}_i) \\ & \sum_i q_i = 1 \\ & q_i \geq 0 \quad i = 1,2,\ldots N \end{aligned} \quad (3)$$

where $J \in \{J_1, J_2\}$.

III. Minimization of The Maximal Estimate Error among Sites.

In this section, we consider problem (3) with J_1 as the cost function, which is denoted by OP I. It is shown that this problem can be solved by investigating a related quasi-convex optimization problem. Then, an efficient Nested-Bisection algorithm is proposed to solve OP I.

By adopting J_1 as the cost function, we are able to partially decouple OP I in terms of objective functions. That is,

$$\max_i f_i(\bar{X}_i) \le \gamma \Leftrightarrow f_i(\bar{X}_i) \le \gamma \quad \text{for all} i \tag{4}$$

where $\gamma \in \mathbb{R}$. Note, however, that the constraint $\sum_i q_i = 1$ still couples the N DTLTI systems. In order to break this constraint, we consider the following optimization problem for each system i where $f_i(\bar{X}_i) \le \gamma$ is treated as a new constraint.

$$\begin{aligned} \underset{q_i,\bar{X}_i}{\text{minimize}} \quad & q_i \\ \text{subject to} \quad & \bar{X}_i = g_{q_i}(\bar{X}_i) \\ & f_i(\bar{X}_i) \le \gamma, \quad q_i \ge 0 \end{aligned} \tag{5}$$

where $\gamma \in \mathbb{R}^+$ can be viewed as a pre-assigned estimate performance. The solution of problem (5) q_i^{opt} implies the smallest probability required for measuring system i for achieving the pre-assigned estimate performance γ. By solving problem (5) for all i, we can obtain the value of $\sum_{i=1}^N q_i^{opt}$. By increasing or decreasing γ, we can drive the value of $\sum_{i=1}^N q_i^{opt}$ equal to 1 such that OP I is solved.

Next, we give a lemma in order to establish that the computation of the solution of problem (5) can be reformulated as the iteration of an Linear Matrix Inequality (LMI) feasibility problem. Without abuse of notation, we remove the subscript i since the following results apply to each of the N systems.

Lemma 1: Assume that $(A, Q^{1/2})$ is controllable and (A,C) is detectable. For any given $q \in \mathbb{R}(0,1]$ and invertible matrices Q and R, the following statements are equivalent:

$a)$ $\exists X \in S_+$ such that $X = g_q(X)$.

$b)$ $\exists K$ and $X \in S_+$ such that $X \ge \Phi(K,X)$.

$c)$ $\exists H$ and $G \in S_+$ such that $\Gamma_q(H,G) \ge 0$.

where

$$\begin{aligned} \Phi(K,X) =& (1-q)(AXA' + Q) \\ & + q(A+KC)X(A+KC)' + qQ + qKRK'. \end{aligned}$$

$$\Gamma_q(G,H) = \begin{bmatrix} G & qGA + qHC & G & GA - qGA & qH \\ (\cdot)' & qG & 0 & 0 & 0 \\ (\cdot)' & (\cdot)' & Q^{-1} & 0 & 0 \\ (\cdot)' & (\cdot)' & (\cdot)' & G' - qG' & 0 \\ (\cdot)' & (\cdot)' & (\cdot)' & (\cdot)' & qR^{-1} \end{bmatrix}$$

The proof of $c)$ takes the change of variable $G = X^{-1}$ and $H = X^{-1}K$. Based on Lemma 1, we immediately obtain the following theorem.

Theorem 1: If $(A, Q^{1/2})$ is controllable and (A,C) is detectable, the solution of the optimization problem (5) can be obtained by solving the following quasi-convex optimization problem in variables (q,G,H,X).

$$\begin{aligned} \underset{q,G,H,Y}{\text{minimize}} \quad & q \\ \text{subject to} \quad & f(Y) \le \lambda, \quad i = 1,2,\ldots N \\ & \begin{bmatrix} Y & I \\ I & G \end{bmatrix} \ge 0 \\ & \Gamma_q(G,H) \ge 0, \quad q \ge 0, \quad G > 0 \end{aligned} \tag{6}$$

Specifically, the solution is obtained by using bisection for variable q and iterating LMI feasibility problems.

Based on Theorem 1, an algorithm to drive the value of $\sum_i q_i^{opt}$ equal to 1 while solving OP I is a Nested Bisection algorithm described as follows.

Algorithm:Nested-Bisection algorithm for OP I

Given $l \le \gamma^{opt}, u \ge \gamma^{opt}$, tolerance $\quad \varepsilon \ge 0$

repeat

1. $\gamma = \frac{l+u}{2}$.
2. For each system T_i, solve optimization problem (6) by bisection algorithm for the variable q_i [13]. Denote the minimized objective value of (6) as q_i^{opt}
3. If $\sum_i q_i^{opt} \le 1, u := \gamma$; else $l := \gamma$.

until $u - l \le \varepsilon$

where γ^{opt} is the optimal objective value of OP I.

Note that the interval $[l,u]$ is guaranteed to contain γ^{opt}, i.e., we have $l \le \gamma^{opt} \le u$ at each step. Therefore, the algorithm is guaranteed to converge to the optimal objective value γ^{opt}.

IV. Minimization of The Average Estimate Error among Sites

In this section, we consider problem (3) with J_2 as the cost function, which is denoted by OP II. Unfortunately, the previous approach does not work and we do not have a general solution yet. In what following, our approach is to exploit the structure of one special class of DTLTI systems (i.e. single-state process with measurement delays) to solve this problem. Exploring this problem in general is a direction of our future research.

A. Modeling of Single-State Systems with Measurement delay

Consider a set of N DTLTI single-state systems to be measured evolving according to the equation

$$x_i[k+1] = a_i x_i[k] + w_i[k] \tag{7}$$

where $x_i[k], v_i[k], w_i[k] \in \mathbb{R}$ and the covariance of w and v are $Q_i \in \mathbb{R}^+$ and $R_i \in \mathbb{R}^+$, respectively. The measurement taken by the single sensor at each time instant is formulated as follows.

$$\tilde{y}_i[k] = \xi_i[k](x_i[k-d_i] + v_i[k])$$

where d_i represents the delay in measurement, which we assume to be fixed and known in this paper.

To deal with delays, which may come from image processing process or wireless communication, an expanded state space system by adding new states corresponding to the measurement delays of system i is defined. The augmented states are defined as follows

$$\begin{aligned} x_i^1[k] &= x_i[k-d_i] \\ x_i^2[k] &= x_i[k-d_i+1] \\ &\vdots \\ x_i^{d_i}[k] &= x_i[k-1] \end{aligned}$$

We then obtain the following compact form for system i with measurement delays.

$$\begin{aligned} X_i[k+1] &= A_iX_i[k] + B_iw_i[k] \\ \tilde{y}_i[k] &= \xi_i[k](C_iX_i[k] + v_i[k]) \end{aligned} \tag{8}$$

where X, A, B, C has the following structure

$$X_i = \begin{bmatrix} x_i^1 \\ x_i^2 \\ \vdots \\ x_i^{d_i} \\ x_i \end{bmatrix}, \quad A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & a_i \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{9}$$

$$C_i = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$$

Note that $x_i[k]$ is the true state of system i at time step k, other states included in vector $X_i[k]$ are dummy variables for handling delays.

B. Solutions of OP II

By exploiting the special structure of above model, we are able to obtain the closed-form fixed point of the MARE. We present this result in the following theorem. Subscript i is removed without any abuse of notation.

Theorem 2: Consider a MARE (i.e. $\bar{X} = g_q(\bar{X})$) in $\mathbb{R}^{n\times n}$ where q is given and (A,B,C) have the structure presented in (9). Then MARE has a unique positive-semidefinite fixed point $\bar{X}$ as

if $a = 1$,

$$\bar{X} = \begin{bmatrix} x_1 & x_1 & \cdots & x_1 \\ x_1 & x_2 & \cdots & x_2 \\ \vdots & & \ddots & \vdots \\ x_1 & x_2 & \cdots & x_n \end{bmatrix} \tag{10}$$

where

$$x_j = \frac{Q + \sqrt{Q^2 + 4qQR}}{2q} + (j-1)Q \quad j = 1, \cdots, n$$

if $0 < a < \sqrt{\frac{1}{1-q}}$ and $a \neq 1$,

$$\bar{X} = \begin{bmatrix} x_1 & ax_1 & \cdots & a^{n-1}x_1 \\ ax_1 & x_2 & \cdots & a^{n-2}x_2 \\ \vdots & \vdots & \ddots & \vdots \\ a^{n-1}x_1 & a^{n-2}x_2 & \cdots & x_n \end{bmatrix} \tag{11}$$

where

$$x_1 = \frac{Ra^2 - R + Q + \sqrt{(Ra^2 - R + Q)^2 - 4(a^2 - 1 - a^2q)QR}}{2(1 + a^2q - a^2)}$$

$$x_j = a^{2(j-1)}x_1 + \frac{1 - a^{2(j-1)}}{1 - a^2}Q \quad j = 2, 3, \ldots, n$$

Note that if $a \geq \sqrt{\frac{1}{1-q}}$, MARE can not converge to a steady state value [12]. Note, however, that MARE as ARE cannot be solved in closed-form without any special structure and this is a main limitation of our approach.

By using the closed-form fixed point of the MARE, we can transform OP II into a simplified form which can be solved by standard techniques. As an example, we consider the linear mapping $f_i(\bar{X})$ in J_2 as

$$f_i(\bar{X}_i) = V'\bar{X}_iV \tag{12}$$

where $V = [0, 0, \cdots, 0, 1]'$ has the compatible dimension. Note that by ignoring the penalty of the delay d_i, this linear mapping $f_i(\bar{X}_i)$ only takes the estimate of the state of system i into consideration. Due to the space constraint, we restrict our treatment to consider the case where $a_i = 1$ for all i. The more general case (i.e. $a_i \neq 1$) can be easily explored in parallel.

Corollary 1: 1) If (A_i, B_i, C_i) has the structure in (9) and the linear mapping f_i in J_2 is defined as (12), then the solution of the OP II can be obtained by solving the following convex optimization problem

$$\begin{aligned} \underset{q_i}{\text{minimize}} \quad & \sum_i \frac{Q_i + \sqrt{Q_i^2 + 4q_iQ_iR_i}}{2q_i} + d_iQ_i \\ \text{subject to} \quad & \sum_i q_i = 1 \quad q_i \geq 0 \quad i = 1, 2, \ldots N \end{aligned} \tag{13}$$

Moreover, the solution (i.e. q_i's) is independent from delays. 2) In the special case where $R_i = 0$ for all systems (i.e. no measurement noise), the optimization problem (13) can be solved by LMI as follows

$$\begin{aligned} \underset{q_i, t}{\text{minimize}} \quad & t \\ \text{subject to} \quad & \sum_i q_i = 1 \quad q_i \geq 0 \quad i = 1, 2, \ldots N \\ & \begin{bmatrix} t - \sum_i d_iQ_i & Q_1^{1/2} & \cdots & Q_{N-1}^{1/2} & Q_N^{1/2} \\ Q_1^{1/2} & q_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ Q_{N-1}^{1/2} & 0 & \cdots & q_{N-1} & 0 \\ Q_N^{1/2} & 0 & \cdots & 0 & q_N \end{bmatrix} \geq 0 \end{aligned}$$

The proof is straightforward to obtain from the closed-form fixed point of the MARE.

V. Generation of a Deterministic Scheduling Sequence

After obtaining the optimal probability distribution, we can make the sensor visit each DTLTI system on-line according to a scheduling sequence which is generated randomly w.r.t this optimal distribution. Instead of using an actual random schedule, it may be of interest to derive deterministic schedules based on the optimal random one. In this section we are interested in finding a deterministic scheduling sequence to make the measure of estimation error small. In particular, we consider sequences of minimal consecutiveness.

Definition 1: Let $\{s[k]\}_{k=1}^{L}$ be a set of sequences with length L, satisfying that each element $s[k]$ in the sequence is taken value from an element set $\mathbb{K} = \{a_i | i \in \{1,2,\cdots,N\}\}$ and the number of occurrences of each value a_i in the sequence is n_i. Then the sequence of minimal consecutiveness is the solution of the following optimization problem

$$\min_{\{s[k]\}_{k=1}^{L}} \max_{i,j \in \{1,2,\cdots,N\}} \{j-i+1 | j \geq i, s[i] = s[i+1] = \cdots = s[j]\}$$

Note that the minimal consecutiveness sequence is not unique. Given the optimal distribution, we know that the optimal stochastic scheduling sequence would be compatible with the optimal distribution. Then we can heuristically generate a deterministic sequence of minimal consecutiveness based on the optimal stochastic one. As an example, suppose that we plan to generate a scheduling sequence with length $L = 13$ from element set $\{1,2\}$ w.r.t an independent and identically distribution

$$\begin{aligned} Prob(s[k] = 1) &= 0.3 \\ Prob(s[k] = 2) &= 0.7 \end{aligned}$$

The optimal scheduling sequence would have approximately four 1's and nine 2's. Then a sequence of minimal consecutiveness is

$$\{s[k]\}_{k=1}^{13} = 2212212212212$$

which has the minimal consecutiveness 2. The intuition for concentrating on sequences of minimal consecutiveness is that, under this class of sequences, the sensor visits all sites in the shortest time compatible with the optimal distribution. Thus, it avoids temporary build-up of error covariance due to possibly long strikes/visits to one location.

Note that, in our off-line scheduling strategy, the number n_i of occurrences of each value in the sequence can be obtained approximately compatible with the optimal probability distribution (i.e the example presented above). In practice, we generate this deterministic scheduling sequence off-line and implement it to the sensor.

VI. Examples and Simulations

In this section, we present some examples and simulations.

Example A

Consider a single sensor for measuring two DTLTI systems which are located in physically different places. We solve OP I by using Nested-Bisection algorithm. The DTLTI systems are presented in controllable canonical form as follows.

$$A_1 = \begin{bmatrix} 0 & 1 \\ -0.49 & 1.4 \end{bmatrix} \quad , C_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}' \quad Q_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ -0.72 & 1.7 \end{bmatrix} \quad , C_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}' \quad Q_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$R_1 = 0.5 \quad R_2 = 1$$

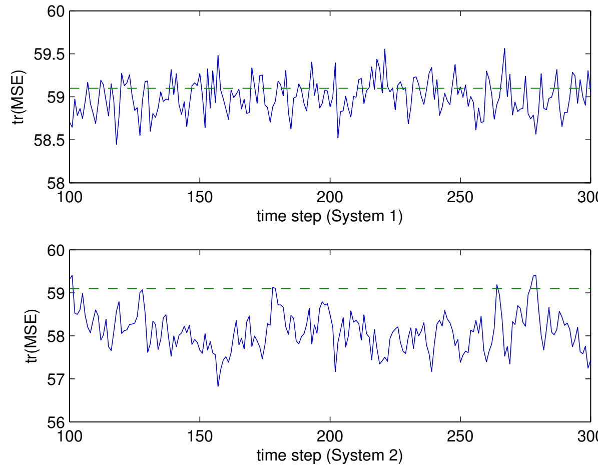

Fig. 1. Empirical state error covariances for DTLTI system 1 and 2 with zero initial condition. The curves are obtained by averaging 5000 Monte Carlo simulation for $k = 1,2,\cdots,300$

The following cost function are considered in this example.

$$J(\bar{X}_1, \bar{X}_2) = \max_{i \in [1,2]} trace(\bar{X}_i)$$

By running the Nested-Bisection algorithm, the optimal probability distribution is $[0.674, 0.326]$ and the minimized objective value of OP I is 59.1. We preassign this optimal probability distribution to the sensor and obtain the empirical error state covariance of each system (as shown in Fig.1) by accordingly generating random tracking sequences. it is shown that the error state covariances are fluctuating with average values 58.7 and 57.5 which are tightly upper bounded by the optimal objective value 59.1 (dashed line).

Next, we generate a scheduling sequence of minimal consecutiveness with length 300 according to the optimal distribution $[0.674, 0.326]$, we let the sensor to take measurement from these two systems under this sequence and the average value of the trace of the error covariance turns out to be 55.53. We next compare this strategy with sliding window algorithm [10]. By running the sliding window algorithm with size 1 for scheduling, the percentages of visits to event 1 and 2 are about 0.673 and 0.327, respectively and the average value of the trace of the error covariance turns out to be 55.61 which is close but worse than 55.53.

Note that solvability of the OP II for this example is not clear since the systems do not have the special structure assumed in (9). Since these exist only two systems considered in this example, we are able to solve the OP II by appropriately gridding $[q_1, q_2]$. The minimized objective value of OP II is 53.6 and the solution $[q_1, q_2]$ is $[0.480, 0.520]$ which is quite different with the solution of OP I.

Example B

In this example, we verify our results in section IV. Consider three random-walk vehicles in an area and a single sensor equipped with a camera is used for tracking their $1-D$ positions. The dynamics of their positions are evolving as (7) with $a_i = 1$. But they are subject to different process

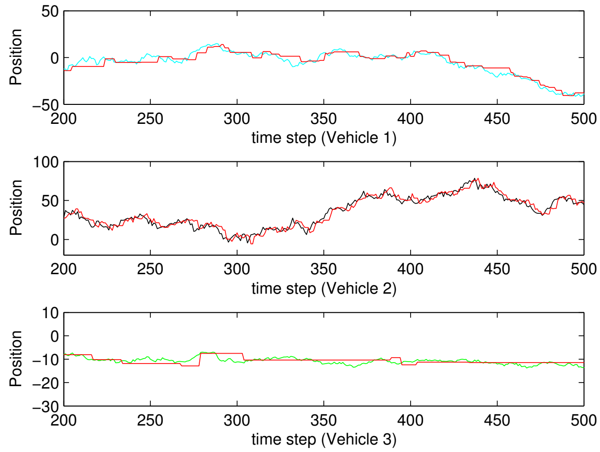

Fig. 2. Positions of random-walk vehicles and tracking paths: Less varying vehicle gets less attention from the camera while the tracking performance is good.

noises, measurement noises and delays. Here we assume that these vehicles have 1, 2 and 2 time-step measurement delays, respectively. We have

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}'$$

$$A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad C_i = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}' \quad i = 2, 3$$

$$Q_1 = 3 \quad Q_2 = 10 \quad Q_3 = 0.2$$

$$R_1 = R_2 = 5 \quad R_3 = 1$$

Remember that Q and R are variances of random variables w and v which have zero means. By taking the linear mapping f_i in (12), the solution of OP II is $[q_1, q_2, q_3] = [0.3395, 0.4945, 0.1660]$ with the minimized objective value 20.7. In Fig.2, the tracking paths (red curves) are shown in comparison of actual time-varying positions of random-walk vehicles. The flat segments of red curves imply that no measurement is taken in this time slot and the estimator simply propagates the state estimated of the previous time-step. It is shown that the tracking path of vehicle 3 (green curve) has most flat segments as a result of smallest *visiting frequency*. In comparison, the tracking paths of vehicle 1 (cyan curve) and vehicle 2 (black curve) match the actual positions much better even though the flat segments in tracking path of vehicle 2 is much visible.

VII. Conclusion

In this paper, we have presented a sub-optimal strategy for sensor scheduling problem. Firstly, we consider a stochastic strategy where the sensor visits each site randomly according to some probability distribution. By minimizing the maximal estimate error among N sites, the optimal distribution can be obtained by the proposed Nested-Bisection algorithm based on solving a set of quasi-convex optimization problems. By minimizing the average estimate error over all sites, we can transform the scheduling problem into a convex form by exploiting the closed-form fixed point of the MARE. Furthermore, we propose a deterministic strategy based on the optimal stochastic one. That is, generate a scheduling sequence of minimal consecutiveness instead of generating it randomly according to the optimal distribution. Finally, we present some examples and simulation results.

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