2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

Robust SDC Parameterization for a Class of Extended Linearization Systems

Sam Nazari and Bahram Shafai
Department of Electrical and Computer Engineering
Northeastern University, Boston, MA
Email: nazari.s@neu.edu, shafai@ece.neu.edu

Abstract

We consider nonlinear regulation of systems with parametric uncertainty. Under mild conditions, these systems can be brought into a psuedo-linear form known as extended linearization. Under this formulation, conventional linear control synthesis methods can be applied. One popular technique that mimics the LQR method of optimal linear control is referred to as the State-Dependent Riccati Equation (SDRE) approach. SDRE control relies on a non-unique factorization of the system dynamics known as the State Dependent Coefficient (SDC) parameterization. Under system uncertainty, each SDC parameterization will produce its own radius of stability in a region of interest in the state space. In this paper a method to compute the radius of stability in a special class of systems is used to obtain the SDC parameterization which results in the maximum radius of stability for the original nonlinear system in the region of interest. It is shown that the problem of finding the maximum radius of stability from a hyperplane of SDC parameterizations can be reduced to constrained minimization of the spectral norm of a comparison system.

1. Introduction

For the infinite-horizon, autonomous, nonlinear regulator problem that is affine in input

$$\dot{x} = f(x) + g(x)u \tag{1}$$

We seek the control, u, that minimizes

$$\frac{1}{2}\int_{t_0}^{\infty} (x^T Q(x)x + u^T R(x)u)\ dt \tag{2}$$

over the infinite horizon. Here $x \in \mathbb{R}^n$ represents the state vector, $u \in \mathbb{R}^m$, denotes the control, both $f(x)$ and $g(x)$ are C^k functions with the mappings $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$. $Q(x)$ is a continuous positive semi-definite state weighing matrix. The variable $R(x)$ represents a penalty associated with the control effort.

There are various methods for analysis and design of nonlinear systems [1]. In the method of *extended linearization* [2], also known as *State-Dependent-Coefficient* (SDC) representation, Eq (1) is factored into a linear-like structure with State-Dependent matrices. A continuous, nonlinear matrix-valued function, $A(x)$, can always be obtained by mathematical factorization if:

Condition 1. $f(0) = 0$

Condition 2. $f(x)$ is continuously differentiable, that is

$$f(x) \in C^1$$

If these conditions are met, $f(x)$ can be factored as:

$$f(x) = A(x)x \tag{3}$$

Note that, as shown by Cloutier in [3], the factorization in Eq (3) is non-unique for $n > 1$. Denoting $g(x) = B(x)$, the system in Eq (1) becomes:

$$\dot{x} = A(x)x + B(x)u \tag{4}$$

Eq (4) possesses a desirable linear structure with SDC matrices $A(x)$ and $B(x)$. Under this formulation, conventional linear control synthesis methods can be applied. One popular technique that mimics the LQR method of optimal linear control is referred to as the *State-Dependent Riccati Equation (SDRE)* approach or the *State-Dependent LQR* methodology (SDLQR). This approach consists of three steps:

1. Bring the system in Eq (1) to SDC form (Eq 4).
2. Solve the state dependent matrix Riccati equation:

$$\begin{aligned} S(x)A(x) + A^T(x)S(x) - \\ S(x)B(x)R^{-1}(x)B^T(x)S(x) + Q(x) = 0 \end{aligned} \tag{5}$$

3. Form the control pointwise:

$$u(x) = -R^{-1}(x)B^T(x)S(x)x \quad (6)$$

This method requires that the pair $(A(x), B(x))$ be pointwise *stabilizable* in the linear sense $\forall x$. One sufficient check for this condition is to form the controllability matrix just as it is done for linear systems and then to check that the controllability matrix has full rank in the domain of interest.

Ultimately, the SDRE control procedure, or SDLQR, leads to a closed-loop system matrix

$$A(x)_{cl} = A(x) - B(x)K(x) \quad (7)$$

that is pointwise Hurwitz stable $\forall x$. The origin at $x = 0$ of an SDRE controlled system is locally asymptotically stable as shown in [3,4].

Partly due to its ease of application and design flexibility, SDRE has enjoyed widespread success in a number of applications. It has been utilized to explore optimal treatment modalities for HIV & AIDS, Missile Guidance, Satellite & Spacecraft design and to control the growing film thickness in high pressure chemical vapor deposition (HPCVD). Furthermore, SDRE has been applied to the design of advanced flight control systems, utilized in automotive control systems and has applications in process control.

As shown in [3], SDRE has been extended to nonlinear H_∞ control. In [5] the robustness properties of Extended Linearization systems are discussed and a result pertaining to the closed loop stability under matched disturbances is presented. Nonlinear systems with parametric uncertainty have been reported by many authors (see [6] and references therein). The author of [7] addresses parametric uncertainty of the SDRE control method, albeit through a design study. In [8], the stability region of convergence is addressed through a method that involves a comparison system which is overvalued.

Because the choice of $A(x)$ is non-unique, different factorizations may be considered which can lead to different robustness characteristics. Under system uncertainty, estimating the radius of stability of the SDRE controlled system is difficult since the closed-loop system equations are not available explicitly. Consequently, difficulties may arise for the designer wishing to assess the robustness and performance tradeoffs associated with each parameterization. In this paper, it is shown that maximizing the radius of stability under SDC parameterization for a multivariable SDRE controlled system corresponds to minimizing the constrained spectral norm of a parameterized comparison system. Our result is based on the work in [9] and is intended to address the problem of choosing the most robust parameterization arising in a hyperplane defined by two or more valid SDC parameterizations.

2. Robust SDC Parameterization

Cloutier has shown in [3] that for the multivariable case there always exists an infinite number of SDC parameterizations. This is because the state vector x has, by definition, at least two components x_1 and x_2 in the multivariable case. Suppose there exists a nonlinear scalar term $f_i(x)$ in one of the state equations. Then in that state equation at least two representations can be found corresponding to $f_i(x)/x_1$ and $f_i(x)/x_2$. Using any two valid SDC representations $A_1(x)$ and $A_2(x)$, an infinite number of parameterization can be constructed from the convex set

$$A(x,\alpha) = \alpha A_1(x) + (1-\alpha)A_2(x), \quad \alpha \in [0,1] \quad (8)$$

In general for $k+1$ valid SDC parameterizations α will be of dimension k and $A(x,\alpha)$ will be the hyperplane

$$\begin{aligned} A(x,\alpha) &= (1-\alpha_k)A_{k+1}(x) \quad (9) \\ &+ \sum_{i=1}^{k}\left(\prod_{j=i}^{k}\alpha_j\right)(1-\alpha_{i-1})A_i(x) \end{aligned}$$

where $\alpha_0 \triangleq 0$.

Parametric uncertainty in linear systems can be expressed as [10]:

$$\dot{x} = [A + D\Delta E]x$$

Now let the SDC parameterization of a nonlinear system be represented by

$$\dot{x} = A(x,\alpha)x + B(x)u \quad (10)$$

then it can also be put into the robust formulation. If there is uncertainty in the system parameters, the nonlinear SDC parameterization simply includes a term for system uncertainty.

Definition 1. Let the SDC parameterization with structured uncertainty be represented by

$$\dot{x} = [A(x,\alpha) + D\Delta E]x \quad \forall x \in \mathbb{R}^n \quad (11)$$

where it is assumed that $A(x)$ is pointwise stable or stabilized according to Eqn (7). Then Eqn (11) is called the *parameterized robust SDC* representation of the nonlinear system, where D and E are $n \times r$ and $s \times n$

dimensional structured matrices, $\alpha \in [0,1]$ and Δ is an unknown uncertainty matrix confined to a region of interest.

To our knowledge, no systematic approach has been introduced for choosing α that will result in the SDC parameterization that is maximally robust with respect to parametric uncertainty.

3. SDC Comparison System

This section is devoted to overvalueing the SDC parameterization by using a comparison system possessing Metzlerian structure.

3.1. Preliminary Analysis

In order to address the problem of maximizing the radius of stability for a family of SDC parameterizations under uncertainty using SDRE, a region of interest in the state space needs to be defined. By overvaluing the SDC parameterization in this region, the overall system is brought into a form that possess special properties. The resulting *comparison system* will be Metzlerian and will be used to ascertain the parameter that robustifies the SDC parameterization.

Theorem 1. *[11] The matrix $M : \mathbb{R}^n \to \mathbb{R}^{m\times m}$ defines an overvaluing system of Eq (10) with respect to the vector norm $p(x)$ if and only if the following inequality is verified for each corresponding component:*

$$D^+ p(x) \le M(x,\alpha)p(x) \qquad \forall x \in \mathbb{R}^n$$

Where D^+ is the upper Dini derivative operator [1]. Denote I_i and I_j as the sets of the indices of the rows and columns of the blocks A_{ij} of any matrix $A(x,\alpha)$. If $p_i(x)$ is the maximum of the modulus of each component of x_i, then:

$$\begin{aligned} M(x,\alpha) &= \mu_{ij}(x,\alpha), \qquad \mu_{ij} : \mathbb{R}^n \to \mathbb{R} \\ \mu_{ij}(x,\alpha) &= \max_{s\in I_i}[a_{ss} + \sum_{l\in I_i, l\neq s} |a_{sl}|] \\ \mu_{ij}(x,\alpha) &= \max_{s\in I_j}[\sum_{l\in I_j} |a_{sl}|] \end{aligned}$$

The resulting $M(x,\alpha)$ system is an overvaluing matrix. Then, μ_{ij} is the greatest sum of all components in each row of the block in A. The following lemmas will be of interest in determining the special properties of the system matrix, $A(x,\alpha)$. The proofs are found in [12].

Lemma 1. *Let a psuedo-overvalued matrix $M(x,\alpha)$ of $A(x,\alpha)$ be defined with respect to the vector norm $p(x)$. Then any one of the following conditions hold:*

i) The off diagonal elements $\mu_{ij}(x,\alpha), (i \neq j)$ of $M(x,\alpha)$ are nonnegative.

ii) If $\lambda_A(x)$ denotes any of the n eigenvalues of $A(x,\alpha)$, $\lambda_M(x)$ any of the k eigenvalues of $M(x,\alpha)$ and $\lambda_m(x)$ the maximal real part of $\lambda_M(x)$, then the following holds:

$$\Re(\lambda_A(x)) \le \Re(\lambda_M(x)) \le \lambda_m(x) \in \mathbb{R} \qquad \forall x \in \mathbb{R}$$

iii) When all the real parts of the $\lambda_M(x)$ are negative, or even if M satisfies the Koteliansky conditions then $M(x,\alpha)$ admits an eigenvector $u_m(x)$, called the importance vector of $M(x,\alpha)$, whose components are strictly positive and which is associated with the real, maximal and negative eigenvalue $\lambda_m(x)$.

Now the *robust comparison system* can be formed.

Definition 2. Suppose there exist a vector norm $p(x)$ and a matrix $M(x,\alpha)$ connected with Eq (11) such that the off diagonal elements of $M(x,\alpha)$ are all nonnegative and that the equality in Theorem 1 is satisfied along the solution of Eq (11) . Then the system

$$\dot{z} = (M(x,\alpha) + D\Delta E)z, \qquad \forall z \in \mathbb{R}^k_+$$

is a *robust comparison system* of Eq (11) in the sense that $z(t) \ge p(x(t))\ \forall t$.

3.2. Metzlerian Systems

Metzlerian systems or continuous time positive system have been studied by many researchers (see [13] and references therein) and are found in variety of applications.

Definition 3. The matrix $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ is a Metzlerian matrix if $a_{ii} \le 0\ \forall i$ and $a_{ij} \ge 0\ i \neq j$ with $i = j = 1,2,\ldots,n$.

Remark 1. The necessary condition for stability of Metzlerian matrices is $a_{ii} < 0$ which is assumed throughout this paper.

This leads to the discussion of systems which posses this important structure.

Definition 4. A system

$$\begin{aligned} \dot{z} &= Mz + Nu \qquad z(0) = z_0 &\quad (12) \\ y &= Wz &\quad (13) \end{aligned}$$

is called a Metzlerian system if M is a Metzler matrix, and $N \ge, W \ge 0$ are nonnegative matrices.

Theorem 2. *A Metzlerian system is asymptotically stable if and only if one of the following equivalent conditions are satisfied:*

i) All eigenvalues of the Metzlerian matrix M have negative real parts.

ii) All coefficients $a_i (i = 0, \ldots, n-1)$ of the characteristic polynomial $\Delta(\lambda) = det(\lambda I - M) = \lambda^k + a_{n-1}\lambda^{k-1} + \ldots + a_0\lambda + a_0$ are positive.

iii) All leading principal minors of the matrix $-M$ are positive.

iv) The matrix M is nonsingular and $-M^{-1} > 0$.

v) There exists a diagonal matrix P such that $M'P + PM < 0$.

By the above properties the overvalued system matrices $M(x,\alpha)$ and M will possess Metzlarian structures. The following theorem, which is proven in [13], can be used to estimate the region of stability under system uncertainty.

Theorem 3. *Suppose $\dot{z} = (M + D\Delta E)z$, where M is Metzlerian stable, $D \geq 0$ and $E \geq 0$ are given structured matrices of appropriate dimensions, say $n \times r$ and $s \times n$, and Δ is an unknown uncertainty matrix confined to a certain set of interest. Then, the real and complex stability radii coincide and are given by the following formulas, depending on the characterization of Δ:*

(1) let $\|\cdot\| = \|\cdot\|_2$ be the Euclidean norm and let $\Delta = \mathbb{R}^{r\times s}$. Then

$$r_R(M,D,E) = r_C(M,D,E) = \frac{1}{\|EM^{-1}D\|_2} \quad (14)$$

(2) let Δ be defined by the set $\Delta = \{P \circ \Delta : p_{ij} \geq 0\}$ with $\|\Delta\| = \max\{|\delta_{ij}| : \delta ij \neq 0\}$ where $[P \circ \Delta]_{ij} = p_{ij}\delta_{ij}$ denotes the Schur product. Then

$$r_R(M,D,E) = r_C(M,D,E) = \frac{1}{\rho(EM^{-1}DP)} \quad (15)$$

It is assumed that the $A(x,\alpha)$ system matrix used for overvaluing is stable. Note, however, that this assumption does not place a general constrain on the method. If necessary, stability can be achieved by closing the loop and using $A_{cl}(x)$ in the overvaluing operation instead.

4. Stability Radius Optimization

For a family of robust comparison systems parameterized by α, obtaining the parameter that corresponds with the maximally robust SDRE parameterization is a constrained minimization problem. Specifically, minimizing the spectral norm of the denominator of Eqn (14) subject to one of the equivalent stability conditions from Theorem 2 expressed in terms of α. This produces the parameter α^* that leads to the largest radius of stability. This is captured in the following Theorem.

Theorem 4. *For the nonlinear system of the form*

$$\dot{x} = f(x) + g(x)u$$

with structured parametric uncertainty given by

$$\dot{x} = [A(x,\alpha) + D\Delta E]x \quad \forall x \in \mathbb{R}^n$$

where $A(x,\alpha)$ is a valid parameterized SDC representation on the hyperplane

$$A(x,\alpha) = (1-\alpha_k)A_{k+1}(x) + \sum_{i=1}^{k}(\prod_{j=i}^{k}\alpha_j)(1-\alpha_{i-1})A_i(x)$$

let

$$\Omega = \{x \in \mathbb{R}^n : x \in (X_L, X_U)\}$$

be the domain of interest with X_L and X_U defining the lower and upper bound of the domain respectively and let the psuedo-overvalued constant matrix, $M(\alpha)$, representing the robust comparison system

$$\dot{z} = (M(\alpha) + D\Delta E)z \quad \forall z \in \mathbb{R}^k_+$$

be Metzlerian stable, then the SDC parameterization corresponding to the largest radius of stability

$$r_R(M(\alpha),D,E) \leq r_R^*(M(\alpha^*),D,E)$$

in the original system is given by $A(x,\alpha^*)$, where α^* is the parameter that minimizes the spectral norm of $EM(\alpha)^{-1}D$ subject to one of the equivalent stability conditions in Theorem 2 expressed in terms of α, e.g:

$$\alpha^* = \underset{\alpha\in[0,1]}{\operatorname{argmin}} \|EM^{-1}(\alpha)D\|_2$$

Subject to

$$det(M(\alpha)) \neq 0$$
$$-M(\alpha)^{-1} > 0$$

Proof. Note that maximizing the radius of stability with respect to the parameter α

$$\max_{\alpha} r_R(M(\alpha), D, E)$$

is equivalent to minimizing

$$\min_{\alpha} \|EM(\alpha)^{-1}D\|_2$$

It can be shown that

$$\|EM(\alpha)^{-1}D\|_2 = \max\{\sigma_i\}$$

where σ_i are the singular values of $(EM(\alpha)^{-1}D)^T(EM(\alpha)^{-1}D)$. Now let,

$$T = \left\{\max_{\alpha} \sigma_i; i = 1 \ldots n, \alpha \in [0,1]\right\}$$

be the set whose elements are the largest singular values of

$$(EM(\alpha)^{-1}D)^T(EM(\alpha)^{-1}D) \quad \forall \alpha \in [0,1]$$

then we seek the desired α according to

$$\alpha^* = \arg\inf_{\alpha} T$$

This establishes the proof for Theorem 4. □

The following example illustrates the utility of this approach.

Example 1. For the nonlinear system:

$$\begin{aligned} \dot{x}_1 &= -x_1 + 2x_1^2 x_2 \\ \dot{x}_2 &= -x_2 + u \end{aligned}$$

Let the parametric uncertainty be captured by

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \Delta = \begin{bmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and let:

$$A_1(x) = \begin{bmatrix} -1 & 2x_1^2 \\ 0 & -1 \end{bmatrix} \quad A_2(x) = \begin{bmatrix} 0 & 2x_1^2 - \frac{x_1}{x_2} \\ 0 & -1 \end{bmatrix}$$

with $B = [0 \quad 1]^T$, $Q = I_{2\times 2}$, $R = 1$. Using Eqn (9), the SDC parameterization is

$$A(x,\alpha) = \begin{bmatrix} -\alpha & -2\alpha x_1^2 + (1-\alpha)(2x_1^2 - \frac{x_1}{x_2}) \\ 0 & -1 \end{bmatrix}$$

Although not necessary in general, here we consider overvaluing the closed loop system matrix,

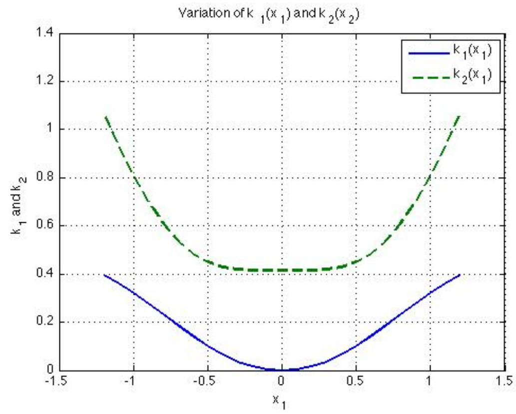

Figure 1: Variation of k_1 and k_2 in S

$A_{cl}(x,\alpha)$, to show that the method is applicable to the closed loop system matrix as well. Since $A(x,\alpha) = A(x_1,\alpha)$, the feedback gains k_1 and k_2 depend on x_1 only. The closed loop matrix $A_{cl}(x,\alpha)$ is:

$$A_{cl}(x,\alpha) = \begin{bmatrix} -\alpha & -2\alpha x_1^2 + (1-\alpha)(2x_1^2 - \frac{x_1}{x_2}) \\ -k_1(x_1) & -1 - k_2(x_1) \end{bmatrix}$$

Let the region of interest be :

$$S = \{x_1, x_2 \in \mathbb{R} : |x_1| < 1.2, |x_2| < 1.2\}$$

The overvaluing matrix $M(x,\alpha)$ for $A_{cl}(x,\alpha)$ is:

$$M(x,\alpha) = \begin{bmatrix} \max_{x\in S}(-\alpha) & \max_{x\in S} |-2\alpha x_1^2 + (1-\alpha)(2x_1^2 - \frac{x_1}{x_2})| \\ \max_{x_1\in S} |-k_1(x_1)| & \max_{x_1\in S}(-1 - k_2(x_1)) \end{bmatrix}$$

The maximum value of k_1 and k_2 can be deduced from their variation in Figure 1. Note that

$$k_1^{max} = 0.396 \qquad k_1^{min} = 0.00 \qquad k_2^{max} = 1.07 \qquad k_2^{min} = 0.414$$

The constant overvaluing matrix $M(\alpha)$ for the domain of interest becomes:

$$M(\alpha) = \begin{bmatrix} -\alpha & \alpha + 1.88 \\ 0.396 & -1.414 \end{bmatrix}$$

Note that $M(\alpha)$ is Metzlerian $\forall \alpha \in (0,1]$ and it is Hurwitz stable for $\forall \alpha \in (0.731, 1]$ [13].

This implies that for any $0.731 < \alpha < 1$ the stability radius can be computed using Theorem 3. The utility of this fact is that even though α^* yields the most robust SDC parameterization with respect to parametric

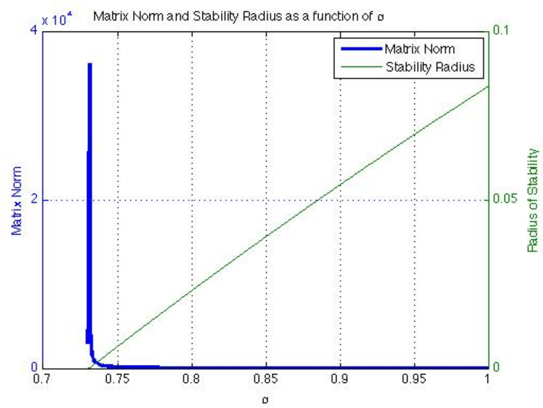

Figure 2: Thick solid line corresponds to radius of stability and thin line corresponds to the matrix norm.

uncertainty, the designer may wish to asses the tradeoff between robustness and closed loop performance by varying α.

In this example, we seek the α that maximizes the radius of stability only. Therefore

$$\alpha^* = \underset{\alpha \in (0.731,1]}{\operatorname{argmin}} \|EM^{-1}(\alpha)D\|_2$$

Figure 2 plots the matrix norm of $EM^{-1}D$ and the corresponding radius of stability $r_R(M,D,E)$ as a function of α. It can be seen that the maximum radius of stability occurs at upper boundary of α, so $\alpha^* = 1$ with a radius of stability of $r_R = 0.084$. Therefore, for this problem $A_1(x)$ corresponds to the parameterization that is maximally robust with respect to parametric uncertainty. Note that when additional constraints are added to the optimization problem, α^* may not necessarily be equal to the upper bound of α.

5. Conclusion

SDC parameterization of nonlinear systems with parametric uncertainty using the SDRE (or SDLQR) methodology was considered. Each SDC parameterization produce its own radius of stability in a region of interest in the state space. A method to compute the radius of stability in a special class of systems was used to obtain the SDC parameterization which results in the maximum radius of stability for the original nonlinear system in the region of interest. It was shown that the problem of finding the maximum radius of stability from a hyperplane of SDC parameterizations is equivalent to minimizing the spectral norm of a constrained comparison system. The method was illustrated with an example.

Acknowledgement

The authors would like to acknowledge the anonymous reviewer for the insightful suggestions.

References

[1] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Prentice Hall, 2002.

[2] B. Friedland, *Advanced Control System Design*. Prentice Hall, 1996.

[3] J.R. Cloutier, C.N. D'Souza and C.P. Mracek, "Nonlinear regulation and nonlinear h_∞ control via the state-dependent riccati equation technique: Part 1, theory; part 2: Examples." *Proceedings of the First International Conference on Nonlinear Problems in Aviation and Aerospace.*, pp. 117–141, 1996.

[4] C. Mracek and J. Cloutier, "Control designs for the nonlinear benchmark problem via the state-dependent riccati equation method." *International Journal of Robust and Nonlinear Control*, vol. 8, pp. 401–433, 1998.

[5] W. Langson and A.G. Alleyne, "Infinite horizon optimal control of a class of nonlinear systems," *Proceedings of the American Control Conference*, pp. 3017–3022, 1997.

[6] A. Zecevic and D. Siljak, "Stabilization of nonlinear systems with moving equilibria," *IEEE Transactions of Automatic Control*, vol. 48, no. 6, 2003.

[7] C. Mracek and J. Cloutier, "A preliminary control design for the nonlinear benchmark problem," *IEEE Conference on Control Applications*, 1996.

[8] E.B. Erdem and A.G. Alleyne, "Estimation of stability regions of sdre controlled systems using vector norms," *Proceedings of the American Control Conference*, 2002.

[9] B. Shafai, S. Nazari, "Stability radius for extended linearization with system uncertainty," *IEEE Conference on Decision and Control*, 2010.

[10] S.P. Bhattacharyya, H. Chapellat, L.H. Keel, *Robust Control: The Parametric Approach*. Prentice Hall, 1995.

[11] W. Perruquetti, J.P. Richard, L.T. Grujic and P. Borne, "On practical stability with the settling time via vector norms," *International Journal of Control*, vol. 62, no. 1, pp. 173–189, 1995.

[12] P. Borne, J.P. Richard, N.E. Radhy, *Nonlinear Systems Vol2: Stability and Stabilization*, A. F. D. Normand-Cyrot, Ed. Chapman and Hall, 1996, vol. Stability, stabilization, regulation using vector norms.

[13] B. Shafai, J. Chen, M. Kothandaraman "Explicit formulas for stability radii of nonnegative and metzlerian matrices," *IEEE Transactions of Automatic Control*, vol. 42, pp. 265–270, 1997.