

# Development of Simplified Statics of Robot Manipulator and Optimized Muscle Torque Distribution based on the Statics

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**Abstract**—Statics for a two-link manipulator is re-derived based on the biarticular muscle coordinate in this paper. Torques of two joint-motors to generate a certain force at the endeffector can be calculated in a simple trigonometric function form. A H infinity optimization algorithm is applied to the redundancy problem and minimizes the necessary torque for each muscle. This algorithm can generate novel torque patterns whose peak values are minimized and the maximum torque of the actuators can be set small. The suggested statics provides not only insight to the robot design, but also some interpretation to our body's muscle activation strategy.

## I. INTRODUCTION

Force control in robotics has been an important issue. It can be extended to motion control [1] and also used for stable interaction with the environment using the concept of impedance [2]. Generally, force control uses Jacobian to define the relationship between actuator torque/force and endeffector force. This relationship is so complicated that it is unable to provide insight to understand the whole system. For this reason, there has been research to simplify the relationship in order to make it more straightforward [1], [3]. In this paper, we suggest a simple and straightforward statics using the biarticular muscle torque.

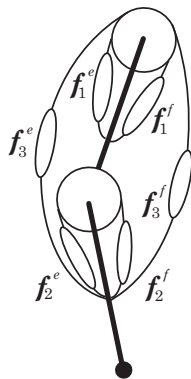


Fig. 1. Two-Joint Manipulator with Muscle Model

Figure 1 shows the 3 pairs of muscles: flexors and extensors of two monoarticular muscle pairs and one biarticular muscle pair. Biarticular muscle described as  $f_3^{e,f}$  in Figure 1 is said to be effective for the simplification of the complicated relationship [4], [5]. Three pairs of muscles activated in an

antagonistic way with a phase difference can generate a well-shaped force hexagon at the endeffector [6].

These characteristics have been applied to robotics and several robots mimicking animals have been developed based on the biarticular muscle system [7], [8]. The feature of these robots lies in the simplicity in the control algorithm.

In order to clarify the simplification of statics by the biarticular muscle system, we have developed novel statics based on the biarticular muscle system and it can be a substitute of Jacobian [9]. The suggested statics consists of simple sine functions that can relate the necessary muscle torques to a specific desired force. It also could reveal the relation between the required muscle torques and the lengths of the links, and the phase relationship between the torque pattern and the manipulator configuration.

Our previous study, however, did not utilize all muscle torques. In order to avoid the redundancy problem, we ignored the monoarticular muscle torque in the second joint. This paper addresses this problem proposing a novel statics based on our recent research [10] that optimizes the muscle torque distribution to minimize the infinity norm of each muscle torque.

First, in Section II we derive statics representation of a general two-joint manipulator which has two actuators located in two joints. This statics is rederived using trigonometric functions rather than Jacobian. The statics suggested in [9] is used to derive this simple statics.

Then, the optimal distribution algorithm in [10] is applied to this statics in Section III. The solution to this optimization problem can be given in an analytical way, since the statics derived in Section II is simple enough. Simulation results and experimental results are shown and some interpretation that can be obtained by the suggested simple statics is also provided.

## II. DERIVATION OF TRIGONOMETRIC FUNCTION REPRESENTATION OF STATICS FOR A TWO-JOINT MANIPULATOR

### A. Simplified Statics based on Biarticular Muscle Coordinate

Let us define the muscle torques that are generated by  $f_i^{e,f}$  as  $\tau_i^m$  and the resultant two joint torques as  $T_1^j, T_2^j$ , then the relation between these two kinds of torques is defined as the following. The superscript  $j$  means the joints and  $m$  means the muscles.

$$\begin{pmatrix} T_1^j \\ T_2^j \end{pmatrix} = \begin{pmatrix} \tau_1^m + \tau_3^m \\ \tau_2^m + \tau_3^m \end{pmatrix}, \quad (1)$$

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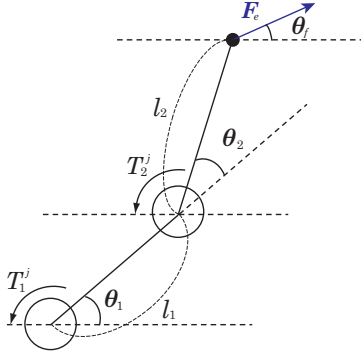


Fig. 2. Configuration of Two-link Manipulator

Figure 2 is the configuration of a two-link manipulator, where two actuators are located in two joints. Equation (2) shows statics that represents the balance between forces applied at the endeffector and joint torques; the force  $F_e$  in Figure 2 is described as  $\mathbf{F}_e = (f_x, f_y)^T$ .

$$\begin{pmatrix} T_1^j \\ T_2^j \end{pmatrix} = J^T \begin{pmatrix} f_x \\ f_y \end{pmatrix} \quad (2)$$

$$J = \begin{pmatrix} -l_1 \sin \theta_1 - l_2 \sin(\theta_1 + \theta_2) & -l_2 \sin(\theta_1 + \theta_2) \\ l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2) & l_2 \cos(\theta_1 + \theta_2) \end{pmatrix} \quad (3)$$

When the magnitude and direction of  $F_e$  are given as  $F_e = (F \cos \theta_f, F \sin \theta_f)$ , the biarticular muscle coordinate in Equation (1) simplifies this statics as the following equation [9].

$$\begin{aligned} \tau_1^{m*} &= Fl_1 \sin(\theta_f - \theta_1), & \tau_2^{m*} &= 0 \\ \tau_3^{m*} &= Fl_2 \sin(\theta_f - \theta_{12}), \end{aligned} \quad (4)$$

where  $\theta_{12} = \theta_1 + \theta_2$  and the superscript  $m^*$  means the required muscle torque under the condition of  $\tau_2^{m*} = 0$ .

Even though one redundant degree of freedom  $\tau_2^{m*}$  is set to 0 to simplify the problem, Equation (4) is the most simplified form of the statics considering the biarticular muscle torque. We proposed an optimized distribution algorithm [10] in order to deal with this redundancy problem so that the statics in Equation (4) can be modified based on the algorithm.

### B. Trigonometric Function Representation of Statics for General Two-joint Actuator

In order to apply the optimized distribution, the statics in Equation (2) is transformed into the trigonometric functions.

By substituting the required muscle torque in Equation (4) to Equation (1), the statics in Equation (2) is expanded as the following.

$$\begin{aligned} T_1^j &= Fl_1 \sin \theta_f + Fl_2 \sin(\theta_f - \theta_2) \\ T_2^j &= Fl_2 \sin(\theta_f - \theta_2), \end{aligned} \quad (5)$$

where  $\theta_1$  is set to 0 which means  $\theta_f$  is defined as the angle with regard to the direction of the first link. Figure 3 describes the parameters that will be used in the remainder of the paper.

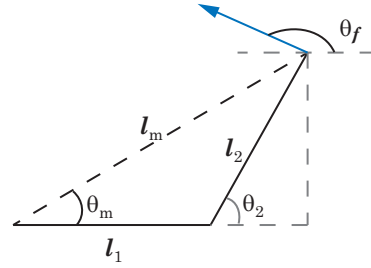


Fig. 3. Parameters for Statics of Two-link Manipulator

Statics in Equation (5) is simplified as follows using the combination of sine functions with the parameters of  $l_m$  and  $\theta_m$  in Figure 3.

$$\begin{aligned} T_1^j &= Fl_m \sin(\theta_f - \theta_m) \\ T_2^j &= Fl_2 \sin(\theta_f - \theta_2) \end{aligned} \quad (6)$$

Detailed derivation process is explained in Appendix A.

Note that Equation (6) can relate the required torques  $T_1^j, T_2^j$  to the desired force described by  $F$  and  $\theta_f$  in a much simpler way than the conventional Jacobian. It also can provide insight of the statics of a two-link manipulator.

### C. Generic Characteristic in Manipulator Force Exertion Revealed by the Suggested Statics

Figure 4 is the required torques  $T_1^j$  and  $T_2^j$  in order to generate a force of 1N at the endeffector with the direction of  $\theta_f$  under the configuration  $l_1 = l_2 = 1m, \theta_2 = \frac{\pi}{3}$ . Necessary torques are illustrated with regard to the angle  $\theta_f$ .

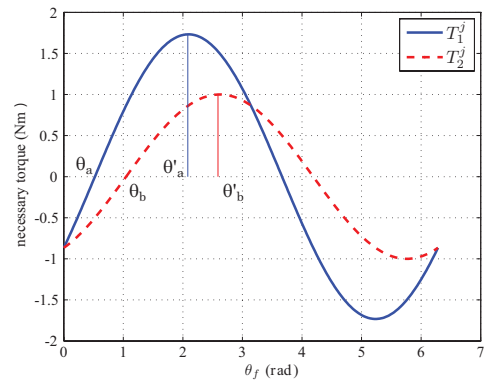


Fig. 4. Necessary Torques to Generate a Force at the Endeffector

This figure provides some general points that can be revealed only by this simplified statics.  $T_1^j$  becomes same with  $T_2^j$  when the force direction  $\theta_f$  is set to 0 regardless of the parameters  $l_1, l_2, \theta_2$ , since  $l_m \sin \theta_m$  is same as  $l_2 \sin \theta_2$  as shown in Figure 3. There are four specific angles  $\theta_a, \theta_b, \theta'_a, \theta'_b$ ;  $\theta_a (= \theta_m)$  is the angle where  $T_1^j$  cannot

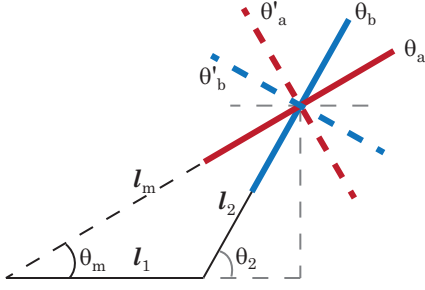


Fig. 5. Specific Angles in Force Exertion of Two Joint-motor Manipulator

contribute to the force at the endeffector since the moment arm of the torque  $T_1^j$  to the force is zero (Figure 5 explains this characteristic),  $\theta_b (= \theta_m)$  is the same angle for the case of  $T_2^j$ , while  $\theta'_a (= \frac{\pi}{2} + \theta_m)$  is the angle where  $T_1^j$  contribute most to the force and  $\theta'_b (= \frac{\pi}{2} + \theta_2)$  is the same angle for  $T_2^j$ .

Figure 5 shows these angles in terms of the robot configuration;  $\theta_a$  is same with  $\theta_m$ , the direction in which the first joint and the endeffectors are aligned and  $\theta_b$  is identified with  $\theta_2$ , the direction of the second link.  $\theta'_a$  and  $\theta'_b$  are the angles  $\frac{\pi}{2}$  away from  $\theta_a$  and  $\theta_b$ . This relationship is in good agreement with the explanation of Figure 4.

Magnitude  $F l_m$  and  $F l_2$  in Equation (6) also explain some points. In the range of  $0 \leq \theta_2 \leq \frac{2\pi}{3}$ ,  $l_m$  is greater than  $l_2$  which means the larger torque is required for  $T_1^j$  than  $T_2^j$ . When  $\theta_2 \geq \frac{2\pi}{3}$ , smaller  $T_1^j$  is good enough to generate a specific force at the endeffector.

This point shows a significant difference between the statics strategies of the general two-joint actuator manipulator and the biarticulated actuator manipulator. Comparing Equation (4) with Equation (6), it should be noticed that the magnitude of the necessary torques is constant in the case of the biarticulated actuator manipulator. This indicates that the output of the motor installed in the first joint can be utilized efficiently regardless of the configuration of the manipulator with the biarticular muscle actuator; the biarticular muscle torque can improve the efficiency of the actuators in terms of force generation.

For example, in the worst case where  $\theta_2$  is set to 0, the maximum torque required for  $T_1^j$  is  $F(l_1 + l_2)$ , while the maximum torque required for  $\tau_1^m$  is still  $F l_1$  to generate the same force at the endeffector.

### III. NOVEL STATICS FOR BIARTICULAR MUSCLE SYSTEM BY OPTIMIZATION OF INFINITY NORM OF ACTUATOR OUTPUT

#### A. Infinity Norm Minimization of Biarticular Muscle System

Recently we have developed an optimization algorithm for the biarticular muscle system, which optimizes the distribution of the necessary muscle torques, minimizing the necessary maximum torque for each actuator. The problem formulation is as follows.

- 1) Minimize  $\max \{|\tau_1^m|, |\tau_2^m|, |\tau_3^m|\}$

- 2) subject to  $\begin{cases} T_1^j = \tau_1^m + \tau_3^m \\ T_2^j = \tau_3^m \end{cases}$
- 3) under the assumption  $\max \tau_1^m = \max \tau_2^m = \max \tau_3^m$ .

The solution is given as follows [10]:

**Case 1** when  $T_1^j T_2^j \leq 0$

$$\begin{cases} \tau_1^m = \frac{T_1^j - T_2^j}{2} \\ \tau_2^m = \frac{T_2^j - T_1^j}{2} \\ \tau_3^m = \frac{T_1^j + T_2^j}{2} \end{cases} \quad (7)$$

**Case 2** when  $T_1^j T_2^j > 0$  and  $|T_1^j| \geq |T_2^j|$

$$\begin{cases} \tau_1^m = T_1^j - \frac{T_2^j}{2} \\ \tau_2^m = \frac{T_2^j}{2} \\ \tau_3^m = \frac{T_2^j}{2} \end{cases} \quad (8)$$

**Case 3** when  $T_1^j T_2^j > 0$  and  $|T_1^j| < |T_2^j|$

$$\begin{cases} \tau_1^m = \frac{T_1^j}{2} \\ \tau_2^m = T_2^j - \frac{T_1^j}{2} \\ \tau_3^m = \frac{T_1^j}{2} \end{cases} \quad (9)$$

However, in our previous research, the torque  $T_1^j$  and  $T_2^j$  required to generate a certain force at the endeffector are calculated based on Jacobian, As a result, the calculation is so complicated that we could not obtain any simple analytical solution for each muscle torque, particularly the conditions for the cases.

In this paper, based on the novel statics derived in Equation (5) and (6), an analytical solution to this optimization problem is given so that we can obtain insight on the biarticular muscle statics from it.

#### B. Application of Minimization Algorithm to Trigonometric Statics Representation

Conditions for three cases are analyzed at first. Figure 3 gives a significant guidance to solve these inequalities. As is said in previous section,  $l_m \sin \theta_m$  is same with  $l_2 \sin \theta_2$  that can be induced from the figure. This represents  $T_1^j = T_2^j$  when  $\theta_f = 0$  regardless of  $l_1, l_2$ , and  $\theta_2$ . The other point we should notice is that  $\theta_2 \geq \theta_m$  also with any configuration, which can be induced from the figure leading to the fact that  $T_1^j$  becomes positive earlier than  $T_2^j$  ( $\theta_a \leq \theta_b$  in Figure 4 with any value of  $\theta_2$ ).

The angle where two graphs of  $T_1^j$  and  $T_2^j$  intersect can be calculated by the subtraction  $T_1^j - T_2^j$ . As calculated in Appedix B,  $T_1^j - T_2^j = l_1 \sin \theta_f$  and the intersection occurs at  $\theta_f = 0$  and  $\pi$  regardless of the values of  $\theta_2, l_1, l_2$ .

Taking these points into consideration, the conditions for three cases in Equation (7), (8), and (9) can be derived as follows. Case 1 holds when  $\theta_m \leq \theta_f \leq \theta_2$  or  $\pi + \theta_m \leq \theta_f \leq \pi + \theta_2$ , Case 2 holds when  $\theta_2 < \theta_f \leq \pi$  or  $\pi + \theta_2 < \theta_f \leq 2\pi$ . Finally Case 3 holds when  $0 < \theta_f < \theta_m$  or  $\pi < \theta_f < \pi + \theta_m$ .

With these conditions and substitutions of the muscle torques, the optimized muscle torque distribution is given as follows.

**Case 1)** when  $\theta_m \leq \theta_f \leq \theta_2$  or  $\pi + \theta_m \leq \theta_f \leq \pi + \theta_2$

$$\begin{cases} \tau_1^m = \frac{F}{2} l_1 \sin \theta_f \\ \tau_2^m = -\frac{F}{2} l_1 \sin \theta_f \\ \tau_3^m = \frac{F}{2} l_1 \sin \theta_f + F l_2 \sin(\theta_f - \theta_2) \end{cases} \quad (10)$$

**Case 2)** when  $\theta_2 < \theta_f \leq \pi$  or  $\pi + \theta_2 < \theta_f \leq 2\pi$

$$\begin{cases} \tau_1^m = \frac{F}{2} l_1 \sin \theta_f + \frac{F}{2} l_2 \sin(\theta_f - \theta_2) \\ \tau_2^m = \frac{F}{2} l_2 \sin(\theta_f - \theta_2) - \frac{F}{2} l_1 \sin \theta_f \\ \tau_3^m = \frac{F}{2} l_1 \sin \theta_f + \frac{F}{2} l_2 \sin(\theta_f - \theta_2) \end{cases} \quad (11)$$

**Case 3)** when  $0 < \theta_f < \theta_m$  or  $\pi < \theta_f < \pi + \theta_m$

$$\begin{cases} \tau_1^m = F l_1 \sin \theta_f + \frac{F}{2} l_2 \sin(\theta_f - \theta_2) \\ \tau_2^m = \frac{F}{2} l_2 \sin(\theta_f - \theta_2) \\ \tau_3^m = \frac{F}{2} l_2 \sin(\theta_f - \theta_2) \end{cases} \quad (12)$$

### C. Simulation Result of Optimized Statics of Biarticular Muscle System

In this section, several torque patterns are simulated based on the suggested optimized statics. Figure 6 and 7 are the torque patterns for muscles under the same configuration in Figure 4, where  $l_1 = l_2 = 1m$ , and  $\theta_2 = \frac{\pi}{3}$ . In order to investigate the optimization result, the muscle torque statics with  $\tau_2^{m*} = 0$  in Equation (4) is also simulated.

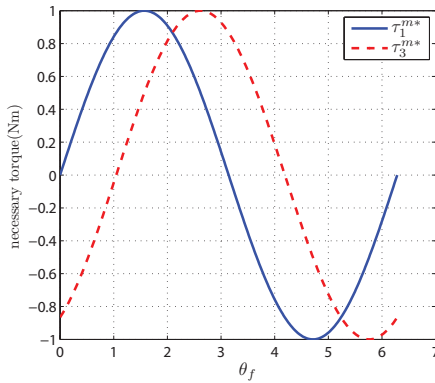


Fig. 6. Muscle Torques with Two Muscle Torques ( $\theta_2 = \frac{\pi}{3}$ )

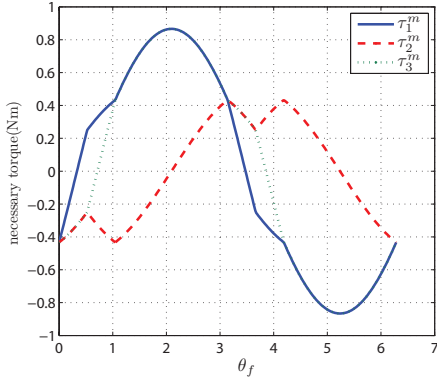


Fig. 7. Muscle Torques Maximum Value Minimized ( $\theta_2 = \frac{\pi}{3}$ )

Figure 6 is the torques  $\tau_1^{m*}$  and  $\tau_3^{m*}$  derived by Equation (4) and Figure 7 is the torques  $\tau_{\{1,2,3\}}^m$  obtained by Equations (10) to (12). Comparing the peaks of all torques, we can verify that  $\max T_1^j > \max \tau_1^{m*} > \max \tau_1^m$ . The maximum

value of  $\tau_1^m$  is the half that of  $T_1^j$  due to the optimization. The algorithm distributes the necessary  $T_1^j$  torque to  $\tau_1^m$  and  $\tau_3^m$  with the same value at its peak so that it can minimize the peak torque.

The problem is that the necessary maximum value of  $T_1^j$  changes with regard to  $\theta_2$ ; if a constant force  $F$  is required at the endeffector with all the angles of  $\theta_2$ , the maximum torque of  $T_1^j$  will be  $F(l_1 + l_2)$  when two links of the manipulator is set aligned with  $\theta_2$  set to 0 and the suggested algorithm minimizes it only by half. In the worst case where  $l_1 = l_2$ , the suggested algorithm minimizes the necessary peak torque only by the same amount with the strategy in Equation (4).

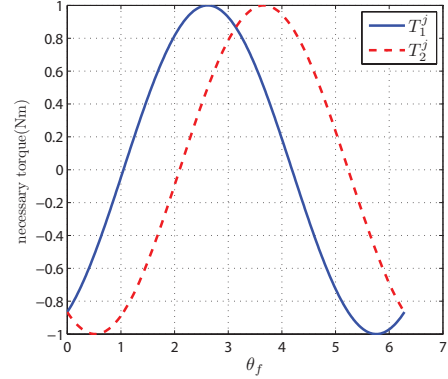


Fig. 8. Joint Torques to Generate Constant Force ( $\theta_2 = \frac{2\pi}{3}$ )

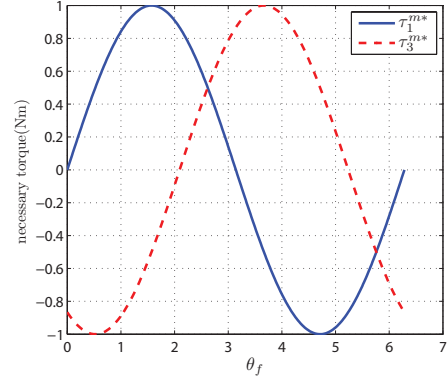


Fig. 9. Muscle Torques with Two Muscle Torques ( $\theta_2 = \frac{2\pi}{3}$ )

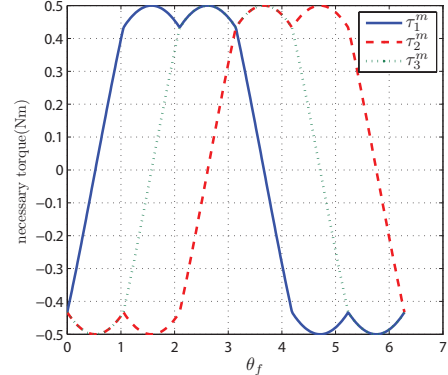


Fig. 10. Muscle Torques Maximum Value Minimized ( $\theta_2 = \frac{2\pi}{3}$ )

A simulation under the setting of  $\theta_2 = \frac{2\pi}{3}$  is conducted and Figures 8 to 10 show the results. Here, the necessary

joint torques  $T_1^j$  and  $T_2^j$  have the same peak torque since  $l_m = l_2$  and the suggested algorithm distributes these peak torques to three muscle torques reducing them by the half value.

Next,  $\theta_2$  is set to  $\frac{5\pi}{6}$  and the same optimization is conducted. Figure 11 to 13 show the results. Since  $\theta_2 > \frac{2\pi}{3}$ , the peak of  $T_1^j$  is less than  $T_2^j$  and the suggested algorithm reduces the peak value of  $T_2^j$  by the half.

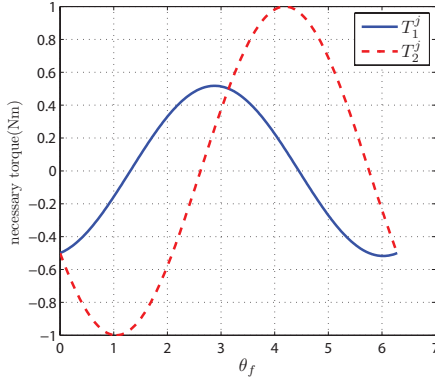


Fig. 11. Joint Torques to Generate Constant Force ( $\theta_2 = \frac{5\pi}{6}$ )

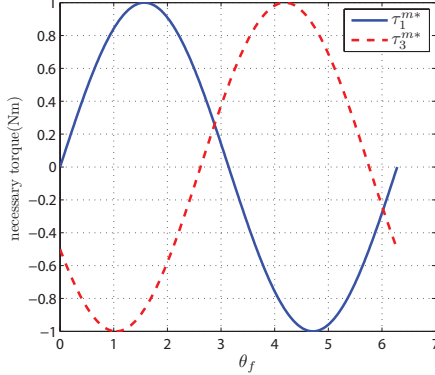


Fig. 12. Muscle Torques with Two Muscle Torques ( $\theta_2 = \frac{5\pi}{6}$ )

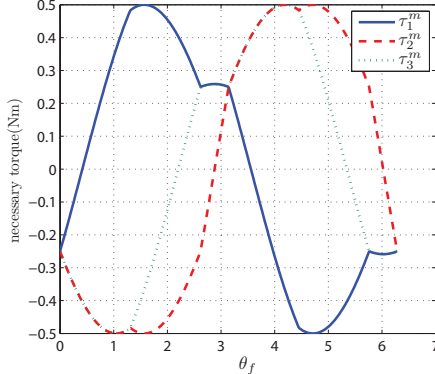


Fig. 13. Muscle Torques Maximum Value Minimized ( $\theta_2 = \frac{5\pi}{6}$ )

These simulation results with various  $\theta_2$  show that the suggested algorithm can minimize the maximum value of necessary torques in all configurations.

#### D. Experiment with Optimized Torque Pattern

We have developed a wire driven robot where the complete 3 pairs of 6 muscles are installed using 6 motors. Torques pattern suggested in this paper are provided to this robot and force at the endeffector is measured under the constraint of the endeffector. In this experiment,  $l_1 = l_2 = 122mm$  and  $\theta_2$  is set to 39.4 degree.

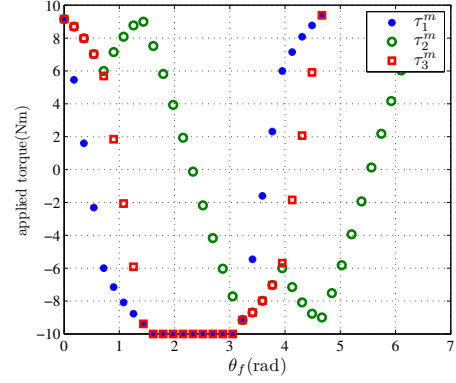


Fig. 14. Three Muscle Torque Patterns Given to the Manipulator

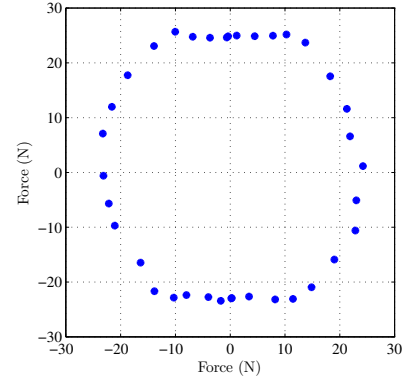


Fig. 15. Measured Force at the Endeffector

Figure 14 shows three torques for  $\tau_1^m$ ,  $\tau_2^m$ , and  $\tau_3^m$ . Figure 15 depicts the measured force. The measurement shows that with the suggested torque patterns, a good force pattern with a constant magnitude and varying direction can be obtained, even though there is some saturation in torques.

#### E. Coincidence with Biological Data

It has been suggested by many groups that antagonistic muscle pairs have their own directions where they play the significant role in the exertion of force at the endeffector in that direction. Some experiments with a human subject show that the activation level of the muscle pairs changes based on this direction [6]. Figure 16 is the activation level of 6 muscles in Figure 1 described by the pairs. The  $x$  axis represents the force direction that is identified with the direction of the force at the hand. This has the same meaning with  $\theta_f$ . The direction is divided into 3 pairs of 6 regions -  $a', b', c', d', e', f'$  which correspond to the direction of the

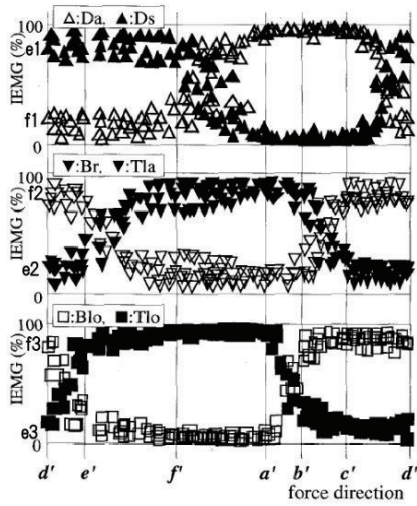


Fig. 16. Muscle Activation Level Measured in Human Arm [6]

first link, the second link and the line connects the shoulders and the hand.

The direction division is same with the direction shown in Figure 17 that is derived in Equation (10) to (12):  $\theta_m, \theta_2, \pi, \pi + \theta_m, \pi + \theta_2, 2\pi$ . This implies that our muscle

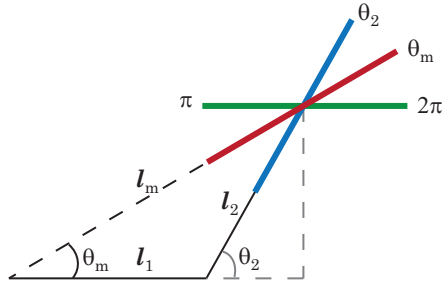


Fig. 17. 6 Major Directions by 3 Pairs of Muscle

also adopts the minimization of maximum value strategy to generate forces.

#### IV. CONCLUSIONS

We propose a simplified statics strategy for a general two-link robot manipulator with two joint-motors and applied it to the optimization algorithm we have developed to minimize the peak values of necessary torques. This process can provide an analytic form of the optimized muscle torque pattern. By simulations and experiments, the efficiency and reduction of the peak value of the necessary torques are verified.

The statics we derived for a robot manipulator can be utilized in various robot designs. For example, the suggested statics indicates that in a robot leg, an actuator needs to be implemented in the hip joint larger than the one in the knee joint to generate a specific force in the horizontal direction at the foot. As our previous research [11], the suggested statics also can provide a novel feedback control due to its simplicity.

#### APPENDIX

##### A. Combination of the Torques for $T_1^j$

The torque to generate a specific external force with the magnitude  $F$  and the direction  $\theta_f$  is given as Equation (5). Two sine functions in  $T_1^j$  can be combined as follows.

$$\begin{aligned} T_1^j &= Fl_1 \sin \theta_f + Fl_2 \sin(\theta_f - \theta_2) \\ &= F(l_1 \sin \theta_f + l_2 \sin \theta_f \cos \theta_2 - l_2 \cos \theta_f \sin \theta_2) \\ &= F((l_1 + l_2 \cos \theta_2) \sin \theta_f - l_2 \sin \theta_2 \cos \theta_f) \\ &= Fl_m \sin(\theta_f - \theta_m), \end{aligned} \quad (13)$$

where  $l_m^2 = (l_1 + l_2 \cos \theta_2)^2 + l_2^2 \sin^2 \theta_2$ ,  $\cos \theta_m = \frac{l_1 + l_2 \cos \theta_2}{l_m}$ , and  $\sin \theta_m = \frac{l_2 \sin \theta_2}{l_m}$  that corresponds with the definition in Figure 3

##### B. Subtraction of $T_1^j - T_2^j$

$$\begin{aligned} T_1^j - T_2^j &= l_m \sin(\theta_f - \theta_m) - l_2 \sin(\theta_f - \theta_m) \\ &= l_m \sin \theta_f \cos \theta_m - l_m \cos \theta_f \sin \theta_m \\ &\quad - l_2 \sin \theta_f \cos \theta_2 + l_2 \cos \theta_f \sin \theta_2 \\ &= l_1 \sin \theta_f + l_2 \sin \theta_f \cos \theta_2 - l_2 \cos \theta_f \sin \theta_2 \\ &\quad - l_2 \sin \theta_f \cos \theta_2 + l_2 \cos \theta_f \sin \theta_2 \\ &= l_1 \sin \theta_f, \end{aligned} \quad (14)$$

since  $\cos \theta_m = \frac{l_1 + l_2 \cos \theta_2}{l_m}$ , and  $\sin \theta_m = \frac{l_2 \sin \theta_2}{l_m}$  as defined in Appendix A.

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