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Decentralized PI Observer-Based Control of Nonlinear Interconnected Systems with Disturbance Attenuation

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Abstract — **We consider the problem of designing decentralized PI observer-based controller for nonlinear interconnected systems using linear matrix inequalities (LMIs). The overall system is composed of linear subsystems and nonlinear time-varying interconnections depending on both time and state. We take advantage of additional degrees of freedom in PI observer to maximize the bound on nonlinear interconnection terms. Also we extend the original problem of robust stabilization by including a disturbance term and provide a sufficient condition for connective stability with disturbance attenuation in the L_2 gain sense. An example is included to validate the theoretical results.**

I. Introduction

LARGE-SCALE interconnected systems can be found in such diverse fields as electrical power systems, space structures, manufacturing processes, control of formation of unmanned vehicles, transportation and communication. In many cases, the subsystems are disconnected and again connected in an unpredictable way to perform programmed task. Under such structural reconfigurations, it is required to guarantee *connective stability* of the overall system or to achieve a desired robust *performance* in the presence of uncertain interconnections by using a proper control strategy.

An important motivation for the design of decentralized scheme is that the information exchange between subsystems is not needed. Therefore, the individual subsystems controllers are simple and use only locally available information. A large body of literature in decentralized control of large-scale systems can be found in [1].

In this paper we consider the problem of designing decentralized controllers in the convex optimization context for interconnected systems with linear subsystems and nonlinear time-varying interconnections. The pioneer work in this context is [3] where the state-feedback controller is designed to guarantee robust stability of the overall systems while maximizing the bounds of unknown interconnection terms. Subsequently, several algorithms are proposed for designing decentralized proportional (P) observer-based controller to robustly stabilize the overall systems ([4]-[6]). In [4] and [5] a sequential two-step design procedure has been introduced to solve the non-convex optimization problem and provide a way to maximize the interconnection bounds. In [6] the problem is solved by avoiding sequential procedure and applied to an industrial utility boiler.

The main contribution of this paper is a modification in terms of a proportional-integral (PI) observer and its solution using an extended LMI. We take advantage of additional degrees of freedom in PI observer to enhance the design objective of maximizing the interconnection bounds. It has been recognized that certain limitation of Luenberger-type P observer can be overcome by using PI observer. This type of observer has an additional integration path, which can improve robustness against parameters variations and disturbances, and can offer design flexibility to achieve other objectives such as steady-state accuracy and stability robustness. The additional degrees of freedom provided by the integral gain has shown to be advantageous in LTR design [8]-[11], attenuation or decoupling disturbances [12], [13], and detecting sensor or actuator fault [14]. PI observer has also been used in a nonlinear setting [7] and for disturbance attenuation in a class of nonlinear systems [12]. The application of PI observer for robust fault diagnosis of descriptor system has been reported in [8]. The adaptive version of PI observer was equally successful for simultaneous parameters, state, and disturbance estimation [15]. We show that the solution of the decentralized PI observer-based control problem reduces to an extended LMI setup which can easily be solved with the advantage that the feasible region is extended by proportional-integral gains leading to a robust solution.

In the second part of the paper, we reformulate the original problem of robust stabilization by including a disturbance term and provide a sufficient condition to guarantee prescribed disturbance attenuation performance while maximizing the interconnection bounds. Using this formulation, it is also possible to minimize the effect of disturbance in L_2 gain sense when a-prior bound on the interconnection terms is given.

The contents of this paper are laid out as follows: Section II introduces the nonlinear interconnected systems and defines the connective stability for these systems. Section III presents the two-step algorithm for designing the decentralized PI observer-based controller. An LMI formulation is provided for robust stability with disturbance attenuation in Section IV. Section V includes numerical example to show the effectiveness of the proposed method. Finally, Section VI provides concluding remarks.

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II. Problem Formulation

Consider a large-scale interconnected system composed of N subsystems represented by:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) + h_i(t,x) \\ y_i(t) &= C_i x_i(t) \end{aligned} \quad \text{for} \quad i = 1,\ldots,N \quad (1)$$

where $x_i \in \Re^{n_i}$ is the state, $u_i \in \Re^{m_i}$ is the control input, $y_i \in \Re^{p_i}$ is the measurement output, $x = [x_1^T,\ldots,x_N^T]^T$ ($n = \sum_{i=1}^N n_i$) is the state of the overall system, and $h_i(t,x) : \Re^{n+1} \to \Re^{n_i}$ is nonlinear interconnection function of the i-th subsystem. The exact expression of $h_i(t,x)$ is unknown but it is assumed to satisfy the following quadratic constraint:

$$h_i^T(t,x) h_i(t,x) \le \alpha_i^2 x^T H_i^T H_i x \quad (2)$$

where $\alpha_i > 0$ are interconnection bounds and $H_i \in \Re^{l_i \times n}$ are constant bounding matrices.

The entire interconnected system (1) can be written as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + h(t,x) \\ y(t) &= Cx(t) \end{aligned} \quad (3)$$

where $A = diag(A_1,\ldots,A_N)$, $B = diag(B_1,\ldots,B_N)$, $C = diag(C_1,\ldots,C_N)$, $u = [u_1^T,\ldots,u_N^T]^T \in \Re^m$ ($m = \sum_{i=1}^N m_i$), $y = [y_1^T,\ldots,y_N^T]^T \in \Re^p$ ($p = \sum_{i=1}^N p_i$), and $h(t,x) = [h_1^T(t,x),\ldots,h_N^T(t,x)]^T$ for which we have the constraint

$$h^T(t,x) h(t,x) \le x^T H^T \Psi^{-1} H x \quad (4)$$

where $H^T = [H_1^T,\ldots,H_N^T]$ is an $l \times n$ matrix ($l = \sum_{i=1}^N l_i$), $\Psi = diag(\psi_1 I_{l_1},\ldots,\psi_N I_{l_N})$ and $\psi_i = \alpha_i^{-2}$.

The following theorem by Siljak and Stipanovic [3] on general LMI-based formulation of the robust stabilization problem represents a basis for the further elaboration on decentralized PI observer-based control design in section III.

Theorem 1: The nonlinear system $\dot{z} = A_f z + h_f(t,x)$, $y = C_f z$ where $z \in \Re^n$ is the state of the system and $h_f(t,z) : \Re^{n+1} \to \Re^n$ is the nonlinearity which satisfies the quadratic inequality

$$h_f^T(t,z) h_f(t,z) \le z^T H_f^T \Psi^{-1} H_f z \quad (5)$$

is robustly stable with degree $\alpha = [\alpha_1,\ldots,\alpha_N]$ if the following problem has a feasible solution.

Minimize $\mathrm{Tr}\,\Psi$

Subject to $X_f > 0$,

$$\begin{bmatrix} X_f A_f + A_f^T X_f & X_f & H_f^T \\ X_f & -I & 0 \\ H_f & 0 & -\Psi \end{bmatrix} < 0 \quad (6)$$

Proof: This result can be proved using the S-procedure i.e. to combine the Lyapunov inequality and constraint (5) in one LMI followed by applying Schur complement formula. The complete proof can be found in [3].

III. Decentralized PI Observer-Based Controller

The objective is to design a decentralized observer based linear controller that robustly regulates the state of the overall system without any information exchange between subsystems, i.e. the local controller u_i is constrained to use only local output signal y_i. This problem is investigated in [4], [5] and [6] with Luenberger-type P observer,

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(C\hat{x}(t) - y(t)). \quad (7)$$

In [5] three approaches are used to solve the problem. As pointed out in the introduction, an observer with additional integral path known as PI observer found useful in various estimation and control scenarios. We apply this type of observer in connection to the decentralized framework to enhance the control objective. Although we can generalize and formulate three equivalent problems based on PI observer, we only concentrate on the third problem and provide a complete solution for it.

We assume that the controller is composed of a PI-observer and a state feedback control law, i.e.

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + L_p(C\hat{x}(t) - y(t)) + Bv(t) \\ \dot{v}(t) &= L_I(C\hat{x}(t) - y(t)) \\ u(t) &= K\hat{x}(t) \end{aligned} \quad (8)$$

where $[\hat{x}^T, v^T]^T \in \Re^{n+m}$ is the augmented observer state. $\hat{x} = [\hat{x}_1^T,\ldots,\hat{x}_N^T]^T$, $\hat{x}_i \in \Re^{n_i}$, $v = [v_1^T,\ldots,v_N^T]^T$, $v_i \in \Re^{m_i}$, $K = diag(K_1,\ldots,K_N)$, $L_I = diag(L_{I1},\ldots,L_{IN})$ and $L_P = diag(L_{P1},\ldots,L_{PN})$ represent the global controller parameter matrices, while triplet (L_{Pi}, L_{Ii}, K_i) determine the local dynamic controllers.

Combining (8) and global system (3), the resulting closed-loop system can be written as:

$$\dot{z}(t) = A_f z(t) + h_f(t,x), \qquad y(t) = C_f z(t) \quad (9)$$

where z is the state of the closed-loop system. Defining

$z = [z_1^T, z_2^T, z_3^T]^T$, $z_1 = x$, $z_2 = \hat{x} - x$ and $z_3 = v$, we have

$$A_f = \begin{bmatrix} A+BK & BK & 0 \\ 0 & A+L_P C & B \\ 0 & L_I C & 0 \end{bmatrix} \quad h_f = \begin{bmatrix} h(t,z_1) \\ -h(t,z_1) \\ 0 \end{bmatrix} \tag{10}$$

$$C_f = \begin{bmatrix} C & 0 & 0 \end{bmatrix}$$

in which A_f and C_f can be rephrased as

$$A_f = \begin{bmatrix} A+BK & BK_X \\ 0 & A_X + L_X C_X \end{bmatrix} \quad C_f = \begin{bmatrix} C_X & 0 \end{bmatrix} \tag{11}$$

where

$$A_X = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} \quad B_X = \begin{bmatrix} B \\ 0 \end{bmatrix} \quad C_X = \begin{bmatrix} C & 0 \end{bmatrix}$$

$$L_X = \begin{bmatrix} L_P \\ L_I \end{bmatrix} \quad K_X = \begin{bmatrix} K & 0 \end{bmatrix} \tag{12}$$

Moreover, the quadratic inequality (6) will be satisfies with

$$H_f = \begin{bmatrix} 2H & 0 & 0 \end{bmatrix}$$

Let us define $Q = diag(Q_1,\ldots,Q_N)$, $P = diag(P_1,\ldots,P_N$, $\bar{P}_1,\ldots,\bar{P}_N)$, $W = KQ = diag(W_1,\ldots,W_N)$ and $V^T = L_X^T P^T =$ $[diag(V_1,\ldots,V_N)^T, diag(\bar{V}_1,\ldots,\bar{V}_N)^T]^T$, where Q_i , P_i , $\bar{P}_i$, W_i , V_i and $\bar{V}_i$ are $n_i \times n_i$, $n_i \times n_i$, $m_i \times m_i$, $m_i \times n_i$, $n_i \times p_i$ and $m_i \times p_i$ matrices respectively.

Substituting (11) and $X_f = diag(Q^{-1}, P)$ into (6), after appropriate congruent transformation, we can obtain

$$\begin{bmatrix} S_1 & BK_X & I & 0 & 2QH^T \\ K_X^T B^T & S_2 & 0 & P & 0 \\ I & 0 & -I & 0 & 0 \\ 0 & P & 0 & -I & 0 \\ 2QH & 0 & 0 & 0 & -\Psi \end{bmatrix} < 0 \tag{13}$$

where $S_1 = AQ + QA^T + BW + W^T B^T$ and $S_2 = PA_X + A_X^T P + VC_X + C_X^T V^T$ with $W = KQ$ and $V = PL_X$.

By taking into account the independency between K_X and L_X in (13), we propose the following two steps algorithm to solve this optimization problem.

Step 1:

Minimize $\mathrm{Tr}\,\Psi$

Subject to $Q > 0$,
$$\begin{bmatrix} S_1 & 0 & 2QH^T \\ 0 & -I & 0 \\ 2HQ & 0 & -\Psi \end{bmatrix} < 0 \tag{14}$$

Step 2:

Minimize $\mathrm{Tr}\,\Delta$

Subject to $P > 0$,

$$\begin{bmatrix} -I & 0 & I & 0 & 0 \\ 0 & S_2 & K_X^T B^T & P & 0 \\ I & BK_X & S_1 & 0 & 2QH^T \\ 0 & P & 0 & -I & 0 \\ 0 & 0 & 2HQ & 0 & -\Psi\Delta \end{bmatrix} < 0 \tag{15}$$

where $\Delta = diag(\delta_1 I_{l_1},\ldots,\delta_N I_{l_N})$ ($\delta_i > 0$), while Q , S_1 , Ψ , and $K_X = \begin{bmatrix} K & 0 \end{bmatrix}$ ($K = WQ^{-1}$) are obtained in step 1.

Theorem 2: The nonlinear interconnected system (3) is robustly stabilized with degree $\alpha = [\alpha_1,\ldots,\alpha_N]$ by the decentralized controller (8) if steps one and two have feasible solutions. Controller parameter are given by $K = WQ^{-1}$, $L_X = P^{-1}V$. The robustness degree bounds are $\alpha_i = 1/\sqrt{\psi_i \delta_i}$.

Proof: The proof follows simply the fact that second LMI in (15) is obtained by applying appropriate permutation into LMI (3) with Γ replaced by $\Gamma\Delta$. Notice that step 2 has to be performed after step 1 and not simultaneously.

The LMI optimization Problems given by (14) and (15) do not pose any restrictions on the size of the matrix variables Q , W , P and V . Consequently, the results of these two optimization problems may yield very large controller and observer gain matrices K and L_X . One can restrict K and L_X by posing constraints on the matrices Q , W , P and V as

$$Q_i^{-1} < \kappa_{Q_i} , W_i W_i^T < \kappa_{W_i} \tag{16}$$

$$P_i^{-1} < \kappa_{P_i} , V_i V_i^T < \kappa_{V_i} \tag{17}$$

Equations (16) and (17) are, respectively, equivalent to

$$\begin{bmatrix} -Q_i & -I \\ -I & -\kappa_{Q_i} \end{bmatrix} < 0 , \begin{bmatrix} -\kappa_{W_i} I & W_i \\ W_i^T & -I \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} -P_i & -I \\ -I & -\kappa_{P_i} \end{bmatrix} < 0 , \begin{bmatrix} -\kappa_{V_i} I & V_i^T \\ V_i & -I \end{bmatrix} < 0 \tag{19}$$

Combining (14) with (18) and (15) with (19), and changing the optimization objective to the minimization of $\sum_{i=1}^{N} (\psi_i + \kappa_{Q_i} + \kappa_{W_i})$ and $\sum_{i=1}^{N} (\delta_i + \kappa_{P_i} + \kappa_{V_i})$ results in the following two optimization problems:

Step 1':

$$\text{Minimize} \sum_{i=1}^{N} (\psi_i + \kappa_{Q_i} + \kappa_{W_i})$$

Subject to (14) and (18) (20)

Step 2':

$$\text{Minimize} \sum_{i=1}^{N} (\delta_i + \kappa_{P_i} + \kappa_{V_i})$$

Subject to (15) and (19) (21)

IV. DISTURBANCE ATTENUATION

Up to know we concentrated on robust stability. However, there are many instances that one would like to attenuate the disturbance while preserving the robust stability. In this section, we formulate the problem in terms of an LMI and provide the complete solution for it.

Let us consider the nonlinear interconnected system with external disturbance w as follows:

$$\begin{aligned} \dot{x}_f(t) &= A x_f(t) + h_f(t, x_f) + B_w w(t) \\ z_f(t) &= C_z x_f(t) + D_{zw} w(t) \end{aligned} \tag{22}$$

where $A = diag(A_1, \ldots, A_N)$, $B = diag(B_1, \ldots, B_N)$, $B_w = diag(B_{w1}, \ldots, B_{wN})$, $C_z = diag(C_{z1}, \ldots, C_{zN})$, $D_{zw} = diag(D_{zw1}, \ldots, D_{zwN})$, $w = [w_1^T, \ldots, w_N^T]^T \in \Re^{m_w}$ ($m_w = \sum_{i=1}^{N} m_{wi}$), $z = [z_1^T, \ldots, z_N^T]^T \in \Re^{p_z}$ ($p_z = \sum_{i=1}^{N} p_{zi}$), and $h_f(t,x) = [h_{f1}^T(t,x), \ldots, h_{fN}^T(t, x_f)]^T$ for which we have the quadratic constraint

$$h_f^T(t,x) h_f(t,x) \le x_f^T H_f^T \Psi^{-1} H_f x_f \tag{23}$$

where $H^T = [H_1^T, \ldots, H_N^T]$ is an $l \times n$ matrix ($l = \sum_{i=1}^{N} l_i$), $\Psi = diag(\psi_1 I_{l_1}, \ldots, \psi_N I_{l_N})$ and $\psi_i = \alpha_i^{-2}$.

We define the L_2 gain of the system (22) from w to z as the quantity

$$\sup_{\|w\|_2 \neq 0} \frac{\|z\|_2}{\|w\|_2}$$

where the L_2 norm of w is $\|w\|_2^2 = \int_0^\infty w^T w \, dt$. Now suppose there exists the quadratic function $V(\xi) = \xi^T X_f \xi$, $X_f > 0$, and $\gamma \ge 0$ such that for all t

$$\frac{d}{dt} V(x_f) + z^T z - \gamma^2 w^T w \le 0 \tag{24}$$

Then it can be shown that the L_2 gain of the system (22) is less than γ [2]. Condition (24) is equivalent to

$$x_f^T (A^T X_f + X_f A) x_f + 2 x_f^T X_f h_f + z^T z - \gamma^2 w^T w \le 0$$

For all x_f and h_f satisfying

$$h_f^T h_f \le x_f^T H_f^T \Psi^{-1} H_f x_f \tag{25}$$

Using the S-procedure one can show that this is true if and only if there exists $a \ge 0$ such that

$$\begin{bmatrix} A\hat{Y}_f + \hat{Y}_f A^T + a H_f^T \Psi^{-1} H_f & \hat{Y}_f C_z^T & B_w & \hat{Y}_f \\ C_z \hat{Y}_f & -\gamma I & D_{zw} & 0 \\ B_w^T & D_{zw}^T & -\gamma I & 0 \\ \hat{Y}_f & 0 & 0 & -aI \end{bmatrix} < 0$$

where $\hat{Y}_f = X_f^{-1}$. It is further equivalent to

$$\begin{bmatrix} A Y_f + Y_f A^T + H_f^T \Psi^{-1} H_f & Y_f C_z^T & bB_w & Y_f \\ C_z Y_f & -\gamma bI & bD_{zw} & 0 \\ bB_w^T & bD_{zw}^T & -\gamma bI & 0 \\ Y_f & 0 & 0 & -I \end{bmatrix} < 0 \tag{26}$$

where $b = a^{-1}$ and $Y_f = b\hat{Y}_f$. Relying on Schur complement formula, (26) can be written as

$$\begin{bmatrix} A Y_f + Y_f A^T & Y_f C_z^T & bB_w & Y_f & H_f^T \\ C_z Y_f & -\gamma bI & bD_{zw} & 0 & 0 \\ bB_w^T & bD_{zw}^T & -\gamma bI & 0 & 0 \\ Y_f & 0 & 0 & -I & 0 \\ H_f & 0 & 0 & 0 & -\Psi \end{bmatrix} < 0 \tag{27}$$

If one is interested in maintaining robust stability with maximum nonlinear interconnection while attenuate the disturbance by a prescribed factor γ, the following theorem can be employed.

Theorem 3: The nonlinear interconnected system (22) is robustly stable with degree $\alpha = [\alpha_1, \ldots, \alpha_N]$ and disturbance attenuation less than γ in the L_2 gain sense if the following problem is feasible.

Minimize $\mathrm{Tr}\,\Psi$

Subject to $Y_f > 0$, $b > 0$, and (27).

It is interesting to point out that from LMI (26), one can also be able to formulate the problem of minimizing L_2 gain of the system from w to z for the case of known bound on nonlinear the interconnection terms.

Using theorem 3 one can follow the procedure discussed in last section and design robust decentralized controller with disturbance attenuation performance.

Theorem 4: Consider the nonlinear interconnected system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + h(t,x) + B_w w(t) \\ z(t) &= C_z x(t) + D_{zw} w(t) \end{aligned} \tag{28}$$

$A = diag(A_1,\dots,A_N)$, $B = diag(B_1,\dots,B_N)$, $B_w = diag(B_{w1},\dots,B_{wN})$, $C_z = diag(C_{z1},\dots,C_{zN})$, $D_{zw} = diag(D_{zw1},\dots,D_{zwN})$, $w = [w_1^T,\dots,w_N^T]^T \in \Re^{m_w}$ ($m_w = \sum_{i=1}^N m_{wi}$), $z = [z_1^T,\dots,z_N^T]^T \in \Re^{p_z}$ ($p_z = \sum_{i=1}^N p_{zi}$), and $h(t,x) = [h_1^T(t,x),\dots,h_N^T(t,x_f)]^T$ for which we have the quadratic constraint

$$h^T(t,x)h(t,x) \le x^T H^T \Psi^{-1} H x \tag{29}$$

where $H^T = [H_1^T,\dots,H_N^T]$ is an $l \times n$ matrix ($l = \sum_{i=1}^N l_i$), $\Psi = diag(\psi_1 I_{l_1},\dots,\psi_N I_{l_N})$ and $\psi_i = \alpha_i^{-2}$. Then the decentralized state-feedback control

$$u = K_D x \tag{30}$$

where $K_D = diag(K_1,\dots,K_N)$ in with $K_i \in \Re^{m_i \times n_i}$ robustly stabilizes the nonlinear interconnected system (28) with degree α and disturbance attenuation less than γ in the L_2 gain sense if the following problem has a feasible solution.

Minimize Tr Ψ
Subject to $Y > 0$, $b > 0$

$$\begin{bmatrix} AY + YA^T + BW + W^T B^T & YC_z^T & bB_w & Y & H^T \\ C_z Y & -\gamma bI & bD_{zw} & 0 & 0 \\ bB_w^T & bD_{zw}^T & -\gamma bI & 0 & 0 \\ Y & 0 & 0 & -I & 0 \\ H & 0 & 0 & 0 & -\Psi \end{bmatrix} < 0 \tag{31}$$

where $Y = diag(Y_1,\dots,Y_N)$, $W = diag(W_1,\dots,W_N)$, Ψ and scalar b are the optimization variables. Furthermore, the controller gain is obtained by $K_D = Y^{-1}W$.

This LMI optimization problems may yield very large controller gain matrices K_D. Similar to the approach outlined in section III, One can restrict K_D by posing constraints on the matrices Y and W as

$$Y_i^{-1} < \kappa_{Y_i}, \; W_i W_i^T < \kappa_{W_i} \tag{32}$$

or equivalently

$$\begin{bmatrix} -Y_i & -I \\ -I & -\kappa_{Y_i} \end{bmatrix} < 0, \; \begin{bmatrix} -\kappa_{W_i} I & W_i \\ W_i^T & -I \end{bmatrix} < 0 \tag{33}$$

and solve the following optimization problem:

$$\begin{aligned} &\text{Minimize} \sum_{i=1}^N (\psi_i + \kappa_{Y_i} + \kappa_{W_i}) \\ &\text{Subject to (31) and (33)} \end{aligned} \tag{34}$$

Remark 1: The above problem can also be extended to the case where the states are not available. Both P and PI observer-based design can be incorporated to this LMI formulation following the procedure outlined in section III.

Remark 2: It should be pointed out that in certain situation where disturbance attenuation is not the objective; rather its estimation places a critical role, the use of PI observer can be beneficial. The advantage of PI observer in connection to fault detection has been demonstrated in [11], [13], and [14] and certainly can be employed here in decentralized framework as well. This will be the subject of future paper.

V. Example

As an example, we consider the system composed of two coupled inverted pendulums as shown in Figure 1. The pendulums are coupled by a sliding spring, the position of which is uncertain. We allow the sliding of the spring to be discontinuous function of both time and state of the system. The normalized model for the motion of two pendulums as in Figure 1 can be described by [1]:

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1 + e\begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} x_1 + e\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_1$$

$$\dot{x}_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_2 + e\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} x_1 + e\begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_2$$

$$y_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} x_1 \qquad y_2 = \begin{bmatrix} 1 & 0 \end{bmatrix} x_2$$

Using the optimization problem (20) and (21), The decentralized PI observer-based controller (8) yields $\alpha_i = 1.40$ with local controller gain $K_i = \begin{bmatrix} 583.29 & 2.97 \end{bmatrix}$

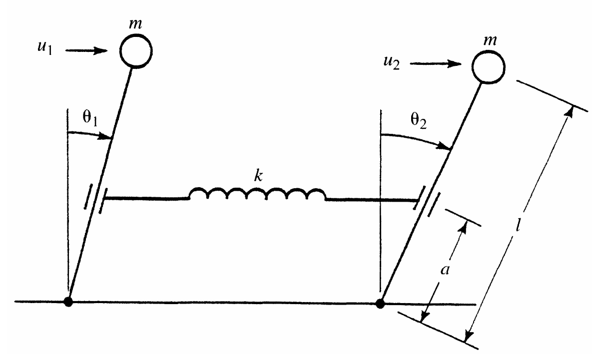

Fig. 1. Inverted Pendulums

and local observer gain $L_{Pi} = \begin{bmatrix} -71.09 \\ -120.87 \end{bmatrix}$ and $L_{Ii} = -17.65$ for $i = 1,2$.

In the presence of disturbance, one can use optimization problem (34) to design a robust decentralized controller to attenuate the disturbance. For inverted pendulums example, the resulting local controllers provide $\alpha_i = 0.34$ with control gains $K_i = \begin{bmatrix} -13.19 & -12.10 \end{bmatrix}$ for $i = 1,2$ with desired disturbance attenuation $\gamma = 0.1$.

VI. Conclusion

In this paper, we presented the design of decentralized PI observer-based controller for linear interconnected systems with nonlinear interconnections satisfying quadratic constraints. The controller structure contains a PI observer and a state feedback control law. The extended LMI problem formulation guarantees the robust stabilization while maximizing the interconnection bounds. Moreover, by including a disturbance term in the problem, we provide an LMI formulation which guarantees a prescribed disturbance attenuation performance while maximizing the interconnection bounds.

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