2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

Control of Synchronization for Multi-Agent Systems in Acceleration Motion with Additional Analysis of Formation Control

Haopeng Zhang, Kalana Pothuvila, Qing Hui, Ran Yang, and Jordan M. Berg

Abstract— **The synchronization control of multi-agent systems plays a significant role in military and civil applications. This paper investigates the acceleration motion of multi-agent systems – unlike other synchronization protocols, we introduce the constant value into the protocol, which acts as the acceleration of the agents. Furthermore, we expand the acceleration protocol into the formation control protocol, by which the multi-agents can achieve the desired formations. Simulations are provided for both protocols, and additionally we adopt the Amigobots as our experiment environment to verify our theoretical results. Compared with the Matlab simulations, we conclude that, under the proposed protocols, the system goes to acceleration synchronization or desired formation with acceptable error.**

I. Introduction

Synchronization control for multi-agent systems has attracted more and more attention and research for its wide applications in military and civil aspects [1]-[4], including UAV's (Unmanned Air Vehicles) and UGV's (Unmanned Ground Vehicles), distributed wireless sensor networks, and swarms of heterogeneous air and space vehicles. The main research in this area has been focusing on synchronization protocol design. Many different design approaches have been proposed to guarantee synchronization of multi-agent systems under different circumstances. Specifically, in [5] an observation-based synchronization protocol was presented and a necessary and sufficient condition was provided to guarantee the system synchronization. [12] adopted the leader-follower mode to investigate the synchronization problem for multi-agent systems. The pinning control approach for the synchronization of multi-agent systems was investigated in [13] in which a general criterion for ensuring network synchronization has been derived.

In this paper, two problems are investigated. Firstly, the averaging problem is discussed in large volumes of literature, but there is still one simple question remaining: can we achieve the desired synchronization value, not just the average for the multi-agent system? The second question is, can we achieve the desired formation for the multi-agent system by a little change in the synchronization protocol? With these questions in mind, we propose a novel synchronization protocol to achieve the acceleration motion in synchronization for the multi-agent system characterized by double integrators. Under our protocol, the agents approach the same acceleration motion by analyzing the zero-state and zero-input effect for the system. Moreover, unlike other synchronization [14] protocols, under which the velocity of the synchronization is the average of the initial velocities of each agent, we can adjust the synchronization velocity by adjusting the constant in our protocol. Furthermore, based on the proposed protocol, we present a formation protocol under which the system can achieve the desired formation. Not merely restricted to one dimension, we expand the protocols into the x-y plane, and even x-y-z space.

Additionally, we adopt the Amigobots as our experiment environment to test the acceleration synchronization protocol and formation protocol for the multi-agent system, but due to the limit of the robots, a one-dimensional experiment is investigated. The corresponding Matlab simulations are also provided which display the exact synchronization or formation for the agents. The robots move into synchronization with acceptable error, thus verifying the theoretical analysis of the protocol. Moreover, the Matlab simulations in the x-y-z space are provided.

The organization of the rest of paper is as follows: the basic graph theory is introduced in Section II, and in Section II, the acceleration synchronization protocol is proposed and fully analyzed; in Section III, we adjust the proposed protocol into the formation protocol. Finally, the Amigobots experiment and Matlab simulation results are shown in Section IV, and Section V concludes the paper.

II. Mathematical Preliminaries

Graph theory is a powerful tool for investigating networked systems. In this paper, we use graph-related notation to describe our network model. More specifically, let $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ denote an undirected graph with the set of vertices $\mathscr{V} \doteq \{v_1, v_2, v_3, ...\}$ and $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ which represents the set of edges. The matrix $\mathscr{A}$ with nonnegative adjacency elements $a_{i,j}$ serves as the weighted adjacency matrix. The node index of $\mathscr{G}$ is denoted as a finite index set $N = \{1, 2, 3, ...\}$. An edge of $\mathscr{G}$ is denoted by $e_{i,j} = (V_i, V_j)$ and the adjacency

This work was supported by the Defense Threat Reduction Agency, Basic Research Award #HDTRA1-10-1-0090, to Texas Tech University.

H. Zhang, K. Pothuvila, Q. Hui and J. M. Berg are with the Department of Mechanical Engineering, Texas Tech University, Lubbock. TX 79409-1021, USA (haopeng.zhang@ttu.edu; qing.hui@ttu.edu; kalana.Pothuvila@ttu.edu; jordan.berg@ttu.edu).

R. Yang is with the School of Information Science and Technology, Sun Yat-Sen University, Guangzhou, Guangdong, 510275, China yangran@mail.sysu.edu.cn.

elements associated with the edges are positive. We assume $e_{i,j} \in \mathscr{E} \Leftrightarrow a_{i,j} = 1$ and $a_{i,i} = 0$ for all $i \in \mathscr{N}$.

The set of neighbors of the node V_i is denoted by $\mathscr{N} = \{V_j \in \mathscr{V} | (v_i, v_j) \in \mathscr{E}, j = 1 : M, j \neq i\}$. The degree matrix of a graph $\mathscr{G}$ is defined as

$$\triangle = [\delta_{i,j}] \tag{1}$$

where

$$\delta_{i,j} = \begin{cases} \sum_{j=1}^{N} a_{i,j}, & \text{if } i = j, \\ 0, & \text{if } i \neq j. \end{cases}$$

The Laplacian matrix of graph $\mathscr{G}$ is defined by

$$L = \triangle - \mathscr{A}. \tag{2}$$

If there is a path from any node to any other node in the graph, then we call the graph *connected*. In this paper, we assume that the topology of the multi-agent system is connected.

Main Result

Given a connected graph, according to Newtonian mechanics, we adopt the same dynamic model as [9] for each agent:

$$\begin{aligned} \dot{x}_i &= v_i \\ m_i \dot{v}_i &= u_i \end{aligned} \tag{3}$$

where x_i, v_i, m_i are the position, velocity, and mass of the node i, respectively. Moreover, for a system of M agents, we assume $m_1 = m_2 = ... = m_M = 1$.

Definition 2.1: The synchronization problem in acceleration motion for multi-agent systems is to find a control law u_i such that

$$\begin{aligned} v_i(t) &= \frac{1}{M}\mathbf{1}\mathbf{1}^{\mathrm{T}} v(0) + bt \\ x_i(t) &= \frac{1}{M}\mathbf{1}\mathbf{1}^{\mathrm{T}} x(0) + \frac{1}{M}\mathbf{1}\mathbf{1}^{\mathrm{T}} v(0)t + \frac{1}{2}bt^2 \end{aligned} \tag{4}$$

I.e., the trajectory of each agent of the multi-agent system is under the same equation of acceleration motion. Here, $x(0) = \begin{bmatrix} x_1(0) & x_2(0) & \cdots & x_M(0) \end{bmatrix}^{\mathrm{T}}$ and $v(0) = \begin{bmatrix} v_1(0) & v_2(0) & \cdots & v_M(0) \end{bmatrix}^{\mathrm{T}}$, and $x_i(0), v_i(0)$ are the initial states of the multi-agent system, $\mathbf{1} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix} \in \mathbb{R}^{M \times 1}$.

The following theorem is one of the main results of the paper.

Theorem 2.1: Given a connected multi-agent system, the system achieves synchronization in acceleration motion as defined in Definition 2.1 under the following synchronization protocol

$$u_i = \sum_{j=1, j \neq i}^{M} a_{i,j}[g(v_j - v_i) + k(x_j - x_i)] + b \tag{5}$$

with $k > 0$, $g > 0$, a_{ij} is the (i,j)th element of the matrix $\mathscr{A}$, and b serves as the acceleration.

With $\mathcal{E}_i = \begin{bmatrix} x_i, v_i \end{bmatrix}^{\mathrm{T}}$ and $\psi_i = \begin{bmatrix} 0, b \end{bmatrix}^{\mathrm{T}}$, under the protocol (5), we obtain the vector form for the multi-agent system

$$\dot{\mathcal{E}}_i = A\mathcal{E}_i + B \sum_{j=1, j \neq i}^{M} a_{i,j}(\mathcal{E}_j - \mathcal{E}_i) + \psi_i \tag{6}$$

where $A = \begin{bmatrix} 0, 1 \\ 0, 0 \end{bmatrix}, B = \begin{bmatrix} 0, 0 \\ k, g \end{bmatrix}$. Furthermore, define $\mathcal{E} = \begin{bmatrix} \mathcal{E}_1 \mathcal{E}_2 \cdots \mathcal{E}_M \end{bmatrix}$ Then, the multi-agent system dynamics become

$$\dot{\mathcal{E}} = \Phi \times \mathcal{E} + \psi \tag{7}$$

where $\Phi = I_M \otimes A - L \otimes B$ and $\psi = 1_M \otimes \begin{bmatrix} 0 \\ b \end{bmatrix}$, where $\otimes$ denotes the Kronecker product.

To prove our main result, we investigate the the zero-state effect and zero-input effect of the system.

A. Zero-input effect of the system

In this subsection, we will study the zero-input effect of the system, in other words, the constant vector ψ is assumed to be a zero vector. The following two lemmas are needed.

Lemma 2.1: The matrix Φ in the system (7) has the following properties:

1. The matrix Φ has the eigenvalue $\lambda = 0$ with algebraic multiplicity of two, which is denoted by $\lambda_{\Phi,1} = \lambda_{\Phi,2} = 0$.
2. Except for $\lambda_{\Phi,1}$ and $\lambda_{\Phi,2}$, all the other eigenvalues of Φ satisfy $Re(\lambda_{\Phi,i}) < 0, i = 3, \cdots, 2M$.

Proof: Based on the fact that the eigenvalues of the matrix L can be denoted by

$$0 = \lambda_{L,1} < \lambda_{L,2} < \lambda_{L,3} < \cdots < \lambda_{L,M}$$

and there exists an orthogonal matrix W such that

$$W^{-1}LW = \mathrm{diag}\{\lambda_{\mathrm{L},1}, \lambda_{\mathrm{L},2}, \cdots, \lambda_{\mathrm{L},\mathrm{M}}\}$$

then it follows that

$$\begin{aligned} &(W^{-1} \otimes I_2)\Phi(W \otimes I_2) \\ &= I_M \otimes A - \mathrm{diag}\{\lambda_{\mathrm{L},1}, \lambda_{\mathrm{L},2}, \cdots, \lambda_{\mathrm{L},\mathrm{M}}\} \otimes \mathrm{B} \\ &= \mathrm{diag}\{\mathrm{A}, \mathrm{A} - \lambda_{\mathrm{L},2}\mathrm{B}, \cdots, \mathrm{A} - \lambda_{\mathrm{L},\mathrm{M}}\mathrm{B}\} \end{aligned} \tag{8}$$

Since the matrix A has two eigenvalues $\lambda_{A,1} = \lambda_{A,2} = 0$, it gives that $\lambda_{\Phi,1} = \lambda_{\Phi,2} = 0$. For $i = 2, 3, \cdots, M$,

$$\det(A - \lambda_{L,i}B) = \det \begin{bmatrix} 0, & 1 \\ -\lambda_{L,i}g, & -\lambda_{L,i}k \end{bmatrix} = \lambda_{L,i}g \neq 0$$

Thus,

$$\begin{aligned} \mathrm{rank}(\Phi) &= \mathrm{rank}((\mathrm{W}^{-1} \otimes \mathrm{I}_2)\Phi(\mathrm{W} \otimes \mathrm{I}_2)) \\ &= \mathrm{rank}(\mathrm{A}) + \sum_{\mathrm{i}=2}^{\mathrm{M}} \mathrm{rank}(\mathrm{A} - \lambda_{\mathrm{L,i}}\mathrm{B}) \\ &= 1 + 2(M-1) \\ &= 2M - 1 \end{aligned} \tag{9}$$

Furthermore, the characteristic polynomial of $A-\lambda_{L,i}B$ is given as follows

$$\begin{aligned} f_i(s) &= \det\begin{bmatrix} s, & 1 \\ \lambda_{L,i}g, & s+\lambda_{L,i}k \end{bmatrix} \\ &= s^2+\lambda_{L,i}ks+\lambda_{L,i}g \end{aligned} \tag{10}$$

Put $f_i(s)=0$, then

$$s=\frac{-\lambda_{L,i}k\pm\sqrt{(\lambda_{L,i}k)^2-4\lambda_{L,i}g}}{2}$$

Since $k,g>0$, it follows that $Re(\lambda_{\Phi,i})<0$ for $i=2,3,\cdots,2M$. ∎

Lemma 2.2: let w_r be the right eigenvector associated with eigenvalue zero, and w_l be the left eigenvector associated with eigenvalue zero. Then,

$$\begin{aligned} w_r &= \frac{1}{\sqrt{M}}\mathbf{1}_M\otimes\begin{bmatrix}1 & 0\end{bmatrix}^{\mathrm{T}} \\ w_l &= \frac{1}{\sqrt{M}}\mathbf{1}_M\otimes\begin{bmatrix}0 & 1\end{bmatrix}^{\mathrm{T}} \end{aligned} \tag{11}$$

and the generalized right eigenvector and the generalized left eigenvector of Φ, denoted by v_r and v_l respectively, are

$$\begin{aligned} v_r &= \frac{1}{\sqrt{M}}\mathbf{1}_M\otimes\begin{bmatrix}-k/g & 1\end{bmatrix}^{\mathrm{T}} \\ v_l &= \frac{1}{\sqrt{M}}\mathbf{1}_M\otimes\begin{bmatrix}1 & k/g\end{bmatrix}^{\mathrm{T}} \end{aligned} \tag{12}$$

Proof: Note that

$$\begin{aligned} \Phi\times w_r &= (\begin{bmatrix}0,1\\0,0\end{bmatrix}\otimes I_M-\begin{bmatrix}0,0\\k,g\end{bmatrix}\otimes L) \\ &\quad\times(\frac{1}{\sqrt{M}}1_{1\times M}\otimes\begin{bmatrix}1\\0\end{bmatrix}) \\ &= \begin{bmatrix}0,1\\0,0\end{bmatrix}\times\begin{bmatrix}1\\0\end{bmatrix}\otimes(I_M\frac{1}{\sqrt{M}}1_{1\times M})+\mathbf{0}_{2M} \\ &= \mathbf{0}_{2M} \\ \Phi\times v_r &= (\begin{bmatrix}0,1\\0,0\end{bmatrix}\otimes I_M-\begin{bmatrix}0,0\\k,g\end{bmatrix}\otimes L) \\ &\quad\times(\frac{1}{\sqrt{M}}1_M\otimes\begin{bmatrix}-k/g\\1\end{bmatrix})^T \\ &= \frac{1}{\sqrt{M}}\mathbf{1}_{M1}\times\mathbf{I}_M\otimes\begin{bmatrix}1\\0\end{bmatrix} \\ &= w_r \end{aligned} \tag{13}$$

Similarly,

$$w_l^T\times\Phi=\mathbf{0}_{2M}^{\mathrm{T}} \tag{14}$$

also, $v_l^T\Phi=w_l$, which completes the proof. ∎

Then, we propose the state-input effect for the multi-agent system.

Lemma 2.3: Consider the dynamic system (7) with $\psi=0$. Then for any $k,g>0$,

$$\lim_{t\to+\infty}\zeta(t)=\quad \lim_{t\to+\infty}\exp(\Phi)\zeta(0) \tag{15}$$

$$=\quad\begin{bmatrix}\frac{1}{N}\mathbf{11}^{\mathrm{T}}x(0)+\frac{1}{N}\mathbf{11}^{\mathrm{T}}v(0)t\\ \frac{1}{N}\mathbf{11}^{\mathrm{T}}v(0)\end{bmatrix} \tag{16}$$

Proof: Based on Lemma 2.1 and 2.2, there must exist a nonsingular matrix P such that Φ can be factored into the Jordan canonical form

$$\begin{aligned} \Phi &= PJP^{-1} \\ &= P\begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \lambda_{\Phi,3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda_{\Phi,2M} \end{bmatrix}P^{-1} \end{aligned} \tag{17}$$

where $P=\begin{bmatrix}w_r, & v_r, & p_3, & \cdots, & p_{2M}\end{bmatrix}, P^{-1}=\begin{bmatrix}v_l, & w_l, & q_3, & \cdots, & q_{2M}\end{bmatrix}^{\mathrm{T}}$ and $p_3,\cdots,p_{2M}$ $(q_3,\cdots,q_{2M})$ are the right (left) eigenvectors or generalized right eigenvectors associated with eigenvalues $\lambda_{\Phi,3},\cdots,\lambda_{\Phi,2M}$, respectively. Since $\lambda_{\Phi,3},\cdots,\lambda_{\Phi,2M}<0$, it follows that

$$\begin{aligned} \lim_{t\to+\infty}\mathcal{E}(t) &= \lim_{t\to+\infty}\exp(\Phi t)\zeta(0) \\ &= \frac{\mathbf{1}_M}{M}\otimes\begin{bmatrix}1 & \frac{-k}{g}\\0 & 1\end{bmatrix}\begin{bmatrix}1 & t\\0 & 1\end{bmatrix}\begin{bmatrix}1 & \frac{k}{g}\\0 & 1\end{bmatrix}\otimes\mathbf{1}_M^{\mathrm{T}} \\ &= \begin{bmatrix}\frac{1}{M}\mathbf{11}^{\mathrm{T}}x(0)+\frac{1}{M}\mathbf{11}^{\mathrm{T}}v(0)t\\ \frac{1}{M}\mathbf{11}^{\mathrm{T}}v(0)\end{bmatrix} \end{aligned} \tag{18}$$

which completes the proof. ∎

B. Zero-state effect of the system

To prove Theorem 2.1, we need to investigate the zero-state effect of the system (7) in this subsection.

Lemma 2.4: Given a connected topology for the multi-agent system, the zero-state effect of the system (7) is

$$\lim_{t\to+\infty}\int_0^t\exp[\Phi(t-\tau)]\psi\,\mathrm{d}\tau=\begin{bmatrix}\frac{1}{2}bt^2\\bt\end{bmatrix}\otimes\mathbf{1}_M \tag{19}$$

Proof: Note that

$$\begin{aligned} &\lim_{t\to+\infty}\int_0^t\exp[\Phi(t-\tau)]\,\mathrm{d}\tau \\ &= P\lim_{t\to+\infty}\exp(J(t))P^{-1} \\ &= [w_r,v_r]\begin{bmatrix}t,\frac{1}{2}t^2\\0,t\end{bmatrix}\begin{bmatrix}v_l^{\mathrm{T}}\\w_r^{\mathrm{T}}\end{bmatrix} \\ &+\begin{bmatrix}p_3,p_4,\dots,p_{2M}\end{bmatrix}diag\{\frac{1}{\lambda_{\Phi,3}},\frac{1}{\lambda_{\Phi,4}},\dots,\frac{1}{\lambda_{\Phi 2M}}\} \\ &\times\begin{bmatrix}q_3 & q_4 & \cdots & q_{2M}\end{bmatrix}^{\mathrm{T}} \end{aligned} \tag{20}$$

It is easy to verify that

$$[w_r,v_r]\begin{bmatrix}t,\frac{1}{2}t^2\\0,t\end{bmatrix}\begin{bmatrix}v_l^{\mathrm{T}}\\w_r^{\mathrm{T}}\end{bmatrix}=\frac{1}{M}(\mathbf{1}_M)(\mathbf{1}_M)^{\mathrm{T}}\otimes\begin{bmatrix}1,\frac{1}{2}t^2\\0,t\end{bmatrix} \tag{21}$$

Next we will prove

$$\begin{aligned} &\begin{bmatrix}p_3,p_4,\dots,p_{2M}\end{bmatrix}diag\{\tfrac{1}{\lambda_{\Phi,3}},\tfrac{1}{\lambda_{\Phi,4}},\dots,\tfrac{1}{\lambda_{\Phi 2M}}\}\begin{bmatrix}p_3^{\mathrm{T}}\\p_4^{\mathrm{T}}\\\dots\\p_{2M}^{\mathrm{T}}\end{bmatrix} \\ &= \tfrac{1}{M}L_M\otimes\begin{bmatrix}-k/g,0\\1,0\end{bmatrix}+S\otimes\begin{bmatrix}0,1/g\\0,0\end{bmatrix} \end{aligned} \tag{22}$$

where

$$L_M = \begin{bmatrix} M & -1 & \cdots & -1 \\ -1 & M & \ddots & \vdots \\ \vdots & \ddots & \ddots & -1 \\ -1 & \cdots & -1 & M \end{bmatrix}$$

and S is a matrix satisfying $SL = \frac{1}{M}L_M$, and all the row sums of S and the column sums of S are zero.
First, we introduce two parameters α and β. Note that

$$\begin{aligned} &Pdiag\{\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\lambda_{\Phi,3}}, \frac{1}{\lambda_{\Phi,4}}, \dots, \frac{1}{\lambda_{\Phi 2M}}\}P^{-1} \\ &\times Pdiag\{\alpha, \beta, \lambda_{\Phi,3}, \dots, \lambda_{\Phi 2M}\}P^{-1} \\ &= \mathbf{0}_{2M} \end{aligned} \tag{23}$$

On the other hand,

$$\begin{aligned} &Pdiag\{\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\lambda_{\Phi,3}}, \frac{1}{\lambda_{\Phi,4}}, \dots, \frac{1}{\lambda_{\Phi 2M}}\}P^{-1} \\ &= \begin{bmatrix} w_r & v_r \end{bmatrix} diag\{\frac{1}{\alpha}, \frac{1}{\beta}\} \begin{bmatrix} v_l & w_l \end{bmatrix}^{\mathrm{T}} \\ &+ \begin{bmatrix} p_3 & \dots & p_{2M} \end{bmatrix} diag\{\frac{1}{\lambda_{\Phi,3}}, \frac{1}{\lambda_{\Phi,4}}, \dots, \frac{1}{\lambda_{\Phi 2M}}\} \\ &\times \begin{bmatrix} q_3 & q_4 & \cdots & q_{2M} \end{bmatrix}^{\mathrm{T}} \end{aligned} \tag{24}$$

and

$$\begin{aligned} &Pdiag\{\alpha, \beta, \lambda_{\Phi,3}, \dots, \lambda_{\Phi 2M}\}P^{-1} \\ &= \Phi + \begin{bmatrix} w_r & v_r \end{bmatrix} \begin{bmatrix} \alpha & -1 \\ 0 & \beta \end{bmatrix} \begin{bmatrix} v_l & w_l \end{bmatrix}^{\mathrm{T}} \end{aligned} \tag{25}$$

it is easy to verify that

$$\begin{aligned} &(\frac{1}{M}L_M \otimes \begin{bmatrix} -k/g, 0 \\ 1, 0 \end{bmatrix} + S \otimes \begin{bmatrix} 0, 1/g \\ 0, 0 \end{bmatrix} \\ &+ \begin{bmatrix} w_r & v_r \end{bmatrix} diag\{\frac{1}{\alpha}, \frac{1}{\beta}\} \begin{bmatrix} v_l & w_l \end{bmatrix}^{\mathrm{T}}) \\ &\times (\Phi + \begin{bmatrix} w_r & v_r \end{bmatrix} \begin{bmatrix} \alpha & -1 \\ 0 & \beta \end{bmatrix} \begin{bmatrix} v_l & w_l \end{bmatrix}^{\mathrm{T}}) \\ &= \mathbf{0}_{2M} \end{aligned} \tag{26}$$

Then, together with (23) and (24), one can obtain (22).
Furthermore,

$$\begin{aligned} &\begin{bmatrix} p_3, p_4, \dots, p_{2M} \end{bmatrix} diag\{\tfrac{1}{\lambda_{\Phi,3}}, \tfrac{1}{\lambda_{\Phi,4}}, \dots, \tfrac{1}{\lambda_{\Phi 2M}}\} \begin{bmatrix} q_3^T \\ \vdots \\ q_{2M}^T \end{bmatrix} \\ &\times \mathbf{1}_M \otimes \begin{bmatrix} 0 \\ b \end{bmatrix} = \mathbf{0}_{2M} \end{aligned} \tag{27}$$

$$\begin{bmatrix} w_r, v_r \end{bmatrix} \begin{bmatrix} t, \frac{1}{2}t^2 \\ 0, t \end{bmatrix} \begin{bmatrix} v_l^{\mathrm{T}} \\ w_r^{\mathrm{T}} \end{bmatrix} \times \mathbf{1}_M \otimes \begin{bmatrix} 0 \\ b \end{bmatrix} = \begin{bmatrix} \frac{1}{2}bt^2 \\ bt \end{bmatrix} \otimes \mathbf{1}_M \tag{28}$$

Now, based on Lemma 2.1 and Lemma 2.2, Theorem 2.1 is obvious. ■

From this point, we can solve the former first question, that is, how we can achieve the desired consensus value, which is defined here as the achievement of the desired velocity for the multi-agent system. This desired velocity is stated by the following theorem.

Theorem 2.2: For the connected system (7), with $k, g > 0$ and a constant $b \neq 0$, $a_{i,j}$ as defined before, the multi-agent system achieves desired velocity v_d under the following control protocol.

$$u_i = \mathcal{C}(t)\{\sum_{j=1, j\neq i}^{M} a_{i,j}[g(v_j - v_i) + k(x_j - x_i)] + b\} \tag{29}$$

where

$$\mathcal{C}(t) \triangleq \begin{cases} 0, & \text{if } t \in [t_d, \infty), \\ 1, & \text{otherwise}, \end{cases} \tag{30}$$

and

$$t_d = \frac{v_d - \frac{1}{N}\mathbf{1}\mathbf{1}^{\mathrm{T}}v(0))}{b} \tag{31}$$

Proof: According to Theorem 2.1, the system goes into acceleration synchronization at time t_s, and the expression of the velocity of each agent is

$$v_i(t) = \frac{1}{N}\mathbf{1}\mathbf{1}^{\mathrm{T}}v(0)) + bt \tag{32}$$

with $t_d > t_s$, and at time t_d,

$$v_i(t_d) = v_d \tag{33}$$

and $u_i(t) = 0$ for $t \geq t_d$, so the desired consensus is achieved. ■

III. Formation control

To address the second question in regard to formation control for multi-agent systems, in this section we propose a formation control protocol which is the transformation of our synchronization protocol presented before.

The formation control protocol for the multi-agent system is:

$$u_i = \sum_{j=1, j\neq i}^{M} a_{i,j}[g(v_j - v_i) + k(x_j - x_i - l_{i,j})] \tag{34}$$

where $l_{i,j}$ is the ideal distance between i and j. Here, we try to control the formation for the multi-agent system via the control of the distance between each agent.

We can obtain the compact vector form of the multi-agent system,

$$\dot{\mathcal{E}} = \Phi \times \mathcal{E} + \psi \tag{35}$$

where $\Phi = I_M \otimes A - L \otimes B$ A,B is as defined before and $\psi = L \otimes \begin{bmatrix} 0 & 0 \\ 0 & g \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & l_{1,2} & 0 & l_{1,3} & \cdots & 0 & l_{1,M} \end{bmatrix}$.

Define $d_{i,j} = x_j - x_i$ and we have the following theorem.

Theorem 3.1: Given a connected multi-agent system (35), under the formation control protocol (34), the system goes to $v_i(t) = \frac{1}{M}\sum v_i(0) d_{i,j}(t) = l_{ij}$.

Proof: For the zero-state effect for the system, because of (21) and (22), we have

$$\begin{aligned} &\begin{bmatrix} w_r, v_r \end{bmatrix} \begin{bmatrix} t, \frac{1}{2}t^2 \\ 0, t \end{bmatrix} \begin{bmatrix} v_l^{\mathrm{T}} \\ w_r^T \end{bmatrix} \times \psi = \frac{1}{M}(\mathbf{1}_M)(\mathbf{1}_M)^{\mathrm{T}} \times L \\ &\otimes \begin{bmatrix} t, \frac{1}{2}t^2 \\ 0, t \end{bmatrix} \times \begin{bmatrix} 0, 0 \\ 0, g \end{bmatrix} = \mathbf{0}_{M\times M} \otimes \begin{bmatrix} t, \frac{1}{2}t^2 \\ 0, t \end{bmatrix} \\ &\times \begin{bmatrix} 0, 0 \\ 0, g \end{bmatrix} = \mathbf{0}_{2M} \end{aligned} \tag{36}$$

Furthermore,

$$[p_3, p_4, \ldots, p_{2M}]\ diag\{\tfrac{1}{\lambda_{\Phi,3}}, \tfrac{1}{\lambda_{\Phi,4}}, \ldots, \tfrac{1}{\lambda_{\Phi 2M}}\}$$
$$\begin{bmatrix} q_3 & q_4 & \cdots & q_{2M} \end{bmatrix}^{\mathrm{T}} \times \psi$$
$$= \left(L \otimes \begin{bmatrix} \frac{-\frac{k}{g}}{M} & 0 \\ \frac{1}{M} & 0 \end{bmatrix} + S \otimes \begin{bmatrix} 0 & \frac{1}{g} \\ 0 & 0 \end{bmatrix} \right) \times \psi$$
$$= \left(S \otimes \begin{bmatrix} 0 & \frac{1}{g} \\ 0 & 0 \end{bmatrix} \right) \times \left(L \otimes \begin{bmatrix} 0 & 0 \\ 0 & g \end{bmatrix} \right)$$
$$\times \begin{bmatrix} 0 & 0 & 0 & l_{1,2} & 0 & l_{1,3} & \cdots & 0 & l_{1,M} \end{bmatrix}^{\mathrm{T}}$$
$$= \tfrac{1}{M} L_M \otimes \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$
$$\times \begin{bmatrix} 0 & 0 & 0 & l_{1,2} & 0 & l_{1,3} & \cdots & 0 & l_{1,M} \end{bmatrix}^{\mathrm{T}}$$
$$= \tfrac{-\mathbf{1}}{M} \textstyle\sum_{j=2}^{M} l_{1,j}$$
$$+ \begin{bmatrix} 0 & 0 & 0 & l_{1,2} & 0 & l_{1,3} & \cdots & 0 & l_{1,M} \end{bmatrix}^{\mathrm{T}} \quad (37)$$

Then, together with lemma 2.3, the theorem can follow immediately. ∎

Furthermore, one can extend our synchronization protocol and formation control protocol into the x-y plane; even x-y-z space. Without losing generality, the X-Y mode is developed here.

X-Y plane mode:

$$\begin{aligned} \dot{x}_i &= v_i^x \\ \dot{y}_i &= v_i^y \\ \dot{v}_i^x &= u_i^x \\ \dot{v}_i^y &= u_i^y \end{aligned}$$

where x_i, y_i is the position, v_i^x, and v_i^y is the x-axis velocity and y-axis velocity, respectively. u_i^x and u_i^y are the control inputs of the agent i respectively. Furthermore, we propose the x-y plane synchronization protocol for the multi-agent system

$$\begin{aligned} u_i^x &= \textstyle\sum_{j=1, j\neq i}^{M} a_{i,j}[g^x(v_j^x - v_i^x) + k^x(x_j - x_i)] + b^x \\ u_i^y &= \textstyle\sum_{j=1, j\neq i}^{M} a_{i,j}[g^y(v_j^y - v_i^y) + k^y(y_j - y_i)] + b^y \end{aligned} \quad (38)$$

where $a_{i,j}$ is defined as before, and g^x, g^y, k^x, k^y are positive constants.

Based on the Theorem (2.1), we can obtain the following corollary.

Corollary 3.1: Given a connected multi-agent system (7), the multi-agent system achieves acceleration synchronization under the control protocol (38).

Furthermore, we propose the following x-y plane formation control protocol for the multi-agent system:

$$\begin{aligned} u_i^x &= \sum_{j=1, j\neq i}^{M} a_{i,j}[g^x(v_j^x - v_i^x) + k^x(x_j - x_i - l_{i,j}^x)] \\ u_i^y &= \sum_{j=1, j\neq i}^{M} a_{i,j}[g^y(v_j^y - v_i^y) + k^y(y_j - y_i - l_{i,j}^y)] \end{aligned} \quad (39)$$

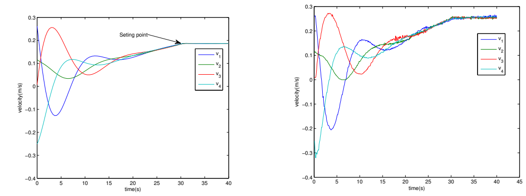
Since the goal is to achieve a particular formation for the multi-agent system, it is necessary for the multi-agent system to achieve the same velocity, so here, we assume $\mathbf{11}^{\mathrm{T}} v^x(0) = \mathbf{11}^{\mathrm{T}} v^y(0)$.

Based on Theorem (3.1), we can arrive at the following corollary.

Corollary 3.2: Given a connected multi-agent system (7), the system achieves the desired formation under the control protocol (39).

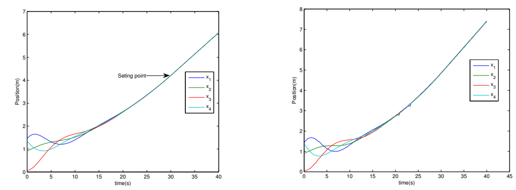
IV. Simulation and Experiments

Given a connected topology, there are four agents in the system and the topology is connected. Under our protocol (5) with $k = 1$, $g = 2$, and $b = 2$, Fig. 1 and Fig. 2 are the Matlab simulation results, i.e., the velocities and positions vs time t, respectively. We can conclude that the system goes into acceleration synchronization with the same equation of motion. Moreover, we apply the protocol to achieve the desired consensus velocity. As Fig. 1 shows, the



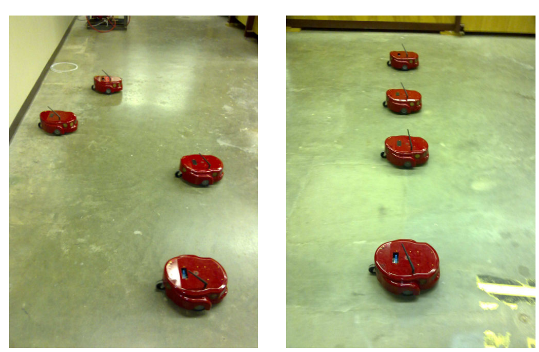
(a) Simulation of Velocity vs Time (b) Experiment of Velocity vs Time

Fig. 1. Comparison of the Simulation and Experimental Results for System's Velocity



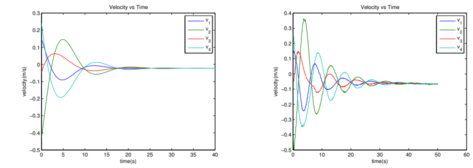
(a) Simulation of Position vs Time (b) Experiment of Velocity vs Time

Fig. 2. Comparison of the Simulation and Experimental Result for System's Position



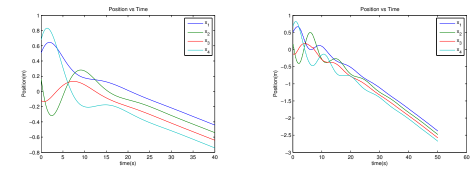
(a) Initial Condition of the Multi-Agent System's synchronization (b) Final Condition of the Multi-Agent System's synchronization

Fig. 3. Amigobots Experiment Set-up



(a) Formation simulation of Velocity vs Time (b) Formation experiment of Velocity vs Time

Fig. 4. Compare of the Simulation and Experiment Result for System's formation



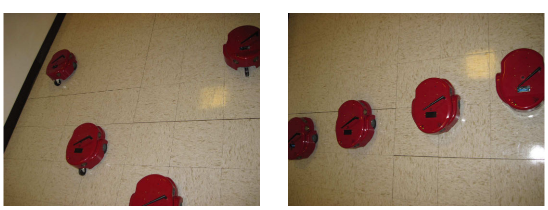
(a) Formation simulation of Position vs Time (b) Formation Experiment of Position vs Time

Fig. 5. Compare of the Simulation and Experiment Result for System's formation

agents achieve the desired velocity $v_d = 0.225m/s$ using our protocol (29).

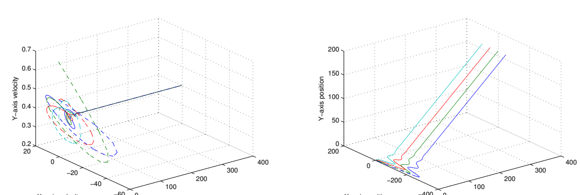
The next goal is for the system to reach the desired formation, which is achieved using proposed formation control protocol (34). The desired formation is one in which the distances between the position of each agent from agent 1 to agent 4 are the same. Fig. 4 displays the agent speeds approaching the average of the initial speed values, and Fig. 5 displays the desired formation. Furthermore, the simulation of the 3D formation is provided in Fig. 7.

In this paper, together with the simulation results, we adopt the Amigobot mobile robots as our experiment environment to verify our theoretical analysis. The same multi-agent graph topology is investigated in the experiment. Under synchronization protocol (29) and formation protocol (34), in Fig. 1, Fig. 2, Fig. 4 and Fig. 5, we compare the experimental results with the Matlab simulations. Fig. 3 and Fig. 6 are photographs of the experiment set-ups having implemented the synchronization and formation protocols, respectively.



(a) Initial Condition of the Multi-Agent System's formation (b) Final Condition of the Multi-Agent System's formation

Fig. 6. Amigobots Experiment Set-up



(a) Multi-Agent System's 3D formation control (b) Multi-Agent System's 3D formation control

Fig. 7. 3D formation control for the multi-agent system

V. Conclusion

This paper investigates the synchronization problem as it relates to acceleration motion as well as the formation control problem for multi-agent systems. Regarding our proposed synchronization protocol, the agents of the system move with the same equation of motion. Also, we provide an application of this protocol, the result being that we can obtain the desired consensus velocity instead of the average of the initial velocities of the agents. Regarding our proposed formation protocol, the desired formation can be achieved. Both control protocols can be extended from a single axis to 3D space, yet no discrepancies are encountered. The Amigobots experiments and the Matlab simulations also verify our theoretical analysis.

References

[1] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proc. IEEE*, vol. 95, 2007, pp 215-233.

[2] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Autom. Control*, vol. 49, 2004, pp. 1520-1533.

[3] L. Schenato and G. Gamba, "A distributed consensus protocol for clock synchronization in wireless sensor network," in *Proc. 46th IEEE Conf. Decision Control*, New Orleans, LA, 2007, pp. 2289-2294.

[4] A. L. Fradkov, B. Andrievsky, and R. J. Evans, "Synchronization of passifiable Lurie systems via limited-capacity communication channel," *IEEE Trans. Circuit Syst. Part I: Reg. Papers*, vol. 56, 2009, pp. 430-439.

[5] Z. Li, Z. Duan, G. Chen, and L. Huang, "Consensus of multi-agent systems and synchronization of comp lex networks: a unified viewpoint," *IEEE Trans. on Circuits Syst. Part I: Reg. Papers*, vol. 57, 2010, pp. 215-233.

[6] R. W. Beard, T. W. McLain, M. A. Goddrich, and E. P. Anderson, "Coordinated target assignment and intercept for unmanned air vehicles", *IEEE Trans. Robot. Autom.*, vol. 18, 2002, pp. 911-922.

[7] J. G. Bender, "An overview of systems studies of automated highway systems", *IEEE Trans. Vehic. Technol.*, vol. 40, 1991, pp. 82-99.

[8] J. R. Carpenter, "Decentralized control of satellite formations", *Int. J. Robust Nonlin. Control*, vol. 12, 2002, pp. 141-161.

[9] G. Xie and L. Wang, "Consensus control for a class of networks of dynamic agents," *Int. J. Robust Nonlin. Control*, vol. 17, 2006, pp. 941-959.

[10] D. Zheng, *Linear Systems Theory*, Beijing, China: Tsinghua Univ. Press, 2002.

[11] W. Ren and R. W. Beard, *Distributed Consensus in Multi-vehicle Cooperative Control Theory and Applications*, Springer-Verlag, London, U.K., 2008.

[12] A. K. Bondhus, K. Y. Pettersen, and J. T. Gravdahl, "Leader follower synchronization of satellite attitude without angular velocity measurements," in *Proc. 44th IEEE Conf. Decision Control*, Seville, Spain, 2005, pp. 7270-7277.

[13] W. Yu, G. Chen and J. Liu, "On pinning synchronization of undirected and directed complex dynamical networks," *Automatica*, vol 45, 2009, pp.429-435.

[14] W. Yu, G. Chen, and M. Cao, "Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems," *Automatica*, vol. 46, pp. 1089-1095, 2010.