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Stability Analysis of Bilateral Teleoperation Systems with Time-Varying Environments

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Abstract— **We present the stability analysis of bilateral teleoperation systems in the face of time varying stiff environments via Integral Quadratic Constraints (IQCs). Numerical cases are given for both arbitrarily fast and slowly varying parametric uncertainties.**

I. Introduction

In the last two decades, the stability problem of bilateral teleoperation systems in the face of a rich class of uncertainties has been considered via numerous methods. The immediate requirement of satisfactory performance levels turned out to be a very challenging problem since obtaining such levels while maintaining stability are, unlike a motion control system, uncompromisable. Hence, the common trade off between performance and stability in control systems is of the utmost importance in teleoperation systems. Therefore, many available control methodologies are being generalized and experimentally tested for a better understanding of the underlying limitations. The most popular methods are the physically motivated energy relation methods, i.e. passivity based approaches that are advocating the strategic energy dissipation schemes. These methods tend to model the bilateral teleoperation system as a 2-port network where human and the environment is modeled to be the source and the load terminations analogous to a circuit. To complete the analogy, the human and the environment have been assumed to be passive Linear Time Invariant (LTI) operators. Therefore, unconditional stability criteria, scattering transforms and wave variables are used for the stability analysis e.g. [1]–[3].

During the teleoperation, when the remote site is explored with a human user in the local site, it is expected that the user experience the changes in the dynamic properties of the environment. The user can touch different objects or skim along a surface to find a soft spot etc. Hence, an LTI assumption of the environment put severe restrictions on the class of systems of interest. On the other hand, one might assume that the environment is passive but arbitrary otherwise hence embed the class of relevant linear passive environments with time varying parameters into the vast passive nonlinear operator class. Thus, passivity based nonlinear stability tests can be invoked. In some cases, if there is a known bound on the parameters and their rate-of-variation, then this covering might lead to conservative assessments of stability.

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Therefore, it might be more beneficial to use the well-known linear frequency domain methods available. However, many classical analysis tools, such as unconditional (absolute) stability criteria, μ-tools from robust control theory etc. require the assumption of LTI operators in general. Seemingly, these shortcomings led the researchers concentrate on the nonlinear counterparts of the passivity theory.

Our motivation is to draw the attention of the teleoperation community to the recent advances in the robust control theory which resolve many of these shortcomings as witnessed in the last decade. In [4] (which is partially presented here together with [5], [6]), our aim is to show that the framework of Integral Quadratic Constraints (IQCs,[7]) utilized here offers a unification of stability analysis (and to some extent controller synthesis) for a rich class of uncertainties. Specifically, in [6], it is shown that the network theoretical frequency domain methodologies can be interpreted as particular cases of IQC theorem.

In this paper, stability analysis with respect to a time-varying parametric uncertainty which models the environment as a stiff spring is considered with frequency dependent multipliers. Two distinct cases namely, parameters with arbitrarily fast and arbitrarily slow variations are presented. A particular advantage of the IQC framework is that whenever additional modeling information is available, the analysis can be refined with ease in a structured manner.

The notation is standard. For a hermitian matrix M, $M \succ 0$ ($M \prec 0$) denotes positive (negative) definiteness. Positive (negative) semi-definiteness is also denoted by $M \succeq 0$ ($M \preceq 0$). The notation $(\star)^*$ denotes the complex conjugate of the right outer factor of a quadratic form. $\mathcal{RH}_\infty$ denotes the set of real rational, proper and stable functions. Consequently $\mathcal{RH}_\infty^{\bullet\times\bullet}$ symbol denotes the real rational proper and stable transfer matrices, where the size of the matrix does not play a role in the discussion. A hat $\hat{}$ denotes the Fourier transform i.e. $\hat{f}(i\omega)$. An upper linear fractional transformation (LFT) is denoted by $\Delta \star G = G_{22} + G_{21}\Delta(I - G_{11}\Delta)^{-1}G_{12}$ for some appropriately partitioned Δ, G. Similarly, a lower LFT is denoted by $G \star \Delta = G_{11} + G_{12}\Delta(I - G_{22}\Delta)^{-1}G_{21}$.

II. Preliminaries

In this section, we briefly recap the IQC theory which is introduced in [7]. Assume a system interconnection given by

$$v = Gw \quad , \quad w = \Delta(v) \tag{1}$$

where $G \in \mathcal{RH}_\infty^{\bullet\times\bullet}$ and Δ is a bounded causal operator.

Definition II.1. *Consider the interconnection depicted in Figure 1a with $G, \Delta \in \mathcal{RH}_\infty^{\bullet\times\bullet}$ of compatible dimensions.*

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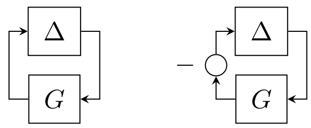
This $G - \Delta$ interconnection is said to be well-posed if $(I - G\Delta)(s)$ has a proper inverse. Moreover, a well-posed interconnection is said to be stable if the inverse of $(I - G\Delta)(s)$ is stable.



(a) (b)

Fig. 1. The general interconnection (left) and the assumed interconnection for passive systems (right)

An Integral Quadratic Constraint (IQC) for the input and output signals of Δ is expressed as

$$\int_{-\infty}^{\infty} \begin{pmatrix} \widehat{\Delta(v)}(i\omega) \\ \hat{v}(i\omega) \end{pmatrix}^* \Pi(i\omega) \begin{pmatrix} \widehat{\Delta(v)}(i\omega) \\ \hat{v}(i\omega) \end{pmatrix} d\omega \geq 0. \quad (2)$$

A bounded operator $\Delta : \mathcal{L}_2^m \to \mathcal{L}_2^n$ is said to satisfy the constraint defined by $\Pi(i\omega)$ if (2) holds for all $v \in \mathcal{L}_2^m$.

The following sufficient stability condition for the interconnection in Figure 1a is the main theorem which forms the basis for the IQC framework.

Theorem II.1 ([7])**.** *Let the system model $G \in \mathcal{RH}_\infty^{n\times m}$ be given and let $\Delta : \mathcal{L}_2^m \to \mathcal{L}_2^n$ be a bounded causal operator. Suppose that*

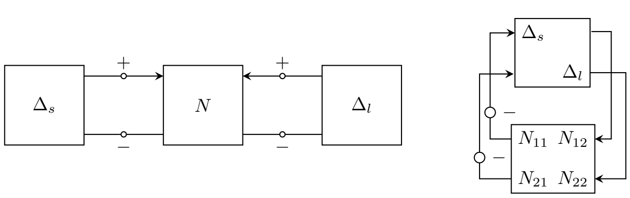
1) *for every $\tau \in [0, 1]$, the interconnection of G and $\tau\Delta$ is well-posed;*
2) *for every $\tau \in [0, 1]$, $\tau\Delta$ satisfies the IQC defined by $\Pi(i\omega)$ which is bounded as a function of $\omega \in \mathbb{R}$;*
3) *there exists some $\epsilon > 0$ such that*

$$\begin{pmatrix} I \\ G(i\omega) \end{pmatrix}^* \Pi(i\omega) \begin{pmatrix} I \\ G(i\omega) \end{pmatrix} \preceq -\epsilon I \text{ for all } \omega \in \mathbb{R}. \quad (3)$$

Then the $G - \Delta$ interconnection in Figure 1a is stable.

In the terminology of network theory, the teleoperation system is considered to be a 2-port network with the human and the environment being seen as the source and the load of the network respectively (Figure 2a). We, instead, turn to a more familiar interpretation depicted in Figure 2b.

Remark 1. The negative feedback is required to correct the sign changes when the flow variable exits one port and enters another with the opposite sign. Hence the differences between Figure 2a and Figure 2b.



(a) (b)

Fig. 2. Two representations of a 2-port network.

A well-known version of the classical passivity theorem, (see e.g. [8, Thm. VI.5.10]) would help us to determine the lowest tolerable level of passivity of the uncertainties for which a given interconnection remains stable. We can also formulate it as a corollary of the general IQC theorem as follows.

Corollary II.2. *The interconnection of $G, \Delta \in \mathcal{RH}_\infty^{\bullet\times\bullet}$ as in Figure 1b is stable if there exist a $p \in \mathbb{R}$ and a positive $\lambda(\omega) \in \mathbb{R}$ such that*

$$\begin{pmatrix} \Delta(i\omega) \\ I \end{pmatrix}^* \begin{pmatrix} -p\lambda(\omega) & \lambda(\omega) \\ \lambda(\omega) & 0 \end{pmatrix} \begin{pmatrix} \Delta(i\omega) \\ I \end{pmatrix} \succeq 0 \quad (4)$$

$$\begin{pmatrix} I \\ -G(i\omega) \end{pmatrix}^* \begin{pmatrix} -p\lambda(\omega) & \lambda(\omega) \\ \lambda(\omega) & 0 \end{pmatrix} \begin{pmatrix} I \\ -G(i\omega) \end{pmatrix} \prec 0 \quad (5)$$

hold for all $\omega \in \mathbb{R} \cup \{\infty\}$.

Remark 2. Note that (4) and (5) are nothing but

$$\Delta(i\omega) + \Delta^*(i\omega) \succeq p\Delta^*(i\omega)\Delta(i\omega), \quad (6)$$

$$G(i\omega) + G^*(i\omega) \succ -pI. \quad (7)$$

The case $p = 0$ recovers the classical passivity theorem.

The idea of replacing the passive operators with known models or at least family of models whose Nyquist curves are bounded by some constraints, has been recently investigated in [9], [10] for nominal LTI environment models in the Bounded Impedance Passivity (or Bounded Impedance Absolute Stability) method. In terms of interconnections, one can see that, these methods boil down to a simple linear fractional transformation on one of the uncertainty blocks. Note that, in scattering domain (4),(5) links the parametrization of the so-called impedance circles of [9] to the test presented.

Lemma II.3 (KYP Lemma)**.** *Let a transfer function $G(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ with no poles on the imaginary axis be given. Then, the following are equivalent:*

1) *For a given symmetric matrix P, the system satisfies*

$$G(i\omega)^* P G(i\omega) \succ 0 \quad \forall\omega \in \mathbb{R} \cup \{\infty\} \quad (8)$$

2) *There exists a symmetric matrix X such that*

$$\begin{pmatrix} I & 0 \\ A & B \\ \hline C & D \end{pmatrix}^T \left(\begin{array}{cc|c} 0 & X & 0 \\ X & 0 & 0 \\ \hline 0 & 0 & P \end{array}\right) \begin{pmatrix} I & 0 \\ A & B \\ \hline C & D \end{pmatrix} \succ 0 \quad (9)$$

holds.

III. Robust Stability Analysis for Time–Varying Environment Models

When modeled as a stiff spring, the stiffness coefficient of the environment plays a crucial role on the achievable performance. But, the changes in this coefficient are seldom taken into account due to the common LTI assumption of the corresponding model. Possibly, e.g. in a teleoperated surgery application, the magnitude and the time variation rates of the parameter might be limited if there is no interaction

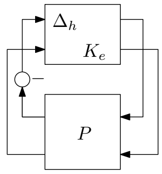


Fig. 3. The uncertain interconnection

with hard surfaces such as bones. Hence the robustness test against an arbitrarily fast varying parameter is expected to be conservative for such an application.

Initially, we let the rate of variation of parameter vary arbitrarily fast to compare this test with and later on, the resulting stability test is modified in the following section for including the uncertainty as a slowly-varying real parametric uncertainty ([11]–[13], cf. [14]) where the rate of variation information is explicitly taken into account by the use of the so-called "swapping lemma".

A. IQC Multiplier for Arbitrarily Fast Varying Parametric Uncertainties

We consider the case that the environment Δ_e is a proportional gain acting against on the position of the remote site, modeling a spring coefficient i.e. $F_e = -(K_e(t) \int \dot{x}_e)$ and we assume that the stiffness coefficient is a time varying real parametric uncertainty. We can assume that the integrator part $\frac{-1}{s}$ is included in the nominal system such that only $K_e(t)$ is treated as the uncertainty. Therefore, we obtain the robustness test for a system $P \in \mathcal{RH}_\infty^{\bullet\times\bullet}$, interconnected to the human and environment operators as depicted in Figure III.

We utilize constant DG-scalings (e.g. [15]–[17]) for the uncertainty $K_e(t) \in [0, \bar{K}]$ for all $t \in \mathbb{R}$, and a frequency dependent positive multiplier $\lambda(\omega)$ for the passive uncertainty via Corollary II.2. One can show that the structured uncertainty block $\hat{\Delta} := \left[\begin{smallmatrix} \Delta_h & 0 \\ 0 & K_e \end{smallmatrix}\right]$ satisfies the following constraint

$$\begin{pmatrix} \hat{\Delta} \\ I \end{pmatrix}^* \begin{pmatrix} -p\lambda(\omega)I & 0 & \lambda(\omega)I & 0 \\ 0 & D & 0 & \frac{\bar{K}}{2}D + iG \\ \lambda(\omega)I & 0 & 0 & 0 \\ 0 & \frac{\bar{K}}{2}D - iG & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{\Delta} \\ I \end{pmatrix} \succeq 0 \tag{10}$$

for all frequencies where $D > 0, G, p$ are scalars and positive $\lambda(\omega) \in \mathbb{R}$ for all ω.

Without loss of generality, we assume a factorization of the form ([7])

$$\lambda(\omega) = \Psi(i\omega)^* M \Psi(i\omega) > 0 \tag{11}$$

Define $\Phi := \mathrm{blkdiag}\{\Psi, 1\}$, then using

$$\mathcal{M} = \begin{pmatrix} -pM & 0 & M & 0 \\ 0 & -D & 0 & \frac{\bar{K}}{2}D + iG \\ M & 0 & 0 & 0 \\ 0 & \frac{\bar{K}}{2}D - iG & 0 & 0 \end{pmatrix} \tag{12}$$

we have a frequency dependent inequality (FDI) of the form given in (3),

$$\begin{pmatrix} \Phi \\ \Phi\tilde{P} \end{pmatrix}^* \mathcal{M} \begin{pmatrix} \Phi \\ \Phi\tilde{P} \end{pmatrix} \prec 0 \tag{13}$$

where $\tilde{P}$ is the system P with the first channel's sign is negated, M is a symmetric matrix and $D > 0, G \in \mathbb{R}$.

Moreover, we introduce the following minimal state-space representations

$$\Psi(s) = \left[\begin{array}{c|c} A_\Psi & B_\Psi \\ \hline C_\Psi & D_\Psi \end{array}\right] \quad \text{and} \quad \begin{pmatrix} \Phi \\ \Phi\tilde{P} \end{pmatrix}(s) = \left[\begin{array}{c|c} \mathcal{A} & \mathcal{B} \\ \hline \mathcal{C} & \mathcal{D} \end{array}\right]. \tag{14}$$

Then by the use of the KYP Lemma, we obtain the LMI counterparts of the FDI (13) and (11):

$$\begin{pmatrix} I & 0 \\ \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{pmatrix}^T \begin{pmatrix} 0 & \mathcal{X} & 0 \\ \mathcal{X} & 0 & 0 \\ 0 & 0 & \mathcal{M} \end{pmatrix} \begin{pmatrix} I & 0 \\ \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{pmatrix} \prec 0 \tag{15}$$

$$\begin{pmatrix} I & 0 \\ A_\Psi & B_\Psi \\ C_\Psi & D_\Psi \end{pmatrix}^T \begin{pmatrix} 0 & Z & 0 \\ Z & 0 & 0 \\ 0 & 0 & M \end{pmatrix} \begin{pmatrix} I & 0 \\ A_\Psi & B_\Psi \\ C_\Psi & D_\Psi \end{pmatrix} \succ 0 \tag{16}$$

Thus, we conclude the stability test as follows:

Theorem III.1. *The teleoperation system transfer matrix $P \in \mathcal{RH}_\infty^{2\times 2}$ interconnected to the uncertainty set $\left[\begin{smallmatrix} \Delta_h & 0 \\ 0 & K_e \end{smallmatrix}\right]$ as described above, is robustly stable if there exist matrices $M, \mathcal{X}, Z$ and scalars $D > 0, G, p$ such that* (15) *and* (16) *holds.*

B. IQC Multiplier for Slowly Varying Parametric Uncertainties

To keep the presentation simpler, we will only treat a system interconnected to time-varying uncertainty, but as in the previous section combining different multipliers for the structured uncertainty sets is straightforward. We will use the results of [11], [13], to include the rate of variation of the parameter K_e into the robustness test to reduce conservatism further. For the convenience of the reader, we include the scalar case of a well known result in adaptive control.

Lemma III.2 (Swapping Lemma)**.** *Assume that $\delta(t)$ is an absolutely continuous, bounded, differentiable function and let the derivative is bounded as $|\dot{\delta}| \leq d$. Assume further that $u \in \mathcal{L}_2$, and a convolution operator $T : \mathcal{L}_2 \to \mathcal{L}_2$ which admits a transfer function such that $T(s) := \big(C(sI - A)^{-1}B + D\big) \in \mathcal{RH}_\infty$ and similarly for the operators*

$$T_c(s) := C(sI - A)^{-1} \quad , \quad T_b(s) := (sI - A)^{-1}B \tag{17}$$

If such a T is chosen then, the commutative property of operator T with a time varying $\delta(t)$ is given by,

$$\delta T u = T\delta u + T_c \dot{\delta} T_b u \tag{18}$$

and thus,

$$\underbrace{\begin{pmatrix} T & T_c \\ 0 & I \end{pmatrix}}_{T_{left}} \underbrace{\begin{pmatrix} \delta \\ \dot{\delta} T_b \end{pmatrix}}_{\Delta_s} = \underbrace{\begin{pmatrix} \delta & 0 \\ 0 & \dot{\delta} I \end{pmatrix}}_{\Delta_x} \underbrace{\begin{pmatrix} T \\ T_b \end{pmatrix}}_{T_{right}} \tag{19}$$

Consider an extended uncertainty block consists of the time-varying parameter $K_e \in [0, \bar{K}]$ and its derivative $|\dot{K}_e| \leq d$ i.e. $\Delta_x = \text{blkdiag}\{K_e, \dot{K}_e\}$ which satisfies the following quadratic constraint with a static multiplier

$$\mathcal{M}_s = \begin{pmatrix} -D_2 & 0 & \frac{\bar{K}}{2}D_2 + iG_2 & 0 \\ 0 & -D_3 & 0 & iG_3 \\ \frac{\bar{K}}{2}D_2 - iG_2 & 0 & 0 & 0 \\ 0 & -iG_3 & 0 & d^2 D_3 \end{pmatrix}$$

i.e.

$$\begin{pmatrix} \Delta_x \\ I \end{pmatrix}^T \mathcal{M}_s \begin{pmatrix} \Delta_x \\ I \end{pmatrix} \succeq 0$$

where $D_i > 0$ and $G_i \in \mathbb{R}$ for $i = 2, 3$. Then, from the positivity of the quadratic constraint, the following also holds true,

$$T_{right}^T \begin{pmatrix} \Delta_x \\ I \end{pmatrix}^T \mathcal{M}_s \begin{pmatrix} \Delta_x \\ I \end{pmatrix} T_{right} \succeq 0$$

Using (19), we have

$$\begin{pmatrix} T_{left}\Delta_s \\ T_{right} \end{pmatrix}^T \mathcal{M}_s \begin{pmatrix} T_{left}\Delta_s \\ T_{right} \end{pmatrix} \succeq 0$$

therefore, we can isolate a frequency dependent multiplier for the parametric uncertainty by

$$\begin{pmatrix} \Delta_s \\ I \end{pmatrix}^T \left[\begin{pmatrix} T_{left} & 0 \\ 0 & T_{right} \end{pmatrix}^T \mathcal{M}_s \begin{pmatrix} T_{left} & 0 \\ 0 & T_{right} \end{pmatrix} \right] \begin{pmatrix} \Delta_s \\ I \end{pmatrix} \succeq 0$$

Since the artificially extended $\dot{K}_e$ uncertainty block plays no role on the plant, we also extend the system with a zero block i.e. $\hat{P} = \begin{bmatrix} P & 0 \end{bmatrix}$ to match the input output numbers. Note that, the loop equations are not affected with this change i.e. let $v \in \mathcal{L}_2$, then

$$(I - P\delta)(v) = (I - \hat{P}\Delta_s)(v)$$

The remaining step is to find the related multiplier factorization for the frequency dependent parts. We initially assumed that $D_2 > 0$, but this constraint can be avoided i.e. we can instead look for positive frequency dependent function of the form $\Psi \hat{D}_2 \Psi \succ 0$ where $\hat{D}_2$ is a symmetric matrix. In a similar fashion G_i multipliers can be replaced with symmetric matrices. Constructing basis transfer matrices Ψ, Ψ_b, Ψ_c is straightforward which follows from the structure of T, T_b, T_c stated in the swapping lemma e.g.

$$\begin{pmatrix} \Psi & \Psi_c \\ \Psi_b & \bullet \end{pmatrix} := \left[\begin{array}{c|cc} A_\Psi & B_\Psi & I \\ \hline C_\Psi & D_\Psi & 0 \\ I & 0 & 0 \end{array} \right]$$

Define

$$\Phi_A = \begin{pmatrix} \Psi & \Psi_c \\ 0 & I \end{pmatrix}, \Phi_B = \begin{pmatrix} \Psi \\ \Psi_b \end{pmatrix} \tag{20}$$

then, the overall frequency dependent multiplier is obtained as:

$$\begin{pmatrix} \Phi_A & \\ & \Phi_B \end{pmatrix}^* \mathcal{M}_d \begin{pmatrix} \Phi_A & \\ & \Phi_B \end{pmatrix} \tag{21}$$

where

$$\mathcal{M}_d = \begin{pmatrix} -\hat{D}_2 & 0 & \frac{\bar{K}}{2}\hat{D}_2 + i\hat{G}_2 & 0 \\ 0 & -\hat{D}_3 & 0 & i\hat{G}_3 \\ \frac{\bar{K}}{2}\hat{D}_2 - i\hat{G}_2 & 0 & 0 & 0 \\ 0 & -i\hat{G}_3 & 0 & d^2\hat{D}_3 \end{pmatrix} \tag{22}$$

Along the lines of the previous case, by the use of KYP Lemma, frequenct dependent constraints are converted to the following LMIs

$$(\star)^T \begin{pmatrix} 0 & \mathcal{X} & 0 \\ \mathcal{X} & 0 & 0 \\ 0 & 0 & \mathcal{M}_d \end{pmatrix} \begin{pmatrix} I & 0 \\ \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{pmatrix}^T \prec 0 \tag{23}$$

$$(\star)^T \begin{pmatrix} 0 & Z & 0 \\ Z & 0 & 0 \\ 0 & 0 & \hat{D}_2 \end{pmatrix} \begin{pmatrix} I & 0 \\ A_\Psi & B_\Psi \\ C_\Psi & D_\Psi \end{pmatrix} \succ 0 \tag{24}$$

$$\hat{D}_3 \succ 0 \tag{25}$$

We can summarize the robustness test as the following

Corollary III.3. *Consider a system interconnected to a time varying uncertainty $K_e(t) \in [0, \bar{K}]$ satisfying the properties stated in Lemma III.2. The interconnection is robustly stable if there exist symmetric matrices $\hat{D}_2, \hat{D}_3, \mathcal{X}, Z$ such that* (23),(24) *and* (25) *holds.*

Hence, one can include the multiplier of passive LTI model of the human into this test as we did in Section III-A.

IV. Numerical Results

In our case study, we use the system taken from [10] with the numerical data given in the appendix. We have a passive uncertainty modeling the human and the time varying K_e modeling the time varying spring in the environment. We select different values of p hence confining the uncertainty set into regions with different size, then testing different $\bar{K}$ and rate of variation bound d via solving the LMIs. The resulting LMIs are solved with the parser YALMIP [18], solvers SeDuMi [19]. The system model is converted to an admittance representation to render the system proper and stable, but we include an integrator into the system model and perturb the integrator transfer function ($\rho = 10^{-5}$ is used) to render the overall nominal system stable. Let

$$Y_m = \frac{1}{M_m s + B_m}, \; Y_s = \frac{1}{M_s s^2 + (B_s + K_v)s + K_p}$$

then the teleoperation system is given by

$$Y_{nom} = \begin{pmatrix} Y_m & -K_f Y_m \\ -Y_m Y_s \mu K_p & \frac{M_m s^2 + B_m s + \mu K_f K_p}{(M_s s^2 + (B_s + K_v)s + K_p)(M_m s + B_m)} \end{pmatrix} \tag{26}$$

then,

$$Y = Y_{nom} \times \begin{pmatrix} 1 & 0 \\ 0 & -\frac{1}{s+\rho} \end{pmatrix}$$

The numerical values are taken from [10] as $M_m = 0.64, M_s = 0.61, B_m = 0.64, B_s = 11, K_v = 80, K_p =$

4000. Note that the product $K_f\mu$ is related to the transparency of the teleoperation system in [10] and for LTI passive human and environment models, the maximum admissible $K_f\mu = 0.127$ with $p \to 0$ computed from the unconditional stability theorem for 2-ports.

A. Parameter with Arbitrarily Fast Variation

We simply use Corollary III.1 by setting $P = Y$. The result is shown in Figure 4 for different values of $\bar{K}$. We see that towards higher values of $\bar{K}$ the performance levels decrease rapidly. Note that, though the values of $\bar{K}$ are quite realistic for physical systems, this test reports very low performance levels.

B. Parameter with Bounded Rate of Variation

As we have derived in the last section, we use the outer factors

$$\Phi_1 = \begin{pmatrix} \Psi_1 & & \\ & \Psi_2 & \Psi_{2c} \\ & 0 & I \end{pmatrix}, \Phi_2 = \begin{pmatrix} \Psi_1 & \\ & \Psi_2 \\ & \Psi_{2b} \end{pmatrix} \tag{27}$$

Then, adding zero columns to Y from the constant scaling case and negating the first channel, $\hat{Y} = \begin{bmatrix} \tilde{Y} & 0 \end{bmatrix}$, the FDI reads as

$$(\star)^* \mathcal{M}_2 \begin{pmatrix} \Phi_1 \\ \Phi_2 \hat{Y} \end{pmatrix} \preceq -\epsilon I \tag{28}$$

where

$$\mathcal{M}_2 = \begin{pmatrix} -pM & 0 & 0 & M & 0 & 0 \\ 0 & -D_2 & 0 & 0 & \frac{\bar{K}}{2}D_2 + iG_2 & 0 \\ 0 & 0 & -D_3 & 0 & 0 & iG_3 \\ M & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\bar{K}}{2}D_2 - iG_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -iG_3 & 0 & 0 & d^2 D_3 \end{pmatrix} \tag{29}$$

and M, D_2, D_3, G_2, G_3 are symmetric matrices. We introduce

$$\Psi_i(s) = \left[\begin{array}{c|c} A_{\Psi_i} & B_{\Psi_i} \\ \hline C_{\Psi_i} & D_{\Psi_i} \end{array}\right] \quad \text{and} \quad \begin{pmatrix} \Phi_1 \\ \Phi_2 \hat{Y} \end{pmatrix}(s) = \left[\begin{array}{c|c} \mathcal{A} & \mathcal{B} \\ \hline \mathcal{C} & \mathcal{D} \end{array}\right]. \tag{30}$$

then again, via KYP Lemma, we have

$$(\star)^T \begin{pmatrix} 0 & \mathcal{X} & 0 \\ \mathcal{X} & 0 & 0 \\ 0 & 0 & \mathcal{M}_d \end{pmatrix} \begin{pmatrix} I & 0 \\ \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{pmatrix}^T \prec 0$$

for the positivity of the frequency dependent multipliers $\lambda > 0$ and $\Psi_2^* D_2 \Psi_2 > 0$,we require

$$(\star)^T \begin{pmatrix} 0 & Z_1 & 0 \\ Z_1 & 0 & 0 \\ 0 & 0 & M \end{pmatrix} \begin{pmatrix} I & 0 \\ A_{\Psi_1} & B_{\Psi_1} \\ C_{\Psi_1} & D_{\Psi_1} \end{pmatrix} \succ 0$$

$$(\star)^T \begin{pmatrix} 0 & Z_2 & 0 \\ Z_2 & 0 & 0 \\ 0 & 0 & D_2 \end{pmatrix} \begin{pmatrix} I & 0 \\ A_{\Psi_2} & B_{\Psi_2} \\ C_{\Psi_2} & D_{\Psi_2} \end{pmatrix} \succ 0$$

and

$$D_3 \succ 0$$

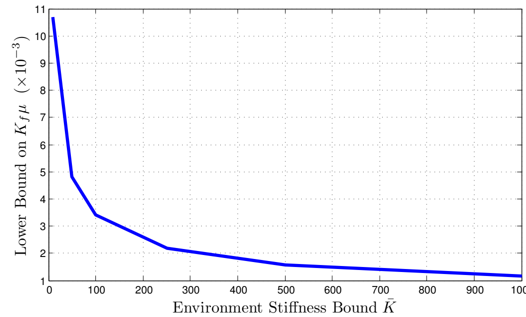


Fig. 4. Lower bound on the maximum achievable performance level with respect to the assumed maximum stiffness coefficient

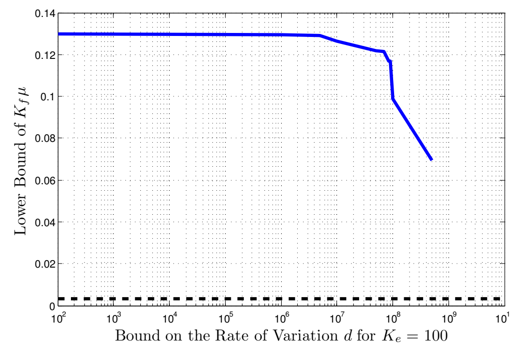


Fig. 5. The transparency vs. increasing rate of variation bound d. The dashed line represents the value obtained by using constant DG-scalings (See Figure 4)

By solving the LMIs we have obtained Figure 5. In this case, we have varied d for fixed $\bar{K} = 100$ and $p = 0.01$ which are moderate values if compared to the previous case. As seen from this result, the dynamic multipliers made it possible to reduce conservatism dramatically. After the value $d \geq 10^7$ the performance levels fall to the value reported by the constant scaling case. As we see from this plot, the conservative results of the test with constant scalings take into account the points that are hardly relevant in a physical experiment for many applications that we have motivated in the introduction. Therefore, it might be beneficial if we can also find such similar bounds for the human operator. Our claim is that the robustness tests with passivity assumption on the respective models suffer from tha same issue. Even the most trivial constraints when used together with passivity assumption might lead to a substantial improvement as we demonstrate in this example.

V. Conclusion

In this paper, we have demonstrated that time varying environments with known rate of variation bounds can easily be incorporated in the analysis of bilateral teleoperation systems. Moreover, we have tried to show the flexibility and power of the IQC framework that is suitable the teleoperation problems. The dynamic multiplier construction is a perfect example to show that one can come up with multipliers to characterize operators that are difficult to deal with by the classical tools. Same line of reasoning is also being pursued for the possible more complicated human/environment models. Our claim is that most of the conservatism is originating from the passivity assumptions of the corresponding operators. It is quite straightforward to modify these results whenever one has a particular multiplier structure for the corresponding uncertainty. Thus, we would like to draw more attention to modeling of human/environment operator problem rather than the oversimplified passivity assumption. Our current focus is on investigating suitable multipliers for various problems of teleoperation and then controller synthesis with dynamic multipliers.

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