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# Thermodynamics-Based Optimization and Control of Vapor-Compression Cycle Operation: Optimization Criteria

Neera Jain, *Student Member, IEEE*, Andrew G. Alleyne, *Senior Member, IEEE*

*Abstract*— **This paper investigates multiple degree of freedom (MDOF) optimization of steady-state vapor-compression cycle (VCC) operation. Five degrees of freedom (DOFs) of the VCC are optimized using an objective function which minimizes the rate of exergy destruction in the cycle. The use of exergy is motivated by its ability to capture the physics of both the first and second laws of thermodynamics in a single property. A case study is considered in which the optimization is applied to a commercial truck transport refrigeration system (TTRS). The results suggest that by using the optimal set points generated by the exergy-based objective function, an increase of 52.5% in COP can be achieved over nominal operation. In particular, the optimization results highlight the regulation of evaporator and condenser pressure as critical parameters in improving the efficiency of steady-state cycle operation.**

## NOMENCLATURE

| | | | |
|---|---|---|---|
| $\alpha$ | heat transfer coefficient | $\omega$ | rotational speed |
| $a$ | aperture | $X$ | exergy |
| $A$ | area | **Subscript** | |
| $C$ | cooling capacity | $a$ | air |
| $\delta$ | displacement | $adb$ | adiabatic |
| $E$ | energy | $c$ | condenser |
| $F$ | fraction of coil surface covered by fins | $dest$ | destroyed |
| | | $e$ | evaporator |
| $h$ | specific enthalpy | $gen$ | generated |
| $k$ | coeff. of conductivity | $H$ | high-temperature reservoir |
| $K$ | flow coefficient | $i$ | interior |
| $\dot{m}$ | mass flow rate | $k$ | compressor |
| $\eta$ | efficiency | $L$ | low-temperature reservoir |
| $P$ | pressure | $o$ | outer |
| $\dot{Q}$ | heat transfer rate | $r$ | refrigerant |
| $\rho$ | density | $sat$ | saturated |
| $s$ | specific entropy | $sys$ | system |
| $T$ | temperature | $v$ | electronic expansion valve |
| $\dot{W}$ | work transfer rate | $vol$ | volumetric |

## I. INTRODUCTION

Vapor-compression cycle (VCC) systems are used to remove heat from a low-temperature ($T_L$) environment and reject it to a high-temperature ($T_H$) reservoir (typically ambient air). They are generally designed to operate optimally at a particular maximum cooling load condition, which is termed the 'design point' of the system. However, in practice, VCC systems are often operated far away from their design point due to either *1)* variations in cooling load, or *2)* over-sizing of a system for its application. As a result, on-off control strategies applied to a fixed-speed compressor are typically utilized to operate VCC systems at partial load conditions which, in turn, compromise operational efficiency.

Electronic actuators have long been advocated as integral to improving the efficiency of VCC systems [1]–[3]. These actuators provide the control engineer with access to the various degrees of freedom (DOFs) of the VCC and consequently, the ability to operate at partial load conditions without cycling the compressor. As a result there is significant potential for improving the off-design point operational efficiency of any given VCC system through optimization and control of these available DOFs.

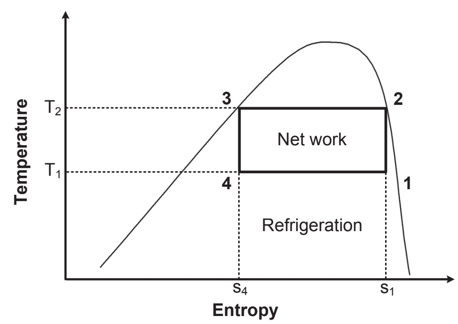


Fig. 1. *Temperature-entropy diagram for the Carnot refrigeration cycle.*

A standard refrigeration cycle consists of four processes of the refrigerant: compression (1 to 2), condensation (2 to 3), expansion (3 to 4), and evaporation (4 to 1), where $1, 2, 3$ and $4$ refer to the transition points of the cycle defined in Fig. 1. The most efficient refrigeration cycle is the Carnot refrigeration cycle (CRC), which assumes isentropic compression and expansion, and isothermal condensation and evaporation (Fig. 1). Although this system is not practically realizable, it provides intuition with regard to how 'close to ideal' a real system is operating. The coefficient of performance (COP) for the CRC (1) is solely a function of $T_1$ and $T_2$ [4] and is bounded above by virtue of (2) and (3) which maintain the necessary temperature gradients during condensation and evaporation, respectively. A higher COP corresponds to more efficient operation.

$$\mathrm{COP}_{Carnot} = \frac{\text{useful refrigeration}}{\text{net work}} = \frac{T_1}{T_2 - T_1} < \frac{T_L}{T_H - T_L} \tag{1}$$

$$T_2 > T_H \tag{2}$$

$$T_1 < T_L \tag{3}$$

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Andrew G. Alleyne is with the Department of Mechanical Science and Engineering at the University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA (phone: 217-244-9993; fax: 217-244-6534; email: alleyne@illinois.edu).

Neera Jain is with the Department of Mechanical Science and Engineering at the University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA (email: njain2@illinois.edu).

The standard vapor-compression cycle (VCC) is derived from the CRC. The VCC assumes *isenthalpic*, rather than isentropic, expansion, as well as the following:

- isobaric condensation and evaporation,
- isentropic compression,
- evaporation of refrigerant to a saturated or superheated vapor state, and
- condensation of refrigerant to a saturated or subcooled liquid state.

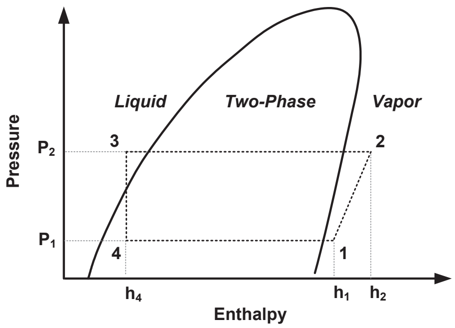


Fig. 2. *Pressure-enthalpy diagram for a standard vapor-compression cycle with four thermodynamic degrees of freedom.*

The COP for the standard VCC (operating at steady-state) is given in (4) [4]. Since enthalpy is an important measurement in the VCC, it is useful to characterize the cycle on a pressure-enthalpy ($P$-$h$) diagram, as shown in Fig. 2.

$$\text{COP} = \frac{\text{evaporator capacity}}{\text{supplied power}} = \frac{\dot{Q}_L}{\dot{W}} = \frac{\dot{m}_r(h_1 - h_4)}{\dot{m}_r(h_2 - h_1)} \qquad (4)$$

Based on the constitutive relationships between pressure, temperature, entropy, etc., the cycle shown in Fig. 2 has four thermodynamic DOFs. The three enthalpies of the cycle, $\{h_1,\, h_2,\, h_3 = h_4\}$, and any one of the following, $\{P_1,\, P_2,\, T_1\}$, uniquely define the remaining thermodynamic states at each of the transition points (denoted $1, 2, 3, 4$ in Fig. 2) given the assumptions listed above. However, to actually compute critical quantities of interest, such as the amount of cooling that is achieved ($\dot{Q}_L$), or the amount of power consumed ($\dot{W}$), there is an additional degree of freedom (DOF) which must be considered: the refrigerant mass flow rate, $\dot{m}_r$ (4). This DOF is a fluid dynamic variable, rather than a thermodynamic one, and cannot be captured in a $P$-$h$ diagram of the cycle.

The distinction between thermodynamic DOFs and fluid dynamic DOFs is an important one. Others have examined the available DOFs in the VCC, generally from a component-based perspective. In [5], five controllable DOFs were identified as "compression work", "valve opening", "effective heat transfer for the two heat exchangers", and the amount of "active charge" in the cycle. The latter, "active charge", is analogous to refrigerant mass flow rate. [5] suggests that the amount of "active charge" should be controlled but asserts that hardware changes would have to be made to a physical system in order to do so. However, this DOF has been largely ignored by others when it comes to optimizing and controlling VCC systems.

Along with a DOF analysis, one must consider possible constraints. In [5], three of the five identified DOFs are constrained, leaving a 2 DOF problem in which the amount of condenser subcooling and evaporator superheat are optimized. Similarly, [6] and [7] investigate set point optimization in refrigeration systems but do not consider any fluid dynamic DOFs. They fix all but one of the remaining thermodynamic DOFs, thereby solving a 1 DOF energy minimization problem in which a variable-speed condenser fan is used to control the condenser pressure. Here we seek to optimize all available DOFs in the cycle without constraining them *a priori*.

Any optimization is only as good as the design of the objective function being solved. Many [7]–[9] have developed objective functions which are empirically-derived and specific to a particular system design. Although different VCC systems use different models and sizes of compressors, valves, and heat exchangers, the processes executed by the components are the same. Here we will develop an objective function that is physics-based, and therefore system and scale independent. More specifically, the emphasis will be on minimizing exergy-destruction as opposed to minimizing power consumption.

Exergy is a property which quantifies the work potential of a given amount of energy at a specified state [10].

$$X = (E - U_0) + P_0(V - V_0) - T_0(S - S_0) \qquad (5)$$

Whereas energy is always conserved, exergy is not. In the presence of irreversibilities (due to friction, etc.), exergy will be destroyed in an amount proportional to the amount of entropy that is generated. Several studies have been conducted on the steady-state exergy analysis of VCC systems [11]–[14]. These references have largely applied exergy analysis to understand, from a design perspective, which components or operating parameters result in the largest destruction of exergy in the cycle. Most recently, [15] performed a multi-objective optimization in which both total exergy destruction and total product cost of a cooling tower assisted refrigeration system were minimized; again, the emphasis was on optimization of the system design parameters.

While component-based exergy analysis has been used to optimize system parameters in different refrigeration system configurations, to the knowledge of the authors, this is the first application of exergy-based optimization to multiple degree of freedom (MDOF) operation of the VCC. The remainder of the paper is organized as follows. Section II describes the development of an exergy-based objective function for the five DOF optimization problem. Section III presents a case study in which the optimization is applied to a truck transport refrigeration system (TTRS) followed by a discussion of the results. Finally, conclusions of this work are summarized in Section IV.

## II. OBJECTIVE FUNCTION DEVELOPMENT

*1) Performance Term:* It is intuitive to characterize the performance of the VCC in the form of some desired cooling capacity produced by the cycle. We define the performance term to be the magnitude of the difference between the desired cooling capacity and the cooling capacity achieved by a given choice of operation set points (6).

$$J_1 = |C_{desired} - C_{achieved}| \tag{6}$$

$$C_{achieved} = \dot{m}_r(h_1 - h_4) \tag{7}$$

*2) Efficiency Term:* We seek to maximize the efficiency of VCC system operation by minimizing the rate of exergy destruction during operation. Exergy can be transferred in three ways: by heat transfer, work, or mass transfer [10]. Since the VCC is closed and cyclic, we consider only exergy transferred by heat transfer and work and apply the exergy balance in rate form as shown in (8) where $T_0$ is an infinite reservoir temperature and $T_j$ is the boundary temperature at which the heat transfer $\dot{Q}_j$ occurs.

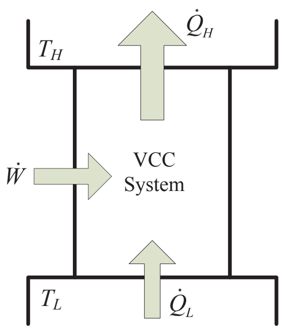


Fig. 3. *Closed system view of the vapor-compression cycle.*

$$\sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j - \left(\dot{W} - P_0\frac{dV_{sys}}{dt}\right) - T_0\dot{S}_{gen} = \frac{dX_{sys}}{dt} \tag{8}$$

During steady-state operation, (8) reduces to

$$\dot{X}_{dest} = -(-\dot{W}) + \left(1 - \frac{T_H}{T_L}\right)\dot{Q}_L \tag{9}$$

where $T_0$ has been replaced by $T_H$ and $T_j$ has been replaced by $T_L$. Equation (9) is evaluated with $T_H$ and $T_L$ in degrees Kelvin. For the VCC, $\dot{Q}_L$ and $\dot{W}$ are given by (10) and (11).

$$\dot{Q}_L = \dot{m}_r(h_1 - h_4) \tag{10}$$

$$\dot{W} = \dot{Q}_H - \dot{Q}_L = \dot{m}_r(h_2 - h_3) - \dot{m}_r(h_1 - h_4) \tag{11}$$

Substituting (10) and (11) into (9) yields

$$\dot{X}_{dest} = \dot{m}_r(h_1 - h_4)\left(1 - \frac{T_H}{T_L}\right) + \dot{m}_r(h_2 - h_1) \tag{12}$$

which is solely a function of $h_1$, $h_2$, $h_4$, and $\dot{m}_r$ (recall that isenthalpic expansion implies $h_3 = h_4$). The fifth DOF, $T_1$, does not appear in (12) but is explicitly characterized in the thermodynamic constraints outlined in Sec. III-A.1.

By virtue of the physical nature of these variables, the following inequalities hold:

$$-1 < \left(1 - \frac{T_H}{T_L}\right) \leq 0 \tag{13}$$

$$\dot{m}_r(h_2 - h_1) > 0 \tag{14}$$

$$\dot{m}_r(h_1 - h_4) > 0. \tag{15}$$

If the cycle operated without any losses due to irreversibility, or equivalently, $\dot{X}_{dest,total} = 0$, then

$$\left|\dot{m}_r(h_1 - h_4)\left(1 - \frac{T_H}{T_L}\right)\right| = \dot{m}_r(h_2 - h_1). \tag{16}$$

However, in reality,

$$\left|\dot{m}_r(h_1 - h_4)\left(1 - \frac{T_H}{T_L}\right)\right| < \dot{m}_r(h_2 - h_1). \tag{17}$$

Therefore, minimizing the rate of exergy destruction is analogous to pushing the system to operate as close as possible to its theoretical maximum efficiency of $\dot{X}_{dest,total} = 0$ which includes first and second law efficiencies.

*3) Complete Objective Function:* We combine $J_1$ and $J_2$ into a single objective function and introduce a weighting factor, $\gamma$, which allows us to weigh the tradeoff between efficiency and performance. The objective function (18) is constructed so that its theoretical minimum value is zero.

$$J = (\gamma)J_1 + (1 - \gamma)J_2 \tag{18}$$

$$J_1 = |C_{desired} - \dot{m}_r(h_1 - h_4)| \tag{19}$$

$$J_2 = \dot{X}_{dest} = \dot{m}_r(h_1 - h_4)\left(1 - \frac{T_H}{T_L}\right) + \dot{m}_r(h_2 - h_1) \tag{20}$$

## III. OPTIMIZATION CASE STUDIES

Based on the objective function developed in Section II, the variables which will be optimized are $h_1$, $h_2$, $h_4$, $T_1$, and $\dot{m}_r$. The objective function is nonlinear; therefore, the function fmincon in the MATLAB Optimization Toolbox will be used to find a solution to the optimization problem [16].

The objective function is entirely based on first principles and therefore independent of the size of the particular system for which the optimization will be conducted. However, in order to obtain meaningful results from the optimization, it is necessary to specify what refrigerant the system is operating with. The following two case studies optimize the cycle for operation with refrigerant R404a.

The inputs to the optimization are the desired cooling capacity ($C_{desired}$), the weighting factor ($\gamma$), the ambient temperature ($T_H$), and the temperature of the cooled environment ($T_L$). An initial guess for each of the optimization variables is also required.

### *A. Case 1: Exergy-Based Optimization with Thermodynamic Constraints*

*1) Constraints:* In this first case study, we consider only thermodynamic constraints on the cycle:

1) $h_1 < h_2$
2) $h_4 < h_1$
3) $T_1 \leq T_L$
4) $T_3 \geq T_H$
5) $T_1 - T_4 \geq 0$
6) $T_3 - T_{3,sat} \geq 0.$

Note that $T_{3,sat}$ is the saturated refrigerant temperature at $P_2$. The first two constraints ensure that compression and evaporation of the refrigerant, respectively, occur. The third and fourth constraints ensure the correct temperature gradients at the outlet of the evaporator and condenser, respectively. The fifth constraint ensures that only refrigerant vapor (and no liquid) is compressed. Finally, the sixth constraint ensures that only refrigerant liquid (and no vapor) is expanded.

The inputs to the optimization are given in Table I. The weighting factor, $\gamma$, was chosen heuristically to sufficiently weight the performance term, $J_1$, to achieve the desired cooling capacity. The optimal solution is given in Table II. An additional metric, the second law (exergetic) efficiency,

TABLE I
OPTIMIZATION INPUTS

| *Input* | *Units* | *Case 1* | *Case 2* |
|---|---|---|---|
| $C_{desired}$ | kW | 10 | 15 |
| $\gamma$ | - | 0.4 | 0.3 |
| $T_H$ | $^\circ C$ | 21.1 | 21.1 |
| $T_L$ | $^\circ C$ | 1.6 | 1.6 |

TABLE II
OPTIMIZATION RESULTS

| | *Units* | *Case 1* | *Case 2* |
|---|---|---|---|
| $C_{desired}$ | kW | 10 | 15 |
| $C_{achieved}$ | kW | 9.99 | 14.99 |
| Optimal Cost, $J$ | kW | 0.252 | 0.369 |
| $h_1$ | kJ/kg | 208 | 219 |
| $h_2$ | kJ/kg | 221 | 232 |
| $h_4$ | kJ/kg | 98.6 | 95.1 |
| $\dot{m}_r$ | kg/s | 0.0911 | 0.121 |

$\eta_{II}$ [17], is introduced to provide a more complete measure of the efficiency of the optimized solution. Intuitively, $\eta_{II}$ tells us how the COP of the system compares to the Carnot (reversible) COP for a cycle operating between two temperature reservoirs, $T_H$ and $T_L$. Table III gives the second law efficiency for each optimization case as well as additional parameters of interest.

$$\eta_{II} = 1 - \frac{\text{exergy destroyed}}{\text{exergy supplied}} = \frac{\text{COP}}{\text{COP}_{Carnot}} \tag{21}$$

TABLE III
ADDITIONAL PARAMETERS OF INTEREST

| | *Units* | *Case 1* | *Case 2* |
|---|---|---|---|
| COP | - | 8.86 | 9.42 |
| $\eta_{II}$ | - | 0.751 | 0.783 |
| $T_1$ | $^\circ C$ | -0.802 | 0.0336 |
| $T_3$ | $^\circ C$ | 22.2 | 21.33 |

The results of the optimization suggest that the optimal operation of the VCC is strongly dependent on $T_H$ and $T_L$. Specifically, it appears to be most efficient to have the outlet temperature of the condenser, $T_3$, approach $T_H$, and similarly, to have the outlet temperature of the evaporator, $T_1$, approach $T_L$. This implies that the system is achieving the maximum possible heat transfer (across each heat exchanger) since the refrigerant nearly reaches thermal equilibrium with the air at the outlet of each heat exchanger.

These results are consistent with the intuition provided by the CRC and (1). Since this solution was found using a numerical search method, it gives us confidence that $J$ is well-posed and that the numerical search method is capable of finding a reasonable solution.

### *B. Case 2: Comparison of Optimal and Commercial System Set Points*

In this case study, data collected on a commercial TTRS [18], again for $T_H = 21.1°C$ and $T_L = 1.6°C$, is used to provide a basis of comparison against the set points generated by the optimization. In practice, the processes of the VCC are realized by a compressor, condenser, expansion valve, and evaporator, and sometimes a refrigerant receiver as is the case for the TTRS considered here. The presence of a receiver constrains the condenser outlet refrigerant condition to be saturated liquid thereby removing $T_1$ as a DOF from the optimization. Therefore, there are 4 remaining DOFs, $h_1$, $h_2$, $h_4$, and $\dot{m}_r$, in this particular example.

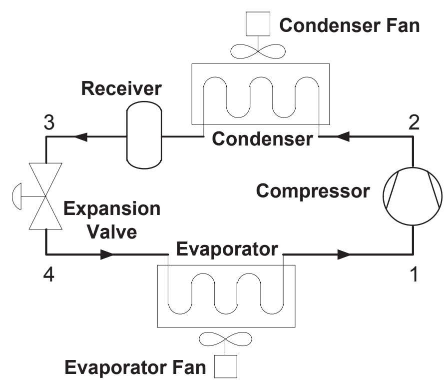


Fig. 4. *Schematic of a VCC system with refrigerant receiver.*

The most common control scheme for VCC systems is on-off control in which a fixed-speed compressor is used to regulate the cooling capacity. We assume that the compressor operates at only one speed; therefore, only one value of $P_1$, the evaporator pressure, can be achieved by the system. Safety requirements are met by maintaining a prescribed amount of evaporator superheat, a temperature which corresponds to the relative degree of vaporization at the evaporator outlet ($T_1 - T_4$); this is regulated by a thermostatic expansion valve (TEV), a mechanical control device. In this framework, no DOFs are actually controlled; instead they are specified by design but then cannot be modulated during system operation.

Since the TTRS examined in this case study is augmented with some additional components beyond those in a standard VCC system (e.g. suction line heat exchanger, throttle valve, etc.) the data was modified to reflect the operation of the 5-component VCC system shown in Fig. 4. The TTRS considered here is operated in an on-off fashion as described above.

*1) Implementation of Design Constraints:* Two types of design constraints are taken into account—thermal design constraints and fluid dynamic ones. The thermal design constraints come from the design of the heat exchangers and corresponding fans. For a prescribed thermodynamic state of the refrigerant and air, and value of $\dot{m}_r$, the design of a heat exchanger determines the maximum possible heat transfer rate across it. The overall heat transfer coefficient, commonly referred to as a UA-value, can be computed using heat transfer correlations and a thermal resistance circuit (Fig. 5) [19].

In order to compute the maximum UA-value for the evaporator and condenser, respectively, the following assumptions are made:

- $\dot{m}_{a,e}$ and $\dot{m}_{a,c}$ are taken to be their maximum possible value based on the design of the evaporator and condenser fans, respectively,
- the refrigerant in each heat exchanger is entirely a two-phase fluid,
- $T_H$ and $T_L$ are constant throughout the condenser and evaporator, respectively, and
- fin heat transfer is one-dimensional.

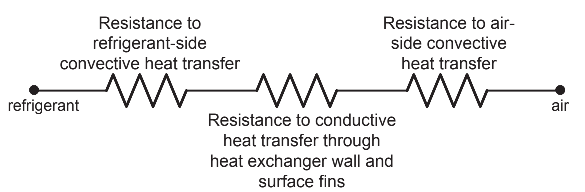


Fig. 5. *Thermal circuit used to compute overall heat transfer coefficient for condenser and evaporator*

The correlations used to compute $\alpha_r$ and $\alpha_a$ for the specific evaporator and condenser in the TTRS are described in [18].

$$\frac{1}{UA} = \frac{1}{\alpha_r A_i} + \frac{t}{kA_o(1-F_a)} + \frac{1}{\alpha_a A_o} \tag{22}$$

$$\dot{Q}_{L,max} = (UA)_{L,max}(T_L - T_4) \tag{23}$$

$$\dot{Q}_{H,max} = (UA)_{H,max}(T_3 - T_H) \tag{24}$$

The constraints are defined in (25) and (26).

$$\dot{Q}_{L,max} \geq \dot{m}_r(h_1 - h_4) \tag{25}$$

$$\dot{Q}_{H,max} \geq \dot{m}_r(h_2 - h_4) \tag{26}$$

In addition to the thermal design constraints, we must ensure that the compressor and electronic expansion valve (EEV) in the system are capable of producing the optimal refrigerant mass flow rate. The fluid dynamic constraints are enforced using validated compressor and EEV models, described in [18]. The refrigerant mass flow rates through the compressor and EEV are described by (27)–(30), where $f_1$ and $f_2$ are empirically-derived nonlinear relationships. During steady-state operation, $\dot{m}_{r,k} = \dot{m}_{r,v}$.

$$\dot{m}_{r,v} = K_v\sqrt{\rho_3(P_2 - P_1)} \tag{27}$$

$$\dot{m}_{r,k} = \eta_{vol}\delta_k\omega_k\rho_1 \tag{28}$$

$$K_v = f_1(a_v, P_1, P_2) \tag{29}$$

$$\eta_{vol} = f_2(\omega_k, P_1, P_2) \tag{30}$$

From (27)–(30) we see that once the optimization variables are specified, there is a unique valve aperture, $a_v$, and a unique compressor speed, $\omega_k$, for which the optimal refrigerant mass flow rate can be achieved. An iterative scheme is used to solve (27)-(30) for $a_v$ and $\omega_k$, respectively, given the optimal values of $h_1$, $h_2$, $h_4$, and $\dot{m}_r$ and ensures that these values are feasible (e.g. $0\% < a_v \leq 100\%$).

*2) Results:* Table IV contains the optimal solution of each of the optimization variables when the design constraints are imposed. Fig. 6 shows the VCC plotted on a $P$-$h$ diagram with the optimized operation set points and the nominal ones used for the TTRS considered in this case study. The COP values presented in Table V are computed with the adiabatic efficiency of the compressor taken into account; the empirical model for $\eta_{adb}$ is described in [18].

TABLE IV
OPTIMIZATION RESULTS WITH DESIGN CONSTRAINTS IMPOSED

| *Parameter* | *Units* | *Value* |
|---|---|---|
| Optimal Cost, $J$ | kW | 0.817 |
| $h_1$ | kJ/kg | 212 |
| $h_2$ | kJ/kg | 233 |
| $h_4$ | kJ/kg | 93.6 |
| $\dot{m}$ | kg/s | 0.129 |

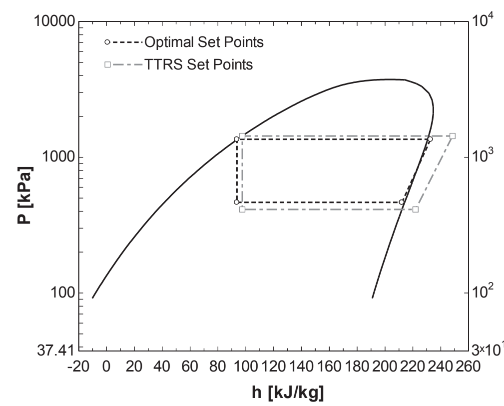


Fig. 6. *Log(P)-h diagram showing optimized cycle and TTRS nominal cycle for 15kW cooling capacity with design constraints imposed.*

$$\text{COP}_{actual} = \eta_{adb}\text{COP} \tag{31}$$

$$\eta_{adb} = f_3(\omega_k, P_1, P_2) \tag{32}$$

Table V shows an increase of 52.5% in COP is achieved by using the optimal set points. From Fig. 6 we see $P_1$ is higher, and $P_2$ lower, in the optimal case, resulting in an overall decrease in pressure differential across the compressor. With the optimal values of $h_1$, $h_2$, and $h_4$, and $\dot{m}_r$, the rate of heat transfer in the evaporator is within 3.79% of its maximum capability (based on the estimated overall heat transfer coefficient); similarly, the condenser heat transfer

TABLE V

COMPARISON BETWEEN NOMINAL AND OPTIMAL TTRS OPERATION PARAMETERS

| | *Units* | *Nominal Case* | *Optimal Case* |
|---|---|---|---|
| Desired Capacity | kW | – | 15.3 |
| Capacity Achieved | kW | 15.3 | 15.2 |
| Power Consumption | kW | 4.72 | 3.07 |
| Superheat | $^{\circ}C$ | 11.3 | 0.575 |
| Second Law Efficiency | - | 0.262 | 0.670 |
| COP | - | 3.24 | 4.95 |

is within 0.559% of its maximum capability. Finally, the optimal solution is characterized by a small superheat value (less than $1^{\circ}C$). These results are consistent with references which assert that operating with a low superheat [20]–[23] and maximum heat transfer through the heat exchangers [5] is optimal.

Nevertheless, the optimal operation set points can only be achieved through regulation of both $P_1$ and $P_2$, highlighting a fundamental tradeoff in the control of VCC systems. Current hardware does not allow for all of the cycle DOFs to be controlled. While adding a variable frequency drive (VFD) to the compressor and fans, and/or replacing a TEV with an EEV, requires an initial monetary investment, it is important to recognize the improvement in efficiency that is then achievable. These results provide concrete motivation for further research in developing optimization and control strategies which specifically utilize all available DOFs of the VCC.

## IV. CONCLUSION

In this paper, the authors investigate the optimal steady-state operation of the VCC from the perspective of its underlying physics. An objective function which minimizes the rate of exergy destruction in the system was derived for the five DOF problem. The optimization variables were chosen as the three enthalpies $h_1$, $h_2$, and $h_3 = h_4$, of the cycle, one cycle temperature, $T_1$, as well as the refrigerant mass flow rate, $\dot{m}_r$. A case study was considered in which the optimization was applied to a commercial truck transport refrigeration system (TTRS). The optimal set points generated by the exergy-based objective function were shown to provide an increase of 52.5% in COP compared against the nominal set points regularly used for the commercial TTRS considered in this paper.

The optimization results highlight that given the ability to optimize all available DOFs of the VCC, a significant improvement in COP can be achieved. Future work will address the control implementation of the optimal operation set points, as well as extending the optimization beyond steady-state operation.


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