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Design of Switching Strategy for Adaptive Cruise Control under String Stability Constraints

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Abstract—**A new switching strategy is proposed in the design of an Adaptive Cruise Control (ACC) system operating in a platooning traffic configuration. The proposed control system enjoys shorter elapsed time than conventional methods while still being able to maintain a safe distance between vehicles during transient process. During the headway control mode of an ACC controller, a constant deceleration is maintained by the following vehicle when approaching the preceding one. In addition, string stability of a platoon of vehicles equipped with such an ACC controller has been taken into consideration and parameter constraints of the controllers are derived to mitigate steady state error propagation. A longitudinal 1-D vehicle model of a passenger car is used for simulation purposes and the results from the proposed design are verified.**

I. Introduction

ADAPTIVE Cruise Control (ACC) systems are currently being deployed by numerous vehicle manufacturers. An ACC system is a driver assistance system that is designed to help driver to improve driving safety and driving comfort during highway operation. ACC is similar to conventional cruise control in most of passenger cars in that it maintains the vehicle's pre-set speed in the absence of preceding in-lane vehicles. However, unlike conventional cruise control, ACC system can automatically adjust speed in order to maintain a proper spacing between the in-lane preceding vehicle and the ACC-equipped vehicle.

In order to achieve such function, each ACC-equipped vehicle utilizes a ranging device, such as a radar or lidar sensor that measures/calculates the distance and relative velocity to the in-lane preceding vehicle [1], [2]. One aspect of the Automated Highway Systems (AHS) program conducted during the late 90s, was the use of ACC systems in a platoon traffic flow. One objective of the AHS program was to improve the traffic capacity on a highway by enabling vehicles to operate together in a tightly spaced platoon. These specially equipped platooning vehicles were demonstrated on a specially designed highway segment in the San Diego region [4].

Due to the rapid increase of ACC equipped vehicles both on highway and local roads, string stability issue has been addressed [3]. The main problem is when many vehicles equipped with an ACC controller forming a vehicle platoon end to end, how the control algorithm is designed to ensure that the spacing error, which is the deviation of the actual range from the desired headway distance, would not amplify as the number of following vehicles increases downstream along the platoon.

Research has been conducted mainly in the following three categories: autonomous control, semi-autonomous control, and centralized control of intelligent vehicles [4],[5],[6]. Autonomous control of a vehicle refers to the methods that solely depend on information collected by sensors on the subject vehicle; while semi-autonomous control, in comparison, refers to control methods that also depend on information sent back and forth among vehicles, which requires vehicle-to-vehicle communication and vehicle networks. The last category is the centralized control which requires not only vehicle-to-vehicle communication but also the vehicle-to-infrastructure communication. While the last two categories don't look realistic, at least in the near future, the first category, the autonomous control with ACC system, is however a very attractive feature right now and is already available in the market [7].

An ACC controller has two operational modes: speed control mode and headway control mode (or spacing control mode in some literatures). The design of ACC controllers often involves the design of a switching logic that decides where and when to switch between the two operational modes in order to ameliorate driving comfort, mitigate the chance of a potential collision with the preceding vehicle while reducing driving load from the driver.

In this paper, we propose a new design of ACC controller by which the following vehicles will approach the equilibrium point fast while maintaining the string stability of the system. A Range vs. Range-Rate chart has been developed and proved to be a very effective way for the design and evaluation of control strategies for ACC [8]. Based on the Range vs. Range-Rate chart, a new switching strategy is proposed that enables an ACC-equipped vehicle to approach the preceding vehicle with a constant deceleration and further reduces elapsed time needed for the following vehicle to reach equilibrium point. A systematic design method for the proposed switching strategy is presented. Secondly, constraints of ACC controller parameters are derived for the following vehicle to maintain string stability at the equilibrium point, when a platoon of ACC controlled vehicles is taken into consideration. Finally, a 1-D vehicle dynamical model is used to verify the theoretical results and the simulation results are also presented.

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II. Problem Formulation

A. Problem under Consideration

The problem under consideration is illustrated in Fig. 1 for two consecutive ACC-equipped vehicles with in a generic multi-vehicle platoon segment.

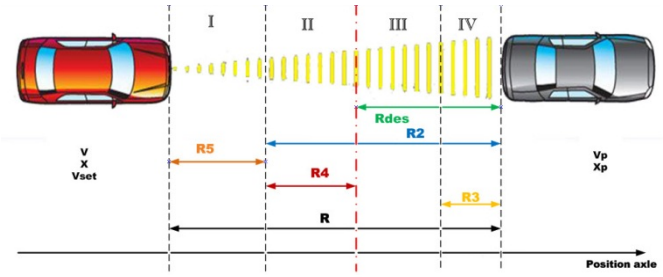

Fig. 1. Definition of variables for the design of an ACC controller.

In the following notation, the sub-index of each variable denotes which vehicle the variable is assigned to in a vehicle platoon. For example, the index of *i*-1 is for preceding vehicle and the index of *i* is for the following vehicle:

v_{i-1}: Velocity of the preceding vehicle.
x_{i-1}: Relative longitudinal position of preceding vehicle.
v_i: Velocity of following vehicle.
x_i: Relative position of following vehicle.
v_{i_set}: Pre-set reference speed of following vehicle by the driver.
R: Actual distance between two vehicles.
R_{des}: Desired headway distance between two vehicles.
R_2: Switching range by which ACC controller switches between the two modes.
R_3: Braking range, in which the following vehicle has to apply maximum braking to avoid potential collision.
R_4: Buffer zone before reaching the equilibrium point.
R_5: Speed control range before switching to headway control mode.
Zone I: Speed control zone.
Zone II: Headway control zone.
Zone III: Moderate braking zone.
Zone IV: Maximum braking zone.

Fig. 1 illustrates basic concepts related to the ACC controller design. The headway control problem can be described as a problem of developing a system that maintains a desired headway distance R_{des} between two consecutive vehicles by modulating the speed of the following vehicle. Once the onboard sensor of the following vehicle detects the presence of a preceding vehicle and the ACC system is engaged, the ACC control strategy starts to take control of the vehicle.

When the following vehicle reaches Zone I, the ACC system determines whether there is enough distance left from the preceding vehicle; then the controller maintains the driver selected speed and operates as a conventional cruise controller. If the set speed is greater than that of the preceding vehicle, the range between the two vehicles will gradually shrink, which implies the following vehicle is catching up with the preceding one.

After the following vehicle reaches Zone II, the ACC controller switches from speed control mode to headway control mode. The controller modulates the speed of the following vehicle to catch up and then maintains the desired headway distance R_{des}, depending on the information available to the following vehicle's onboard controller. In a cut-in event, the following vehicle's ACC controller detects a cut-in vehicle that causes the range within Zones III or IV, it will instantly switch to headway control mode, reduce the following vehicle's speed (e.g., by moderate or maximum braking), and enlarge the range R to be equal to R_{des}.

There are basically two different strategies to control the velocity of the following vehicle, based on different definitions of the desired headway distance R_{des}. One defines $R_{des} = h * v_{i-1}$ and the other is given by $R_{des} = h * v_i$, where h is called headway time with second as its unit. The different definitions of R_{des} will result in different results. In this paper both definitions will be discussed when analyzing string stability of a vehicle platoon, and a brief comparison will be presented based on our simulation results.

B. Range vs. Range Chart Description

Range (R) vs. Range-rate (dR/dt or R_{dot}) chart is developed to describe the problem more efficiently and effectively [8]. Relative position and relative velocity between the two consecutive vehicles can be represented in such chart. Variables are defined as follows:

$$R = x_{i-1} - x_i \,, \tag{1}$$

$$R_{dot} = \frac{dR}{dt} = v_{i-1} - v_i \,. \tag{2}$$

On the R-R_{dot} chart, the values of R are along the vertical axis and the values of R_{dot} are along the horizontal axis. At the equilibrium point, the range R would be equal to the desired headway distance R_{des} and the velocity of the following vehicle v_i would be equal to the velocity of the preceding vehicle v_{i-1}.

In the upper left quadrant shown in Fig. 2, because $R_{dot} < 0$, that means $v_{i-1} < v_i$ the following vehicle is moving faster than the preceding vehicle. In this situation, the trajectories on the R-R_{dot} chart have the tendency to go downward since the value R tends to decrease. In the upper right quadrant, because $R_{dot} > 0$, that means $v_{i-1} > v_i$ the following vehicle is moving slower than the preceding vehicle. In this situation, the trajectories on the R-R_{dot} chart have the tendency to go upward since the value R tends to increase. Possible equilibrium point on the R-R_{dot} chart should be on the vertical axis, where $R = R_{des}$ and $R_{dot} = 0$.

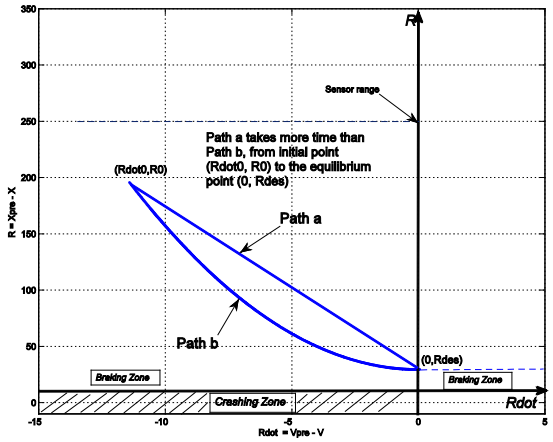

Fig. 2. Different paths with different elapsed time.

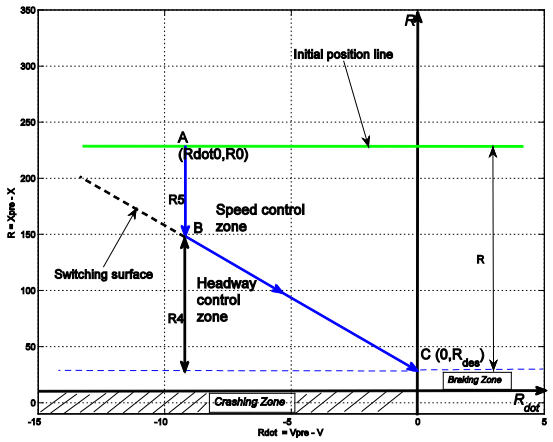

Fig. 3. Linear trajectory in R-R_{dot} chart.

From the definition of R_{dot}, the elapsed time needed to slide from the initial point to the final point is given by

$$t_e = t - t_0 = \int_{R_0}^{R} \frac{dR}{R_{dot}} . \tag{3}$$

From Fig. 2, it is not difficult to see that the elapsed time on path *b* is less than that on path *a*.

III. Switching Logic Design

Switching logic defines where and when the ACC controller switches between the two functional modes, speed control mode and headway control mode.

A. A linear relationship between R and R_{dot}

Trajectories based on linear relationship between R and R_{dot} have been studied as a prototype headway control system [8]. The equation of the trajectory BC is given by

$$R = c \cdot R_{dot} + R_{des} , \tag{4}$$

where c is the slope of the segment BC.

Rewriting (4) as a general first order differential equation and solving it with the initial condition $R = R_0$, we obtain

$$R = R_0 \cdot e^{-t/c} + R_{des}\left(1 - e^{-t/c}\right) . \tag{5}$$

where c is a time constant that decides how fast the system would approach the equilibrium point. From the solution, it is clear that no matter how long it takes, the system would only approach the final position without reaching it because of the exponential term in equation (5). However after $5c$ time, the system will get closer enough to the equilibrium point, over 99% of the whole length of segment BC [8].

B. Constant Decelerations of the Following Vehicle

This approach is to modulate the acceleration/deceleration of the following vehicle to be a constant while the following vehicle is approaching the preceding vehicle. Assume that the preceding vehicle runs at a constant speed during this process.

From the definitions and assumptions above, we have

$$R_{dot} = R_{dot0} + D \cdot t_e , \tag{6}$$

which implies that

$$t_e = \frac{R_{dot} - R_{dot0}}{D} , \tag{7}$$

where R_{dot0} is the initial value of R_{dot} on the trajectory, t_e is the elapsed time needed to reach the equilibrium point where $R_{dot} = 0$, and D is the deceleration level.

Rewriting (6) as $dR = R_{dot0}dt + D \cdot tdt$ and integrating the two sides of it,

$$\int_{R_0}^{R} dR = \int_{0}^{t_e} R_{dot0}dt + \int_{0}^{t_e} D \cdot tdt$$

$$R - R_0 = R_{dot0} \cdot t_e + D \cdot \frac{1}{2} t_e^2 . \tag{8}$$

Substituting t_e expressed in (7) into (8), we have

$$R - \frac{R_{dot}^2}{2D} = R_0 - \frac{R_{dot0}^2}{2D} . \tag{9}$$

Since point $(0, R_{des})$ is on the trajectory of (9), we have $R_{dot} = 0$ and $R = R_{des}$, thus we obtain

$$R_{des} = R_0 - \frac{R_{dot0}^2}{2D} . \tag{10}$$

After the following vehicle reaches equilibrium point, R is expected to be equal to R_{des}, which means (9) can be

rewritten as

$$R = \frac{R_{dot}^2}{2D} + R_{des}\,. \tag{11}$$

From the analysis above, we derive a relationship between R and R_{dot}, which is shown in Fig. 4. Depending on how large the value of deceleration would be, the elapsed time of the deceleration process can be calculated from (11). When a larger deceleration level is selected, the trajectory of the deceleration process is shifting toward the more negative side of R_{dot}.

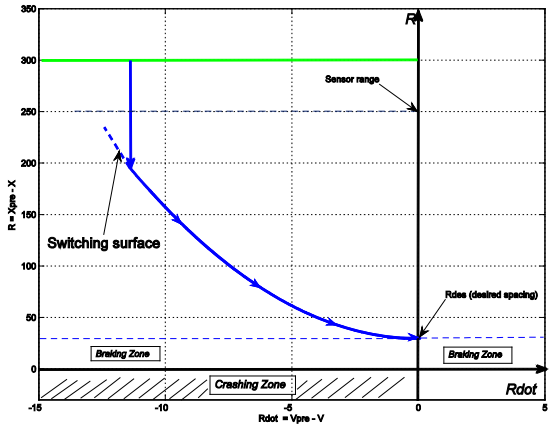

Fig. 4. Trajectory of constant deceleration strategy on R-R_{dot} chart.

C. Design Procedures of the Proposed Switching Logic

Assumptions had been made that a reliable conventional cruise control system had already been available. The simulation has been conducted in a situation in which the preceding vehicle is moving at a constant speed for most of the time. Since a desirable comparison can only be made when the desired headway distance, i.e., R_{des} is fixed in the R-R_{dot} chart. The design process proposed in this paper consists of the following steps:

1. Define the desired headway distance for a selected velocity and headway time.
2. Choose the deceleration to be used. In this paper the deceleration is set to be D=0.2g, where g=9.8 m/s^2.
3. Construct a switching zone. A constant deceleration parabola can be drawn on the R-R_{dot} chart. Above the parabola, a switching zone has to be constructed within which the following vehicle performs switching from speed control to headway control.
4. If the design doesn't involve the use of brake in the approaching process, costing deceleration *0.02g* (calculated from the Simulink vehicle dynamic model we built for simulation) could be used to be the upper limit of the switching zone.
5. Design a Dead Zone around the equilibrium point so that the trajectory won't jitter due to strict boundaries of different zones on the R-R_{dot} chart. This step will be elaborated later in this section.

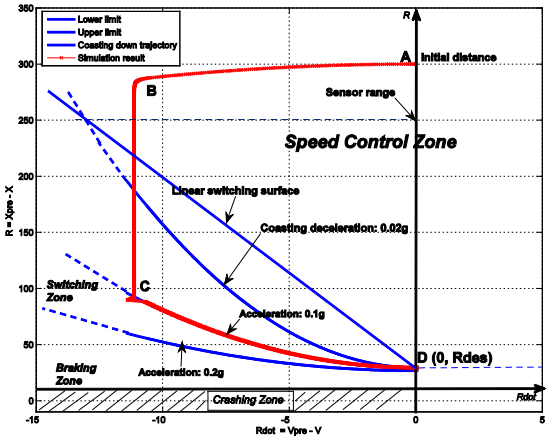

Fig. 5. Example of an adaptive cruise controller design following the proposed switching strategy.

A 1-D longitudinal vehicle dynamic model has been developed based on Matlab/Simulink. Our simulation results are shown in Fig. 5. In this example, h is set to be 1.5sec, v_p is set to be the velocity of a lead vehicle which follows a speed profile, starting roughly at *17.88 m/s, i.e., 45 MPH*. Assuming the preceding vehicle and the following vehicle both start from zero velocity and begin accelerating at time 0, with certain spacing in between. Such situation is indicated by point A in Fig. 5, where R_{dot} is 0 and R has certain value larger than sensor range. Segment AB shows that the final velocity of the following vehicle is greater than that of the preceding vehicle, which leads to a negative R_{dot}. Segment BC is vertical because both vehicles have reached their pre-set target speeds and then maintain each target speed during speed control mode. As the two vehicles get closer and closer, the trajectory hits the switching surface when ACC controller switches from speed control mode to headway control mode.

In this paper, we have proposed a constant deceleration switching surface shown in Fig. 4, with a shorter elapsed time, i.e. faster approaching process. Segment CD in Fig. 5 shows the simulation result and verifies our design purpose.

IV. String Stability Analysis

Consider a platoon of vehicles equipped with ACC controller and operate in headway control mode, as shown in Fig. 6. Because the length of each vehicle is not going to affect the result of the analysis, assumption has been made that the vehicle length is neglected in the following calculation. Relative position of each vehicle in the platoon is defined according to a common reference starting point.

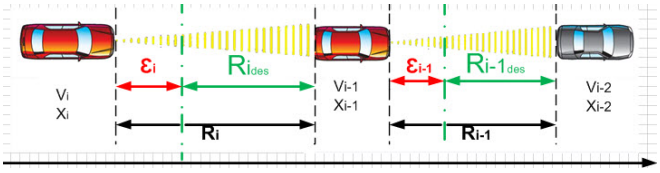

Fig. 6. Vehicles in a platoon.

Mathematically, a desirable characteristic for string stability is often specified as $\|\varepsilon_i\|_\infty \leq \|\varepsilon_{i-1}\|_\infty$, where ε_i is the spacing error of the following vehicle, ε_{i-1} is the spacing error of the preceding vehicle, and $\|\cdot\|_\infty$ denotes the infinity norm. Previous work has proved that if conditions $\|H(s)\|_\infty \leq 1$ and $h(t) > 0$ are satisfied (where $h(t)$ is the impulse response of the error transfer function), string stability is guaranteed [3]. $H(s)$ is the steady state error transfer function defined as:

$$H(s) = \frac{\varepsilon_i(s)}{\varepsilon_{i-1}(s)}. \tag{12}$$

Usually a second order linear system can be used to represent the 1-D dynamic model of a vehicle for longitudinal control, as shown below,

$$y = \frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)} u. \tag{13}$$

By measuring the corresponding time at magnitude 63.2% [9] of the step response of the 1-D vehicle dynamic model used in out paper, the major time constant of the second order system in (13) is estimated to be $\tau_1 = 0.864s$. Using Matlab System Identification Toolbox [10], the other time constant τ_2 is estimated to be 0.002, which is exactly as what has been predicted, namely a much smaller time constant than τ_1. Since the pole corresponding to time constant τ_2 is much farther away from the vertical axis in a complex plane than the pole that corresponds to time constant τ_1 (more than 10 times farther way), its influence to the whole system can be neglected when looking at a large scale in time.

A. Desired Headway Distance based on Velocity of the Preceding Vehicle

As described before, the implementation of desired headway distance based on velocity of the preceding vehicle is straightforward. In this section, two conditions are derived for this type of implementation.

From analysis before, for the *i*-th vehicle in the platoon, equation (13) can be simplified by eliminating the minor pole corresponding to time constant τ_2 as follows

$$x_i(s) = \frac{1}{\tau s + 1} \cdot x_{i_{des}}(s). \tag{14}$$

where τ is the major time constant, $\dot{x}_{i\_des}$ is the desired vehicle speed as the system input and the output $\dot{x}_i$ is the actual vehicle speed. We can rewrite (14) as

$$\tau \ddot{x}_i + \dot{x}_i = k_p \cdot \varepsilon_i + k_d \cdot \dot{\varepsilon}_i, \tag{15}$$

where ε_i is the spacing error of the *i*-th vehicle, k_p and k_d are parameters of the PD controller for headway control, and $(k_p \cdot \varepsilon_i + k_d \cdot \dot{\varepsilon}_i)$ serves as control input[11].

Using the definition of spacing error

$$\varepsilon_i = R_i - h \cdot v_{i-1} = R_i - h \cdot \dot{x}_{i-1}, \tag{16}$$

the error transfer function could be derived as

$$H(s) = \frac{\varepsilon_i(s)}{\varepsilon_{i-1}(s)} = \frac{-k_d h s^2 + (k_d - k_p h)s + k_p}{\tau s^2 + (k_d + 1)s + k_p}. \tag{17}$$

According to the conditions for string stability proved in the previous work given by $|H(j\omega)| = \left|\frac{\varepsilon_i(j\omega)}{\varepsilon_{i-1}(j\omega)}\right| \leq 1$, we obtain the constraints for the design of controller parameters when the desired spacing is defined as a function of the preceding vehicle's velocity:

$$\begin{cases} (2k_p\tau + k_p^2h^2 - 2k_d - 1)(\tau^2 - k_d^2h^2) \leq 0 \\ \tau^2 - k_d^2h^2 \neq 0 \end{cases} \tag{18}$$

or

$$\begin{cases} 2k_p\tau + k_p^2h^2 - 2k_d - 1 \leq 0 \\ \tau^2 - k_d^2h^2 = 0 \end{cases} \tag{19}$$

B. Desired Headway Distance based on Velocity of the Following Vehicle

A different definition of desired headway distance is adopted in this section. The spacing error of this case is given by

$$\varepsilon_i = R_i - h \cdot v_i = R_i - h \cdot \dot{x}_i. \tag{20}$$

Following the similar analysis, we obtain the error transfer function as

$$H(s) = \frac{\varepsilon_i(s)}{\varepsilon_{i-1}(s)} = \frac{k_p + k_d s}{(hk_d + \tau)s^2 + (hk_p + k_d + 1)s + k_p}. \tag{21}$$

According to the same condition $|H(j\omega)| = \left|\frac{\varepsilon_i(j\omega)}{\varepsilon_{i-1}(j\omega)}\right| \leq 1$, we obtain the constraints for the design of controller parameters when the desired spacing is defined as a function of following vehicle's velocity:

$$k_p^2h^2 + 2k_d + 1 - 2k_p(h - \tau) \geq 0. \tag{22}$$

C. Simulation Results

Following the design and analysis above, simulations of an eight-vehicle platoon have been performed to verify the applicability of the proposed design. The results are presented in the following figures. If parameters of the controller are designed to violate constraints concluded above, for example in this simulation parameters are chosen to be $k_p = 0.3, k_d = 9.6, h = 1.5$ and $\tau = 0.864$, the spacing errors of the following vehicles get amplified as shown in Fig. 7, and the velocities of the following vehicles are not going to converge quickly, as shown in Fig. 8.

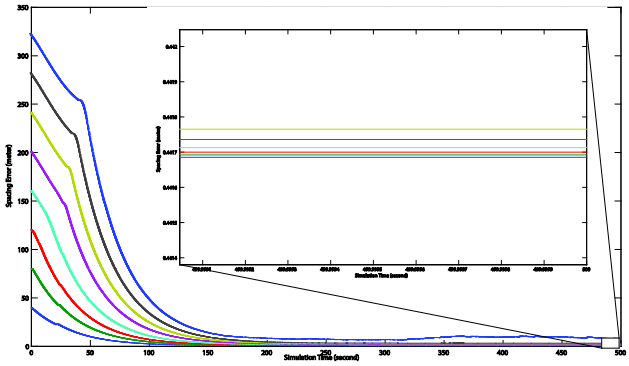

Fig. 7. Amplification of steady state errors along the vehicle platoon with parameters that violate conditions for string stability.

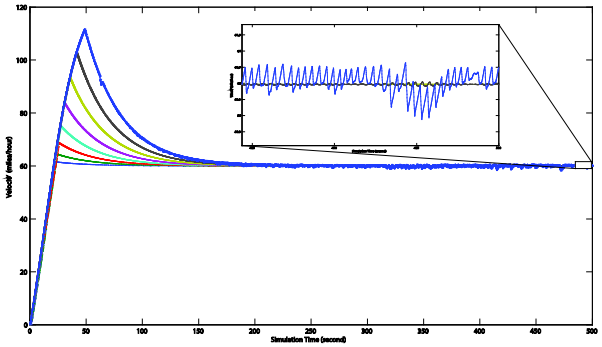

Fig. 8. Speed performance of each vehicle in the platoon with parameters that violate conditions for string stability.

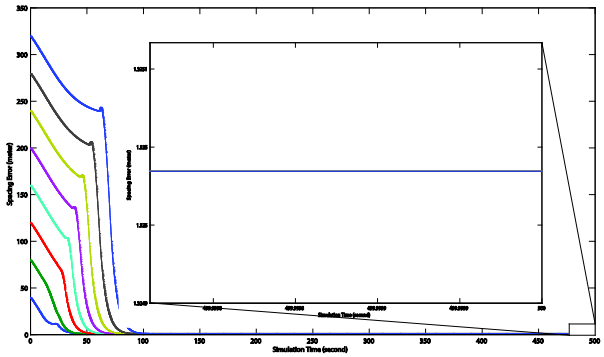

Fig. 9. Amplification of steady state errors along the vehicle platoon with parameters that satisfy conditions for string stability.

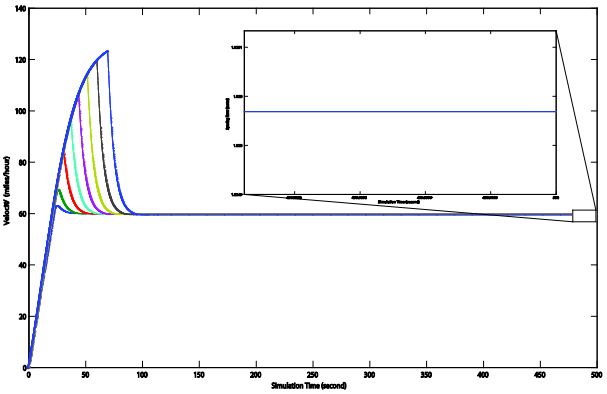

Fig. 10. Speed performance of each vehicle in the platoon with parameters that satisfy conditions for string stability.

While if parameters of the controller satisfy constraints concluded above, for example $k_p = 0.1, k_d = 0.576, h = 1.5$ and $\tau = 0.864$, the spacing errors of the following vehicles do not amplify as shown in Fig. 9 and the velocities of the following vehicles converge considerably fast as illustrated in Fig. 10.

V. Conclusion and Future Work

In this paper, a new switching logic for the systematic design of ACC controllers with shorter transient time has been proposed. Simulations based on the proposed design have been carried out. String stability issues have been taken into consideration and constraints for controller parameters have been derived to mitigate steady state error propagation. Two different definitions of desired headway distance have been discussed and string stability constraints under both definitions have been considered and derived. Simulations based on both definitions have been performed.

The Future directions include the development of new control strategies during the transient period between speed control mode and headway control mode to further eliminate uncomfortable factors. Advanced control approaches such as sliding-mode control might be considered instead of PD control during headway control mode of the controller. Achieving vehicle string stability in noisy environment in terms of sensor noise and model uncertainty might also be an interesting topic to investigate in the future.

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