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A Comparison of Multi-Switch Bang-Bang and Time-Optimal Zero-Vibration Commands for Rest-to-Rest Moves of a Floating Oscillator

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Abstract— **This paper compares two methods for creating multi-switch bang-bang commands to produce rest-to-rest command inputs for a floating oscillator system. The proposed multi-switch, bang-bang commands (MSBB) have constant switching intervals and satisfy zero-vibration (at the end of the maneuver), maximum acceleration, and move distance constraints. The MSBB commands are compared to the Time-Optimal, Zero-Vibration (TO-ZV) commands that produce a vibration-free move for a floating oscillator system. Simulation results show that the proposed MSBB commands are a valid alternative to the TO-ZV command due to the simplicity of the MSBB design procedure, the comparable move times and robustness, and the avoidance of very short switching intervals.**

I. Introduction

Lightly damped systems are difficult to control because their natural response exhibits substantial oscillations and complex controllers may be required to achieve the desired performance. The objective of this paper is to compare methods for creating multi-switch bang-bang control inputs to regulate the position of a floating oscillator system shown in Figure 1. This benchmark system is used to model any system with one rigid-body mode and one flexible mode such as robotic manipulators with flexible links, satellites with flexible appendages, etc. [1].

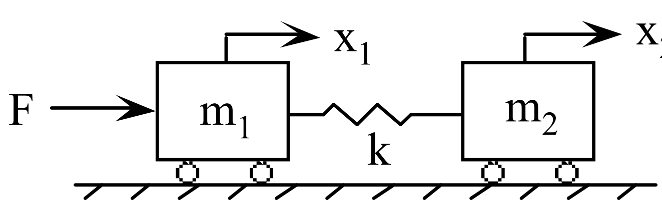

Fig. 1. Benchmark System

Several researchers have addressed the use of a bang-bang type controller for lightly damped or flexible systems previously [2-5]. These types of controllers are necessary when On-Off (bang-bang) actuators are used. Other researchers have previously developed input commands specifically for floating oscillators [6-12]. The major advantage of the proposed control inputs for floating oscillator system presented here is its simplicity and its robustness. The amplitude and duration of the control commands are computed from the desired move distance, the system parameters (masses and spring constant) and the robustness requirements. Since the command signal relies on the system parameters, the robustness of the proposed controller to model uncertainty must be considered. In this paper robustness is measured by the percent overshoot in the system response. For the proposed method, the robustness improves as the duration of the bang-bang command signal increases for a given number of switches.

The proposed multi-switch, bang-bang (MSBB) commands have constant switching intervals and satisfy zero-vibration (at the end of the maneuver), maximum acceleration, and move distance constraints. The proposed commands are compared to the Time-Optimal, Zero-Vibration (TO-ZV) commands that have been shown to produce the minimum-time move for a flexible system [8]. The MSBB command proves to be a valid alternative to the TO-ZV command due to the simplicity of the MSBB design procedure, the comparable move times and robustness, and the avoidance of very short switching intervals.

The remainder of the paper is organized as follows. Section II reviews the TO-ZV commands. Section III presents the MSBB commands and provides a comparison of the MSBB and TO-ZV performance and robustness. Section IV contains conclusions and suggestions for future work.

II. Time-Optimal, Zero-Vibration Commands

Time-Optimal, Zero-Vibration (TO-ZV) commands have been shown to produce the minimum-time move for a flexible system [8]. While input shaping is traditionally used modify commands in real time, they can also be used to create MSBB commands that have specific input values and are designed to a pre-specified move distance. Figure 2 shows how to select the impulse amplitudes to produce a MSBB command.

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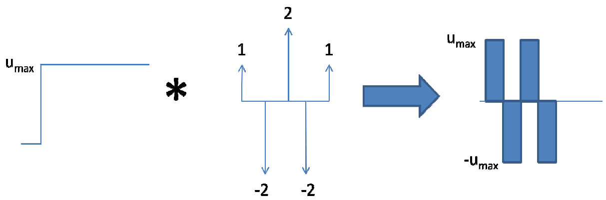

Fig. 2. Using Input Shaping to Produce MSBB Commands.

In addition to the traditional vibration constraints, TO-ZV commands, or any command designed for rest-to-rest moves, must also take into account the move distance. One of the greatest advantages of using input shaping to produce these MSBB commands is that analytic or curve-fit solutions for a wide range of moves can be produced, thereby eliminating the need to perform an optimization for every move distance.

III. Multi-Switch Bang-Bang Commands

The time-optimal, zero vibration (TO-ZV) commands are designed to create a rest-to-rest move for a flexible systems with no vibration upon completion of the move. Previously, a simple method for creating bang-bang commands was presented [13]. Here we compare the performance of these two MSBB commands. Command duration, number of switches in the command and the command's robustness are all considered.

A. Simple MSBB Derivation

The method first detailed in [13] produces vibration-free commands whose switch times are determined by the system's period with input amplitudes that vary based on move distance. For the floating oscillator model shown in Fig. 1, the control objective is to move the 2nd mass (M_2) a specified distance and hold it at that position without oscillation (vibration). In [13], a bang-bang signal was constructed to achieve the desired move distance and zero vibration at the completion of the maneuver. To construct the bang-bang signal, the response of the floating oscillator to a step input is considered to move M_2 half the desired distance. An equal and opposite input will move the 2nd mass the remainder of the desired distance.

For a constant input, $f = F \cdot 1(t)$, where $1(t)$ is the unit step, the motion of the 2nd mass is given by

$$x_2(t) = \frac{1}{2}\left(\frac{F}{M_1+M_2}\right)t^2 - \left(\frac{F}{(M_1+M_2)\omega^2}\right)(1-\cos(\omega t))$$
(1)

when the system starts from rest. The frequency of oscillation is given by $\omega = \sqrt{\frac{k(M_1+M_2)}{M_1 M_2}}$. To achieve half the desired move distance in half the command duration, the requirement on the motion is that

(2)

$$x_2\left(\frac{t_m}{2}\right) = \frac{1}{2}\left(\frac{F}{M_1+M_2}\right)\left(\frac{t_m}{2}\right)^2 - \left(\frac{F}{(M_1+M_2)\omega^2}\right)\times \left(1-cos\left(\omega\frac{t_m}{2}\right)\right) = \frac{x_d}{2} \quad (2)$$

If the move duration, t_m, of the bang-bang signal is chosen so that $\cos\left(\omega\frac{t_m}{2}\right) = 1$ or

$$T = \frac{4q\pi}{\omega} \quad (3)$$

where $q = 1,2,3,\cdots$, then the move distance at time $t = \frac{t_m}{2}$ is

$$x_2\left(\frac{t_m}{2}\right) = \frac{1}{2}\left(\frac{F}{M_1+M_2}\right)\left(\frac{t_m}{2}\right)^2$$
(4)

As a result, the input force can be chosen to be

$$F = (M_1+M_2)a \quad (5)$$

where $\frac{1}{2}a\left(\frac{t_m}{2}\right)^2 = \frac{x_d}{2}$ or

$$a = \frac{4x_d}{\left(\frac{t_m}{2}\right)^2} \quad (6)$$

The duration of the bang-bang signal (equal to the move time) is given by

$$t_m = \frac{4q\pi}{\omega} = 2qT \quad (7)$$

where ω is the frequency of vibration, $T = \frac{2\pi}{\omega}$ is the period of vibration, and $q = 1,2,3,\cdots$. The signal switches at one-half the move time, $\delta = qT$ and the move time is $2q$ times the period of vibration.

Given the desired move distance, x_d, the input acceleration must be chosen to be

$$a = \frac{4x_d}{t_m^2} \quad (8)$$

Given a maximum acceleration, the move time can be adjusted by the choice of q to force the acceleration below the maximum. Given a feasible acceleration (i.e., below the maximum value), the desired acceleration can be achieved using a true bang-bang actuator (i.e., only produces the maximum acceleration) and pulse-width modulation.

In [15], a multi-switch bang-bang signal was constructed assuming that the time between switches is constant. The zero vibration constraints

$$\sum_{i=2}^{\sigma}(-1)^{i-1}\left(\sin\left(\omega t_i\right)+\sin\left(\omega\left(2t_{\sigma+1}-t_i\right)\right)\right) +\sin\left(\omega t_{\sigma+1}\right)\left(\cos\left(\omega t_{\sigma+1}\right)-1\right)=0 \tag{9}$$

$$\sum_{i=2}^{\sigma}(-1)^{i-1}\left(\cos\left(\omega t_i\right)+\cos\left(\omega\left(2t_{\sigma+1}-t_i\right)\right)\right) +\cos\left(\omega t_{\sigma+1}\right)\left(\cos\left(\omega t_{\sigma+1}\right)-1\right)=0 \tag{10}$$

are satisfied with a constant switching interval $\delta=\frac{T}{2\sigma}(l+2\sigma n)$ where $1\le l\le 2\sigma, l\ne\sigma$ and $n=1,2,3,\cdots$. The resulting bang-bang signal requires $2(\sigma+1)$ switches. Given the desired move distance, x_d, the input acceleration must be chosen to be

$$a=\frac{4x_d}{T^2}\frac{\sigma}{(l+2\sigma n)^2} \tag{11}$$

The multi-switch bang-bang reduces to the traditional single switch bang-bang signal if $\sigma=1$, $l=2\sigma$, and $n=q-1$. It follows that the multi-switch bang-bang signal with $2(\sigma+1)$ switches can be defined with

$$\begin{aligned}\delta&=\frac{Tq}{2\sigma}\\ a&=\frac{x_d}{q^2T^2}\sigma\\ t_m&=2Tq\end{aligned} \tag{12}$$

for a desired move distance x_d. Furthermore, the multi-switch bang-bang signal in (11) generalizes the results in [13] and [15].

Given a desired move distance x_d and a maximum acceleration $a_{\max}$, a two-phase design is implemented. First, assuming a bang-bang signal ($\sigma=1$), choose q so that the acceleration is below the maximum using (11)

$$q=ceil\left(\sqrt{\frac{x_d}{a_{\max}T^2}}\right) \tag{13}$$

If the acceleration is at most half of the maximum, a multi-switch bang-bang signal can be employed. The switch parameter σ is computed from the ratio of the maximum acceleration to the achieved acceleration

$$\sigma=floor\left(\frac{a_{\max}}{\frac{x_d}{n^2T^2}}\right)=floor\left(\frac{a_{\max}n^2T^2}{x_d}\right) \tag{14}$$

This approach uses as much acceleration as possible without exceeding the maximum acceleration constraint.

B. Comparison to TO-ZV performance

Given an actuator limit (maximum acceleration), the command duration will increase by $2T$ as q is increased. The simple MSBB command is compared to the TO-ZV command in the Fig. 3 for a maximum acceleration

$$a_{\max}=0.5\frac{m}{s^2}$$

The MSBB command appears to be slightly faster than the TO-ZV command just as the move distance limit is reached for a given value of q. This result is due to the curve-fit that was used to produce the TO-ZV commands. (NOTE: Obviously the MSBB command cannot be faster than the TO-ZV command. We used a curve-fit rather than the full optimization for ease of computation on this proof of concept.) When compared to the TO-ZV command, the MSBB command is slowest just after q is increased. Fig. 4 shows the how much longer the MSBB command is than the TO-ZV command. In general, the longer the move distance, the less of a difference there is between the commands durations of the MSBB and the TO-ZV. Across the move distance range shown, the MSBB command averages less than 10% longer than the TO-ZV command.

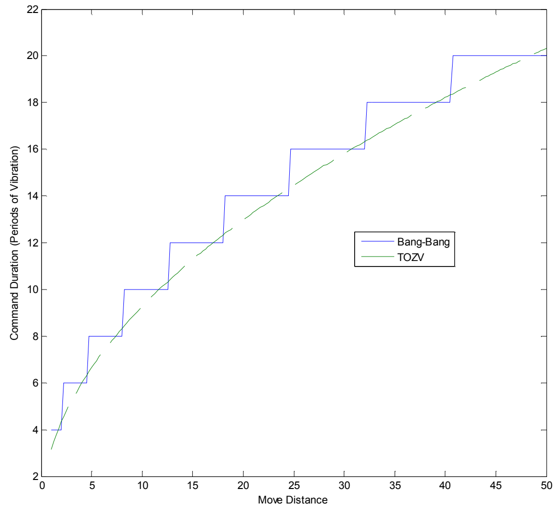

Fig. 3. Comparison of Move Durations, MSBB vs. TO-ZV.

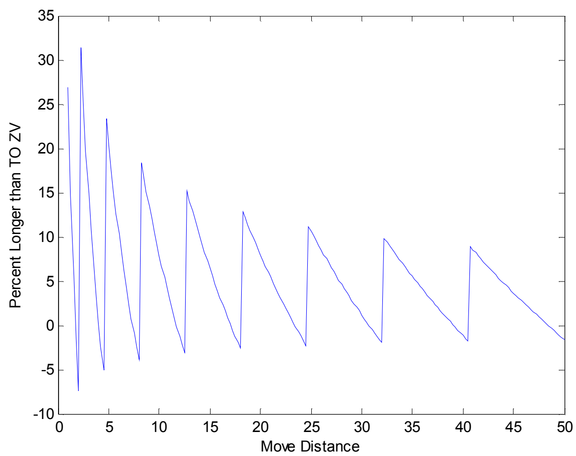

Fig. 4. Percent Difference of Move Durations, MSBB vs. TO-ZV.

To illustrate the relationship between the move distance and the command duration (move time), two move distances are considered, $x_d = 2.5$ and $x_d = 4.5$ for $q=3$ (i.e., command duration = 6 periods of vibration). These moves result in the longest and shortest commands, respectively, for $q=3$ when compared to the TO-ZV commands.

Figs. 5 and 6 show the actuator force versus time for the two move distances. The actuator force (f) is computed as the total mass times the command acceleration. Referring to Fig. 1, $f = (m_1 + m_2)a$. For the simulations, $m_1 = m_2 = 1kg$ so $f = 2a$.

Fig. 5 shows that for the smaller move distance ($x_d = 2.5$), the MSBB command is longer than the TO-ZV command and it uses less than the maximum allowed force. Since the MSBB uses as acceleration in excess of 0.25 m/s^2 or, equivalently, a force in excess of 1N, the number of switches cannot be increased without violating the actuator limit. Fig. 6 shows that for the larger move distance ($x_d = 4.5$), the MSBB command uses the maximum force and is slightly faster than the TO-ZV command. This trend holds for all the "steps" in the MSBB command durations shown in Fig. 3. The MSBB command is longest (as compared to the TO-ZV command) immediately following an increase necessitated by the amplitude constraint in (13) and is shortest when the amplitude limit is reached.

The design takes into account the actuator limit as a maximum allowable acceleration. An additional constraint may be the minimum switching interval. The actuator dynamics will limit how quickly the actuator can be switched from one extreme to another. Figs. 5 and 6 show that the TO-ZV commands require three very short switching intervals at the beginning, middle, and end of the maneuver. In contrast, the MSBB has a fixed switching interval that can be adjusted using the switching parameter σ.

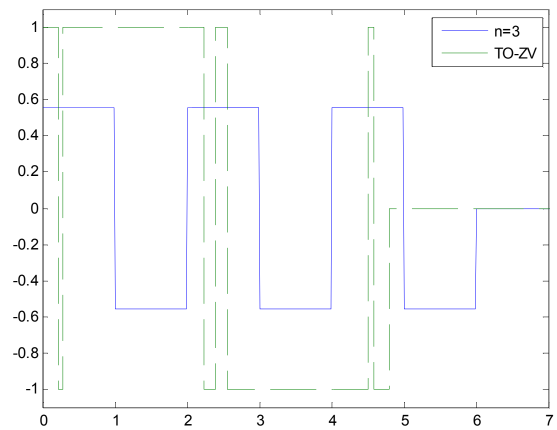

Fig. 5. MSBB and TO-ZV Commands – Move Distance = 2.5.

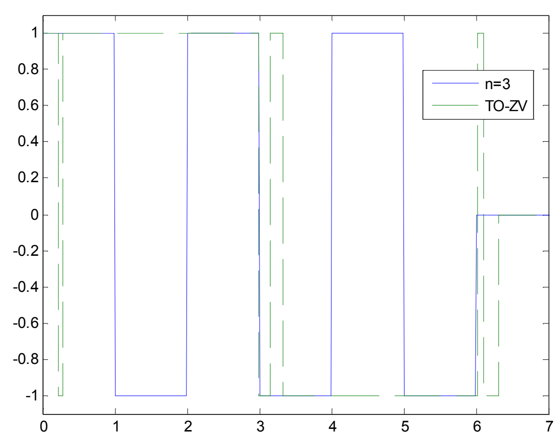

Fig. 6. MSBB and TO-ZV Commands – Move Distance = 4.5

C. *Comparison to TO-ZV robustness*

The advantage of the MSBB approach is its simplicity. However, the trade-off for this simple design is that the robustness is not incorporated explicitly in the design process. To evaluate the robustness of the MSBB command, the command is implemented when the actual frequency of vibration (determined by the spring constant) varies between 70% and 130% of the modeled value.

Fig. 7 shows the percent overshoot of the TO-ZV command for the range of move distances considered in Figures 3 & 4 as the actual frequency varies from the modeled frequency by +/- 30%. With the exception of moves less than 5 units, the TO-ZV command is extremely robust to modeling error. Fig 8 shows the same information for the MSBB command. Comparing Figs. 7 and 8, the MSBB delivers very similar robustness to the TO-ZV command.

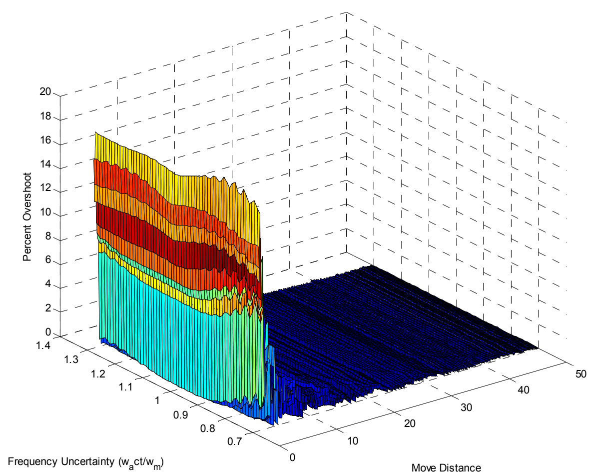

Fig. 7. Robustness Analysis of TO-ZV Commands

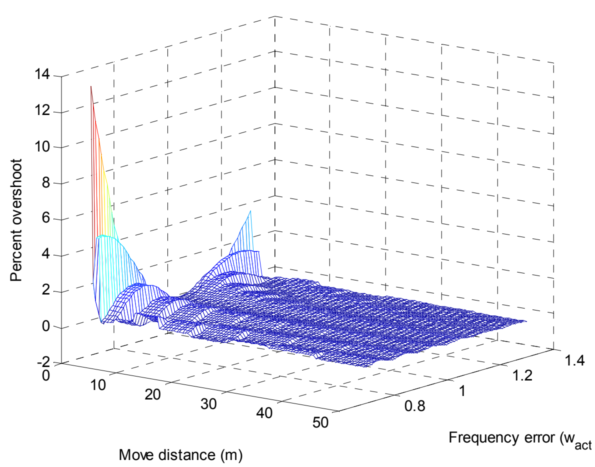

Fig. 8. Robustness Analysis of MSBB Commands

IV. Conclusion

A simple, multi-switch, bang-bang (MSBB) command was proposed. These commands have constant switching intervals and satisfy zero-vibration (at the end of the maneuver), maximum acceleration, and move distance constraints. The proposed commands were compared to the Time-Optimal, Zero-Vibration (TO-ZV) commands that produce a vibration-free move for a floating oscillator system. The MSBB command proved to be a valid alternative to the TO-ZV command due to the simplicity of the MSBB design procedure, the comparable move times and robustness, and the avoidance of very short switching intervals.

In future work, the effect of actuator dynamics will be examined. In particular, the relationship between the minimum switching interval and the time constant of the actuator will be investigated.

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