Adaptive Backstepping-based Synchronization of Uncertain Networked Lagrangian Systems

Wenlin Zhang, Zheng Wang, and Yi Guo

Abstract— In this paper, we study the synchronization problem of networked uncertain Lagrangian systems on directed communication topologies. For the nominal model, we propose a backstepping-based synchronization design for heterogenous Lagrangian systems on digraphs with a spanning tree. We relax the feedback gain constraints on the distributed synchronization control law which encompasses the existing double integrator consensus design as a special case. For the uncertain model, we develop a distributed adaptive redesign assuming that the uncertainty in each agent dynamics can be expressed as a linear parametrization. Simulation results show the effectiveness of the proposed method.

I. INTRODUCTION

In multi-agent cooperative control, distributed synchronization usually refers to steering a specific variable of group members to a common value across the network using local information. In the passed few years, numerous papers have been published on the distributed synchronization problem. For the directed communication topologies, system matrix analysis is very useful for the linear consensus models [1][2], but it only offers analytic conditions for the consensus rather than the design procedure. Originated from the nonlinear control framework [3], constructive approaches are very powerful in the synchronization control design, e.g., passivity-based design [4] [5], backstepping-based design [6], and contraction analysis-based design [7]. The constructive approaches generally rely on the symmetry of the communication topologies and cannot apply to the directed communication topologies with a spanning tree.

For robotic systems, the Lagrangian model represents a typical class of robotic systems, such as ground vehicles, aircrafts, and robot arms. Recently, the synchronization design of Lagrangian systems increasingly attracts research attentions. In [8], the leaderless synchronization design of Lagrangian systems is considered on undirected communication topologies. The authors of [9] consider synchronization of Lagrangian systems with non-holonomic constraints. In [10], the passivity-based synchronization of [5] has been extended from balanced communication topologies to strongly connected topologies. The authors of [11] consider coordinated tracking of Lagrange systems. All of the works mentioned above cannot apply to directed communication topologeis containing a spanning tree, due to the intrinsic symmetry requirement of the nonlinear control design tools.

In this paper, we study the synchronization design for the networked heterogenous Lagrangian systems on general directed communication topologies. Taking advantage of the spectral properties of the Laplacian matrix, we construct a Lyapunov function for the single integrator consensus on directed graphs. Using backstepping in a distributed fashion, we solve the synchronization design for the nominal Lagrangian system model on directed graphs with a spanning tree. The backstepping-based design provides us a control Lyapunov function for the synchronization analysis, which makes the distributed robust and adaptive redesign implementable. To handle the model undertaies, we extend the adaptive control design [12] into the distributed redesign with respect to parametric model uncertainties.

The contribution of the proposed design has two folds. First, our backstepping-based framework overcomes the symmetry loss of directed communication in nonlinear system synchronization, which is challenging for most constructive control designs. The deducted control law for Lagrangian systems encompasses the double integrator consensus problem as a special case. Second, the flexibility of backstepping technique relaxes the lower bound gain condition imposed on the distributed control law [1]. In our method, each agent's control law is independent of the graph Lapalcian matrix, which is more realistic in cooperative control scenarios.

The subsequent sections are organized as follows. Section II introduces the related graph theory and the spectral properties of the graph Laplacian matrix. In section III, we formulate the networked Lagrangian system synchronization and the distributed adaptive redesign problem. Section IV describes how to design the synchronization control law by the backstepping technique. In section V, we develop the distributed adaptive redesign technique. Section VI shows the simulation results. In section VII, we present the conclusion and future works.

II. PRELIMINARIES ON GRAPH THEORY

Given an index set $\mathcal{I} = \{1, 2, ..., n\}$, a digraph (directed graph) $\mathcal{G}$ consists of a triple $(\mathcal{V}, \mathcal{E}, \mathcal{A})$. $\mathcal{V} = \{v_i | i \in \mathcal{I}\}$ is a finite nonempty set of nodes. The edge set $\mathcal{E} = \{e_{ij} = (v_i, v_j) | i, j \in \mathcal{I}\}$. We refer to v_i and v_j as the tail and head of the edge (v_i, v_j). The weighted adjacency matrix $\mathcal{A} = \{a_{ij} | a_{ij} \neq 0 \Leftrightarrow e_{ij} \in \mathcal{E}, a_{ij} = 0 \Leftrightarrow e_{ij} \notin \mathcal{E}\}$. For simplicity, we assume $a_{ii} = 0$ and $a_{ij} \geq 0, i \neq j$. The set of neighbors of node i is denoted by $N_i = \{j : e_{ij} \in \mathcal{E}\}$.

The graph *Laplacian* associated with the graph $\mathcal{G}$ is defined as $\mathcal{L}(\mathcal{G}) = L = \Delta - \mathcal{A}$. The diagonal matrix $\Delta = [\Delta_{ij}]$ where $\Delta_{ij} = 0$ for all $i \neq j$ and $\Delta_{ii} = \sum_{j=1}^n a_{ij}$. The Laplacian matrix always has a zero eigenvalue with the

W. Zhang, Z. Zheng and Y. Guo are with the Electrical & Computer Engineering Dept., Stevens Institute of Technology, Hoboken, NJ 07030, USA. {wzhang5,Zheng.Wang,yguo1}@stevens.edu

right eigenvector of one. We denote as $\lambda_1 = 0, w_r = \mathbf{1} = [1, 1, \ldots, 1]^T$.

A digraph has a spanning tree if there exist at least one node that all the other node could reach it following the edge directions. If a digraph has a spanning tree, then its graph Laplacian matrix has a simple zero eigenvalue associated with an eigenvector $\mathbf{1}$, $\text{rank}(L) = n - 1$ and all of the other eigenvalues have positive real parts [13]. We denote as $\text{Re}(\lambda_k) > 0, k = 2, \ldots, n$.

Lemma 1: If a digraph $\mathcal{G}$ has a spanning tree and the associated graph Laplacian is L, then there exists a symmetric positive definite matrix P satisfying the equation

$$PL + L^T P = Q \geq 0, \quad (1)$$

where Q is a positive semidefinite matrix.

Proof: Let J be the Jordan form of L, i.e., $L = T^{-1}JT$, where T is a full rank matrix associated with the Jordan transformation. If the digraph has a spanning tree, we can choose a suitable T that $J = \text{diag}\{0, J_1\}$, where $-J_1$ is a $(n-1) \times (n-1)$ Hurwitz matrix [13]. So, there exists a symmetric positive definite matrix P_1 satisfying $P_1 J_1 + J_1^T P_1 = Q_1 > 0$ (Theorem 4.6 in [14]). Then, we choose ([15] Chapter 4, Theorem 4.29)

$$P = T^T \text{diag}\{1, P_1\}T, \quad (2)$$

such that

$$PL + L^T P = T^T \begin{bmatrix} 0 & 0 \\ 0 & P_1 J_1 + J_1^T P_1 \end{bmatrix} T$$
$$= T^T \begin{bmatrix} 0 & 0 \\ 0 & Q_1 \end{bmatrix} T = Q \geq 0. \quad (3)$$

■

III. PROBLEM FORMULATION

A. Nominal Model

A group of agents with Lagrangian dynamics are modeled as [16]

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = u_i, \quad i \in \mathcal{I}, \quad (4)$$

where $q_i \in \mathbb{R}^m$ is the vector of the ith agent's generalized configuration coordinates, $u_i \in \mathbb{R}^m$ is the vector of generalized forces acting on the system, $M_i(q_i)$ is the $m \times m$ symmetric, bounded positive definite inertia matrix, i.e., $\exists \alpha, \beta > 0$ such that $\forall i \in \mathcal{I}$

$$\alpha I \leq M_i(q_i) \leq \beta I, \text{and } \frac{1}{\beta}I \leq M_i^{-1}(q_i) \leq \frac{1}{\alpha}I, \quad (5)$$

$C_i(q_i, \dot{q}_i)\dot{q}_i$ is the symmetric matrix of Coriolis and centripetal torques which satisfy

$$\| C_i(q_i, \dot{q}_i) \| \leq c_{max} \| \dot{q}_i \|, \quad (6)$$

for some $c_{max} > 0, \forall i \in \mathcal{I}$, and $g_i(q_i)$ is the vector of gravitational torques. $\dot{M}_i - 2C_i$ is skew symmetric [16], which means

$$\dot{q}_i^T(\dot{M}_i - 2C_i)\dot{q}_i = 0, \quad \forall \dot{q}_i \in \mathbb{R}^m. \quad (7)$$

Assumption 1: The group communication topology is a static digraph which has a spanning tree.

Assumption 2: The group reference velocity v^d and $\dot{v}^d$ are accessible to every agent, where v^d is a bounded differentiable function mapping from $\mathbb{R}$ to $\mathbb{R}^m$.

Problem 1: Under the Assumptions 1 and 2, design a distributed control law $u_i(v^d, \dot{v}^d, q_i, \dot{q}_i, q_j, \dot{q}_j), j \in N_i, \forall i \in \mathcal{I}$ for system (4) so that as $t \to \infty$, $q_i \to q_j, \dot{q}_i \to v^d, \forall i, j \in \mathcal{I}$.

B. Uncertain Model

Due to parameter perturbation or model reduction, model uncertainties usually exists for the Lagrangian systems. From [16][17], the parameter uncertainties of Lagrangian model can be expressed as a linear parametrization, shown as,

$$M_i\ddot{q}_i + C_i\dot{q}_i + g_i + \phi_i(q_i, \dot{q}_i, v^d, \dot{v}^d)\theta_i = u_i, \quad (8)$$

where $i \in \mathcal{I}, \phi_i(q_i, \dot{q}_i, v^d, \dot{v}^d)$ is a known matrix of piecewise continuous function, $\| \phi_i(q_i, \dot{q}_i, v^d, \dot{v}^d) \|$ is bounded, and θ_i is a vector of unknown constants.

Every agent estimates the unknown θ_i by $\hat{\theta}_i$ and constructs an updating law $\dot{\hat{\theta}}_i = \Gamma_i \tau_i(v^d, \dot{v}^d, q_i, \dot{q}_i, q_j, \dot{q}_j, \hat{\theta}_i), j \in N_i$, where the adaptation gain matrix Γ_i is positive definite. Our goal is to design a distributed adaptive control $u_i(v^d, \dot{v}^d, q_i, \dot{q}_i, q_j, \dot{q}_j, \hat{\theta}_i), j \in N_i$ with the updating scheme τ_i to synchronize the uncertain systems (8).

Problem 2: Under Assumptions 1 and 2, assume $\hat{u}_i(v^d, \dot{v}^d, q_i, \dot{q}_i, q_j, \dot{q}_j), j \in N_i$ is the distributed control law for the nominal systems (4) to synchronize, design an additional distributed control $\tilde{u}_i(v^d, \dot{v}^d, q_i, \dot{q}_i, q_j, \dot{q}_j, \hat{\theta}_i), j \in N_i$ and the update scheme $\tau_i(v^d, \dot{v}^d, q_i, \dot{q}_i, q_j, \dot{q}_j, \hat{\theta}_i), j \in N_i$, such that the control $u_i = \hat{u}_i + \tilde{u}_i$ ensures for the uncertain system (8) to achieve $q_i \to q_j$, and $\dot{q}_i \to v^d, \forall i, j \in \mathcal{I}$, as $t \to \infty$.

IV. BACKSTEPPING DESIGN FOR THE LAGRANGIAN SYSTEM SYNCHRONIZATION

In this section, we develop the backstepping-based synchronization design for the nominal Lagrangian systems. We first present the results in Theorem 1, then we illustrate how to deduct the control law by backstepping.

A. Main Theorem

Theorem 1: Under Assumptions 1 and 2, Problem 1 is solved with the distributed control law

$$u_i = M_i\dot{v}^d + C_i v^d + g_i + k_1(v^d - \dot{q}_i)$$
$$-k_0 M_i \sum_{j \in N_i} a_{ij}(\dot{q}_i - \dot{q}_j)$$
$$-k_0(C_i + k_1) \sum_{j \in N_i} a_{ij}(q_i - q_j), \quad (9)$$

where $i \in \mathcal{I}, k_0, k_1 > 0$. That is, for system (4), as $t \to \infty$, $q_i \to q_j, \dot{q}_i \to v^d, \forall i, j \in \mathcal{I}$.

The proof is given in Section IV-C.

Remark 1: Theorem 1 encompasses as a special case the exsiting popular double integrator consensus design in the literature [1] [18], when $M_i = I, C_i = 0$. All these existing

consensus algorithms have feedback gain conditions on each agent's control law when the communication topologies are directed graphs with a spanning tree. These feedback gain conditions depends on the graph Laplacian matrices, which means each agent has to know the knowledge of the group. In contrast, (9) is truly distributed since the gains k_0 and k_1 only need to be positive. As long as the communication topology has a spanning tree, the control law of each agent does not depend on any other group information.

Remark 2: When the relative velocity measurement is unavailable for the control law design, we can remove the term $k_0 M_i \sum_{j \in N_i} a_{ij}(\dot{q}_i - \dot{q}_j)$ in (9) through increasing the feedback gain k_1, following the same procedure of Theorem 2 in [19].

B. Backstepping-based Control Design

For convenience, let $p_i = \dot{q}_i$ and we rearrange the coordinates of the model (4) in a compact form as

$$
\begin{aligned}
&q = [q_1^T, \ldots, q_n^T]^T, \ p = [p_1^T, \ldots, p_n^T]^T, \\
&M = \mathrm{diag}\{M_1(q_1), \ldots, M_n(q_n)\}, \\
&C = \mathrm{diag}\{C_1(q_1, \dot{q}_1), \ldots, C_n(q_n, \dot{q}_n)\}, \\
&g = [g_1(q_1), \ldots, g_n(q_n)]^T, \ u = [u_1^T, \ldots, u_n^T]^T,
\end{aligned}
$$

then we obtain

$$\dot{q} = p \tag{10a}$$
$$\dot{p} = -M^{-1}Cp - M^{-1}g + M^{-1}u. \tag{10b}$$

We also denote the group reference as

$$v = [v^{dT}, \ldots, v^{dT}]_{1 \times nm}^T. \tag{11}$$

In the following, we design the synchronization control law using backstepping for the nominal system (10).

1) Step 1: For the first subsystem (10a), we take p^* as a virtual control input,

$$\dot{q} = p^*. \tag{12}$$

Let

$$p^* = -k_0(L \otimes I_m)q + v. \tag{13}$$

where $k_0 > 0$ is the feedback gain, and v is defined in (11), we have the Lyapunov function as

$$V_0 = \frac{1}{2}q^T\left((L^T P L) \otimes I_m\right)q, \tag{14}$$

where P is defined in (2). Taking derivative of V_0 along with the trajectories of $\dot{q} = -k_0(L \otimes I_m)q + v$ gives

$$\dot{V}_0 = q^T\left((L^T P L) \otimes I_m\right)(-k_0(L \otimes I_m)q + v). \tag{15}$$

Apparently, $(L \otimes I_m)v = 0$, since v is a vector with identical elements. We have

$$\dot{V}_0 = -\frac{k_0}{2}q^T\left((L^T Q L) \otimes I_m\right)q \leq 0, \tag{16}$$

where $Q = PL + L^T P$ is a positive semidefinite matrix defined in (3). From Lemma 1, matrix Q has an eigenvector $\mathbf{1}$ associated with the simple eigenvalue 0, and all the other

eigenvalues are positive. It means Q and L have the same Null space. From the invariance properties of eigenspace [20], $(L \otimes I_m)q \in \mathrm{Null}\{Q \otimes I_m\} \Rightarrow q \in \mathrm{Null}\{L \otimes I_m\} \Rightarrow (L \otimes I_m)q = 0$. Therefore, we can rewrite the derivative (16) as

$$\dot{V}_0 \leq -\frac{k_0 \lambda_2(Q)}{2} \parallel \tilde{q} \parallel_2^2 . \tag{17}$$

where $\tilde{q} = (L \otimes I_m)q$, $\lambda_2(Q)$ denotes the smallest positive eigenvalue of Q. From (14), we also have

$$\frac{\lambda_{min}(P)}{2} \parallel \tilde{q} \parallel_2^2 \leq V_0 \leq \frac{\lambda_{max}(P)}{2} \parallel \tilde{q} \parallel_2^2 \tag{18}$$

where $\lambda_{min}(P)$ and $\lambda_{max}(P)$ denote the smallest and largest positive eigenvalue of matrix P, respectively. According to Theorem 4.10 in [14], $\tilde{q} = 0$ is exponentially stable, which means $q_i \to q_j$ exponentially, as $t \to \infty, \forall i, j \in \mathcal{I}$. From (12)(13), $\dot{q}_i \to v^d$ as $\tilde{q} \to 0, \forall i \in \mathcal{I}$.

Remark 3: The transformation (1) manipulates the spectral structure of L so that Q has a zero eigenvalue associated with eigenvector $\mathbf{1}$, and all the other eigenvalues are positive. That is, matrix Q has a similar spectral as an undirected graph Laplcacian matrix. This property enables the construction of the Lyapunov function (14).

2) Step 2: Following the backstepping procedure, we define the error signal

$$z = p - p^*, \tag{19}$$

and we want to regulate z to zero, so that when q_i synchronizes, p_i will achieve the group reference v^d.

Differentiating both sides of (19) yields

$$\dot{z} = \dot{p} - \dot{p}^* = -M^{-1}C(z + p^*) + M^{-1}u - M^{-1}g - \dot{p}^*$$

Then, we consider the system,

$$\dot{q} = z + p^* \tag{20a}$$
$$\dot{z} = -M^{-1}C(z + p^*) + M^{-1}u - M^{-1}g - \dot{p}^*, \tag{20b}$$

with the Lyapunov function

$$V = V_0 + \frac{k}{2}z^T M z, \tag{21}$$

where $k > 0$ is a free parameter to be chosen. Taking derivative of (21) with respect to the trajectories of (20) gives

$$
\begin{aligned}
\dot{V} &= q^T((L^T P L) \otimes I_m)\dot{q} + kz^T M\dot{z} + \frac{k}{2}z^T \dot{M}z \\
&= q^T((L^T P L) \otimes I_m)z + q^T((L^T P L) \otimes I_m)p^* \\
&\quad + \frac{k}{2}z^T(\dot{M} - 2C)z \\
&\quad + kz^T M(-M^{-1}Cp^* - M^{-1}g + M^{-1}u - \dot{p}^*).
\end{aligned}
$$

Since M and C are block diagonal matrices, we have

$$z^T(\dot{M} - 2C)z = \sum_{i=1}^{n} z_i^T(\dot{M}_i - 2C_i)z_i = 0. \tag{22}$$

The second equation in (22) is from (7). Then, it follows that

$$
\begin{aligned}
\dot{V} &= q^T((L^T P L) \otimes I_m)z + q^T((L^T P L) \otimes I_m)p^* \\
&\quad + kz^T(-Cp^* - g + u - M\dot{p}^*).
\end{aligned} \tag{23}
$$

In (23), let

$$u = Cp^* + g + M\dot{p}^* - k_1 z, \quad k_1 > 0, \qquad (24)$$

we have

$$\dot{V} \leq -\frac{k_0}{2}\lambda_2(Q) \parallel (L \otimes I_m)q \parallel_2^2$$
$$+q^T((L^T PL) \otimes I_m)z - kk_1 z^T z$$
$$\leq -\frac{k_0}{2}\lambda_2(Q) \parallel (L \otimes I_m)q \parallel_2^2 -kk_1 z^T z$$
$$+\frac{1}{2\epsilon_1}q^T\left((L^T Q_1 L) \otimes I_m\right)q + \frac{\epsilon_1}{2}z^T z, \qquad \epsilon_1 > 0,$$

where $Q_1 = PLL^T P$. Since Q_1 is positive semidefinite, rank $n-1$ and has $\mathbf{1}$ as an eigenvector with the simple eigenvalue 0, we obtain,

$$\dot{V} \leq -\frac{\epsilon_1 k_0 \lambda_2(Q) - \lambda_n(Q_1)}{2\epsilon_1} \parallel (L \otimes I_m)q \parallel_2^2$$
$$-\frac{2kk_1 - \epsilon_1}{2}z^T z \qquad (25)$$

where $\lambda_n(Q_1)$ denotes the largest eigenvalue of Q_1. Choosing suitable k_0, k, k_1 and ϵ_1 that

$$\epsilon_1 > \frac{\lambda_n(Q_1)}{\lambda_2(Q)k_0} \text{ and } k_1 > \frac{\epsilon_1}{2k} \qquad (26)$$

yields $\dot{V} \leq 0$. This finishes the backstepping design procedure.

C. Proof of Theorem 1

With the control law (24), we have the derivative of the Lyapunov function (21) negative semidefinite with respect to the trajectories of system (10), by choosing parameters as (26). Denote

$$x = [((L \otimes I_m)q)^T, z^T]^T, \qquad (27)$$

where z is defined in (19). Apparently, $\parallel x \parallel_2 = 0$ means $q_i = q_j, \dot{q}_i = v^d, \forall i, j \in \mathcal{I}$. From (21), we have

$$V \geq \min\{\frac{1}{2}\lambda_{min}(P), \frac{k\alpha}{2}\} \parallel x \parallel_2^2$$
$$V \leq \max\{\frac{1}{2}\lambda_{max}(P), \frac{k\beta}{2}\} \parallel x \parallel_2^2$$

where P is defined in (2), and α, β are defined in (5) as the lower and upper bound for the norm of the inertia matrix $M_i(q_i)$, respectively. $\lambda_{min}(P)$ and $\lambda_{max}(P)$ represent the smallest and largest eigenvalue of P, respectively. Meanwhile, from (25), we have

$$\dot{V} \leq -k_3 \parallel (L \otimes I_m)q \parallel_2^2 -k_4 z^T z$$
$$\leq -\min\{k_3, k_4\} \parallel x \parallel_2^2$$

for some $k_3, k_4 > 0$. According to Theorem 4.10 in [14], the nominal system (20) is exponentially stable at $x = 0$. Denote $a_1 = \min\{\frac{1}{2}\lambda_{min}(P), \frac{k\alpha}{2}\}, a_2 = \max\{\frac{1}{2}\lambda_{max}(P), \frac{k\beta}{2}\}$ and $a_3 = \min\{k_3, k_4\}$, the convergence rate is estimated as

$$\parallel x \parallel_2 \leq \left(\frac{a_2}{a_1}\right)^{\frac{1}{2}} \parallel x_0 \parallel_2 e^{-\frac{a_3}{2a_2}t}. \qquad (28)$$

In distributed form, (24) is written as (9). Particularly, k_0, k_1 only need to be positive in (9), since for any $k_0, k_1 > 0$, we can always find a large enough k to satisfy (26).

This finishes the proof of Theorem 1.

Remark 4: From the estimation (28), a_1, a_2 depend on the system properties such as graph Laplacian structure and the inertial matrix. To increase the convergence rate, we have to increase simultaneously the two parameters k_0, k_1 in the control law, because it is the minimum of k_3 and k_4 in a_3 that decides the convergence rate.

V. DISTRIBUTED ADAPTIVE REDESIGN TO ACCOUNT FOR MODEL UNCERTAINTIES

Backstepping design procedure provides a control Lyapunov function which leads to the adaptive redesign [12] to account for model uncertainties.

Theorem 2: Under Assumptions 1 and 2, Problem 2 is solved under the control law

$$u_i = \hat{u}_i - \phi_i \hat{\theta}_i, \qquad (29a)$$
$$\dot{\hat{\theta}}_i = \Gamma_i \phi_i^T z_i, \qquad (29b)$$
$$z_i = \dot{q}_i - v^d + \sum_{j \in N_i}(q_i - q_j), \ \forall i \in \mathcal{I},$$

where $\hat{u}_i$ is the nominal control law (9), Γ_i is the adaptive gain matrix. That is, for system (8), as $t \to \infty$, $q_i \to q_j, \dot{q}_i \to v^d, \forall i, j \in \mathcal{I}$.

Proof: For convenience, we rearrange the coordinates of (8) in the compact form the same as the one in the backstepping design in Section IV-B, we obtain

$$\dot{q} = p \qquad (30a)$$
$$\dot{p} = -M^{-1}Cp - M^{-1}g + M^{-1}(u + \Phi\theta) \qquad (30b)$$

where $\Phi = diag\{\phi_1, \ldots, \phi_n\}$ and $\theta = [\theta_1^T, \ldots, \theta_n^T]^T$. Then, following the backstepping procedure, we have

$$\dot{q} = z + p^* \qquad (31a)$$
$$\dot{z} = -M^{-1}C(z + p^*) - M^{-1}g - \dot{p}^* + M^{-1}(u + \Phi\theta) \qquad (31b)$$

where $z = p - p^*$, p^* is defined in (13). For system (31), we consider the Lyapunov function

$$V_a = V_0 + \frac{k}{2}z^T M z + \frac{k}{2}\tilde{\theta}^T \Gamma^{-1}\tilde{\theta} \qquad (32)$$

where $\Gamma = diag\{\Gamma_1, \ldots, \Gamma_n\}$ is the adaptation gain, and $\tilde{\theta} = \theta - \hat{\theta}$ is the estimation error. Calculate derivative of V_a with respect to the trajectories of system (31), take $u = \hat{u} + \tilde{u}$, we have

$$\dot{V}_a = q^T((L^T PL) \otimes I_m)z + q^T((L^T PL) \otimes I_m)p^*$$
$$+kz^T(-Cp^* - g + \hat{u} - M\dot{p}^*)$$
$$+kz^T(\tilde{u} + \Phi\theta) + k\tilde{\theta}^T \Gamma^{-1}\dot{\tilde{\theta}}$$

If choosing $\hat{u}$ as the nominal control law (9), and $\tilde{u} = -\Phi\hat{\theta}$, we have

$$\dot{V}_a \leq -k_3 \parallel (L \otimes I_m)q \parallel_2^2 -k_4 z^T z + kz^T \Phi\tilde{\theta} + k\tilde{\theta}^T \Gamma^{-1}\dot{\tilde{\theta}}.$$

for some positive k_3, k_4. With $\dot{\tilde{\theta}} = -\Gamma\Phi^T z$, we obtain

$$\dot{V}_a \leq -k_3 \parallel (L \otimes I_m)q \parallel_2^2 -k_4 z^T z \leq 0.$$

Since v^d and $\parallel \phi_i \parallel$ are bounded, the trajectories $(L \otimes I_m)q, z, \tilde{\theta}$ are all bounded. According to the Invariance-like Theorem (Theorem 8.4 in [14]), we can conclude $\parallel (L \otimes I_m)q \parallel_2^2 \to 0$ and $z^T z \to 0$ as $t \to \infty$, which is equivalent to $q_i \to q_j, \dot{q}_i \to v^d$, as $t \to \infty, \forall i, j \in \mathcal{I}$. ∎

Remark 5: When the model uncertainties cannot be expressed as a linear parametrization, we can apply the distributed Lyapunov redesign [19] for the robustification, but Lyapunov redesign only ensures the bounded convergence performance.

Remark 6: To improve the transient performance, we can incorporate the adaptive redesign and nonlinear damping together [12], as

$$\tilde{u}_i = -\phi_i \hat{\theta}_i - \gamma_i z_i \parallel \phi_i \parallel_2^2, \qquad (33)$$

Take derivative of V_a in (32) with respect to the trajectories of system (31), and choose $u_i = \hat{u}_i + \tilde{u}_i$, $\hat{u}$ as the nominal control law (9), and $\tilde{u}_i$ as the redesign (33), we have

$$\dot{V}_a \leq -k_3 \parallel (L \otimes I_m)q \parallel_2^2$$
$$-k_4 z^T z - k \sum_{i=1}^{n} \gamma_i \parallel z_i \parallel^2 \parallel \phi_i \parallel^2$$
$$+k \sum_{i=1}^{n} z_i^T \phi_i \tilde{\theta}_i + k \sum_{i=1}^{n} \tilde{\theta}_i^T \Gamma_i^{-1} \dot{\tilde{\theta}}_i,$$

for some $k_3, k_4 > 0$. Still choose $\dot{\hat{\theta}}_i = -\Gamma_i \phi_i^T z_i$, we have

$$\dot{V}_a \leq -k_3 \parallel (L \otimes I_m)q \parallel_2^2 -k_4 z^T z$$
$$-k \sum_{i=1}^{n} \gamma_i \parallel z_i \parallel^2 \parallel \phi_i \parallel^2 \leq 0.$$

Following the same procedure as the proof of Theorem 2, we can prove the convergence of synchronization. The damping term $-\gamma_i z_i \parallel \phi_i \parallel_2^2$ will fortify the adaptive process and smoothen the transient trajectories. We will show this point in the simulation.

VI. SIMULATIONS

As shown in Fig. 1, assume the group has four two-link manipulators and the communication topology is a digraph. For convenience, we set the link weights $a_{ij} = 1, j \in N_i$, $i, j \in \{1, 2, 3, 4\}$.

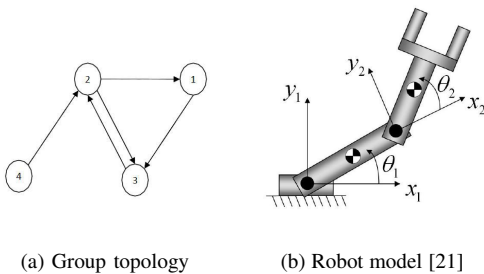(a) Group topology   (b) Robot model [21]

Fig. 1.   Group of two-link robots

As shown in Fig. 1(b), the two-link manipulator model [21] has two degrees-of-freedom, the generalized coordinates

$q = [\theta_1, \theta_2]^T$. The detailed modeling process is discussed in [16]. We choose the parameters as the Table 5.1 in page 129 of [21] and we set the reference velocity $v^d = [0 \ 2\sin(\pi t)]^T$, 0 for Angle 1st and $2\sin(\pi t)$ for Angle 2nd. The nominal control law is chosen as (9) with $k_0 = k_1 = 5$. The synchronization performance is shown in Fig. 2.
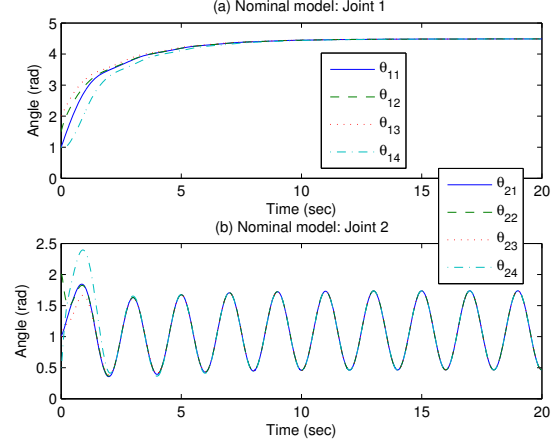

Fig. 2.   Joint angle trajectories: Synchronization of the nominal model. The group reference $v^d = [0 \ 2\sin(\pi t)]^T$.

In the uncertain model, we perturbed 10% of the model parameters, Fig. 3 shows the synchronization performance is sensitive to the model uncertainties: synchronization cannot be achieved.
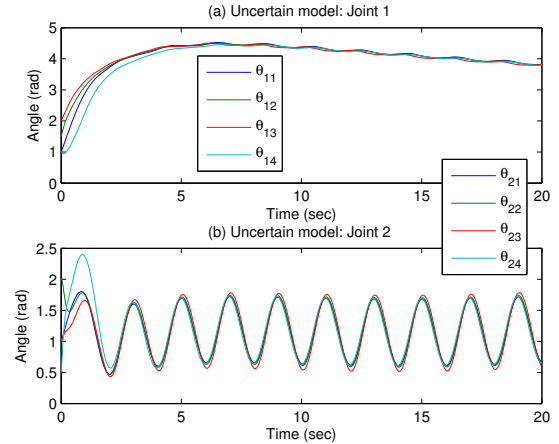

Fig. 3.   Joint angle trajectories: Synchronization of the uncertain model. The group reference $v^d = [0 \ 2\sin(\pi t)]^T$.

To represent the model uncertainties in the linear parametric form, we choose the method presented in Page 1578 of [16]. After the adaptive redesign (29) is adopted, the angle trajectories are shown in Fig. 4. In the updating law (29b), we choose the gain $\Gamma_i = 50$. We observe from Fig. 4 that the synchronization is achieved after the parameter adaptation, but the transient adjustment trajectories are quite erratic due to the adaptive process.
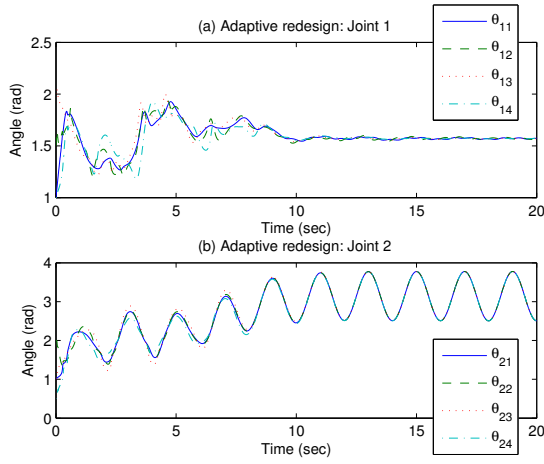
Fig. 4. Joint angle trajectories: Synchronization of the uncertain model using adaptive redesign. The group reference $v^d = [\ 0\quad 2\sin(\pi t)\ ]^T$.

To improve the transient performance, we add the nonlinear damping as (33) with $\gamma_i = 20$. Fig. 5 shows that transient trajectories become smoother during the parameter adaptation.
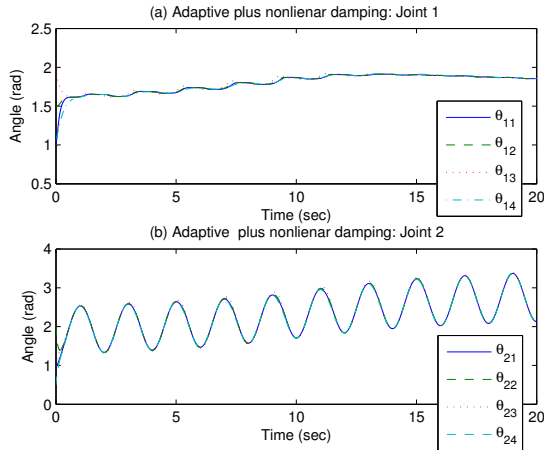

Fig. 5. Joint angle trajectories: Synchronization of the uncertain model using adaptive redesign plus nonlinear damping. The group reference $v^d = [\ 0\quad 2\sin(\pi t)\ ]^T$.

VII. CONCLUSIONS AND FUTURE WORKS

In this paper, we develop a distributed backstepping-based synchronization design for uncertain networked Lagrangian systems on directed communication topologies. To account for model uncertainties, we apply adaptive control in a distributed fashion. Backstepping-based design overcomes the symmetry loss of directed graph structure and relax the feedback gain conditions of the synchronization control. Future work will extend to switching topology and sampled data scenario.

REFERENCES

[1] W. Ren and E. Atkins, "Distributed multi-vehicle coordinated control *via* local information exchange," *Int. J. Robust Nonlinear Control*, vol. 17, pp. 1002–1033, 2007.

[2] Z. Qu, J. Wang, and R. Hull, "Cooperative control of dynamical systems with application to autonomous vehicles," *IEEE Trans. Autom. Control*, vol. 53, pp. 894–911, May 2008.

[3] P. Kokotović and M. Arcak, "Constructive nonlinear control: a historical perspective," *Automatica*, vol. 37, no. 5, pp. 637 – 662, 2001.

[4] M. Arcak, "Passivity as a design tool for group coordination," *IEEE Trans. Autom. Control*, vol. 52, pp. 1380–1390, Aug. 2007.

[5] N. Chopra and M. Spong, "Passivity-based control of multi-agent systems," *In Advances in Robot Control: From Everyday Physics to Human-Like Movements, Springer-Verlag*, pp. 107–134, 2006.

[6] R. Mellish, S. Napora, and D. A. Paley, "Backstepping control design for motion coordination of self-propelled vehicles in a flowfield," *Int. J. Robust. Nonlinear Control*, vol. 00, pp. 1–13, 2010.

[7] S.-J. Chung and J.-J. Slotine, "Cooperative robot control and concurrent synchronization of Lagrangian systems," *IEEE Transactions on Robotics*, vol. 25, pp. 686 –700, June 2009.

[8] W. Ren, "Distributed leaderless consensus algorithms for networked Euler-Lagrange systems," *International Journal of Control*, vol. 82, pp. 2137–2149, Nov. 2009.

[9] H. Yu and P. Antsaklis, "Passivity-based output synchronization of networked Euler-Lagrange systems subject to nonholonomic constraints," in *American Control Conference (ACC), 2010*, pp. 208 –213, july 2010.

[10] Y.-C. Liu and N. Chopra, "Synchronization of networked robotic systems on strongly connected graphs," *49th IEEE Conference on Decision and Control (CDC)*, pp. 3194 –3199, 2010.

[11] J. Mei, W. Ren, and G. Ma, "Distributed coordinated tracking with a dynamic leader for multiple Euler-Lagrange systems," *IEEE Transactions on Automatic Control*, 2011, to appear.

[12] M. Krstić, I. Kanellakopoulos, and P. Kokotović, *Nonlinear and Adaptive Control Design*. John Wiley & Sons, Inc., 1995.

[13] W. Ren and R. Beard, *Distributed Consensus in Multi-vehicle Cooperative Control Theory and Applications, Series: Communications and Control Engineering*. Springer-Verlag, London, 2008.

[14] H. K. Khalil, *Nonlinear Systems*. Prentice Hall, Upper Saddle River, New Jersey, 2002.

[15] Z. Qu, *Cooperative Control of Dynamical Systems, Application to Autonomous Vehicles*. Springer-Verlag London, 2009.

[16] R. Ortega and M. Spong, "Adaptive motion control of rigid robots: a tutorial," in *Proceedings of the 27th IEEE Conference on Decision and Control*, pp. 1575 –1584 vol.2, Dec. 1988.

[17] R. J. Luquette, *Nonlienar Control Design Techniques for Precision Formation Flying at Lagrange Points*. PhD thesis, University of Maryland, 2006.

[18] W. Ren, "On concensus algorithms for double-integrator dynamics," *IEEE Trans. Autom. Control*, vol. 53, pp. 1503–1509, Jul. 2008.

[19] W. Zhang, Z. Wang, and Y. Guo, "Robust consensus for uncertain multi-agent systems on directed communication topologies," *49th IEEE Conference on Decision and Control (CDC)*, pp. 6317 –6322, 2010.

[20] J. D. Meiss, *Differential Dynamical Systems*. SIAM Mathematical Modeling and Computation, 2007.

[21] S.-J. Chung, *Nonlinear Control and Synchronization of Multiple Lagrangian Systems with Application to Tethered Formation Flight Spacecraft*. PhD thesis, Department of Aeronautics and Astronautics, MIT, June 2007.